Understanding the World Housing Boom

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Abstract

This paper develops an open economy growth model with land and housing services to account for international evidence on the behavior of housing prices since 1970. An 18-country panel data set on housing prices and other aggregate variables indicates a "multiplier" effect in which housing price growth varies much more than one-for-one with overall economic growth, both across countries and within countries over time. The key insight is that a low elasticity of substitution between housing services and other consumption predicts precisely such strong price responses to changes in permanent income. To the extent such changes are highly persistent, housing prices can have a "bubbly" appearance in which prices rise faster than income for an extended period, then collapse and grow more slowly than the overall economy. The model also suggests that exogenous changes in world real interest rates have a modest impact on housing price growth, and do not explain. Finally, in contrast to the U.S. case, variation (cross-sectionally and over time) in hours of work appears to be more important than variation in productivity in accounting for variation in permanent income growth and hence in housing prices.

With the rapid growth of housing prices since the mid-1990's in most of the world's advanced economies, as well as the recent dramatic downturns, increased attention has been given to the causes and effects of fluctuations in housing prices and investment. Much of the focus has been on credit markets, and in particular the subprime mortgage market in the United States. Price appreciation in the U.S. was not, however, out of line with that of other advanced economies—in fact the inflation-adjusted housing price growth since 1997 was slightly below the average of 18 countries according to data from the Bank for International Settlements (BIS).[1] Nor was this the first time housing prices experienced such a sustained appreciation, as similar (albeit less sustained) episodes occurred in many countries in the 1970s and again in the 1980s.

Figure 1 depicts the behavior of inflation-adjusted housing prices since 1970 for those 18 countries. The key question for this paper is to what extent we can rationalize these movements both in the cross-section (e.g. over some given time period, the cross-sectional variation in prices) and time series (within a given country, or on average across many countries). Among other things, the chart shows the substantial upward movement in prices across 15 of the 18 countries since the mid-1990s (the exceptions being Germany, Japan, and Switzerland). We can also see that the U.S. is pretty much in the middle of the pack throughout the 35-year period covered by the dataset.

This paper highlights a key fact about medium frequency house price fluctuations, namely that their amplitude greatly exceeds the amplitude of fluctuations in overall economic growth—both across countries and over time. This fact is not an outgrowth of recent financial innovation, but appears to be relatively stable over time. The paper then explores the extent to which a relatively simple model, in particular one devoid of financial frictions or bubbles, can account for this stylized fact. The model is a stochastic growth model with two goods (housing services and non-housing consumption) and three inputs (capital, labor, and land). Two key features of the model are a non-unit cross elasticity of substitution between housing and other goods[2], and greater land intensity in the production of housing services. Both assumptions are justified empirically, and are essential in delivering the result that house prices indeed should move substantially more than one for one with growth. Exogenous changes in interest rates also affect house prices, but the impact is essentially one for one with overall economic growth.

[1] Those countries are: The U.S., Japan, the U.K., Germany, France, Italy, Canada, Spain, Australia, Netherlands, Belgium, Sweden, Switzerland, Denmark, Norway, Finland, New Zealand, and Ireland.

[2] This parameter has been featured in many studies related to housing (e.g. Li et al, 2008; Piazzesi and Schneider, 2007; Flavin-Nakagawa, 2004.). At the same time, many others have assumed, presumably for convenience or tractability, a value of one for this elasticity (e.g. Iacoviello-Neri, 2006, Kiyotaki et al, 2007). Kahn (2008) provides evidence based on both aggregate and microeconomic data that this elasticity is considerably less than one.

1 Background

Housing has grown as a share of expenditures and wealth in many developed economies. In the U.S., the real value of housing wealth, as measured by flow of funds data, has grown an average of 4.6 percent since 1952. This compares with 3.4 percent growth of private net worth excluding real estate, and 3.5 percent growth of personal consumption expenditures over the same time period. Figure 2 plots the ratio of nominal housing wealth to nominal consumption expenditure. This ratio has increase by more than 50 percent since 1952. Figure 3 plots the much more volatile ratio of housing wealth to total net worth. While the enormous volatility of non-real estate wealth (mostly the stock market) hinders precise inferences about relative trends, the upward drift of this ratio is apparent, and not just the result of the runup in real estate wealth over the last decade . The bottom line is that real estate has gone from 27 percent of net worth in 1952 to 39 percent by 2008.[3]In OECD data, the ratio of housing services in total expenditures

One possible explanation for the relative increase in housing prices is a simple income effect, or non-homogeneity in preferences. As people get wealthier, they may prefer to have more of their consumption coming from housing services, the price of which will tend to rise because of its being relatively intensive in land, a fixed factor. The (nominal) share of housing services in U.S. GDP has gone from 7.5 percent in 1952 to over 10 percent in 2005. The share of housing services in consumer expenditures has gone from 12.2 percent to 14.6 percent over the same period, but data going back to 1929 (Figure 4) show little long-term trend over a period in which wealth has grown enormously, but a positive association with the relative price of housing services (except in recent years). This paper will focus on productivity growth—in particular, productivity growth in the production of goods other than housing services. The relatively large share of land and structures, two inputs usually thought to be less amenable to technical progress, in the value of housing makes this story plausible. This paper will argue that the timing of low-frequency changes in both housing prices and productivity suggests that this mechanism is important.

In fact, in the model it will be the price of land (endogenously driven by changes in real growth and interest rates) that is the key driver of housing prices. There is some evidence that in fact the increase in housing wealth does not stem from an increase in the value of houses per se, but rather from the increase in the value of the land upon which they are built. First, a price index that include the value of land, the Conventional Mortgage Home Price Index, has increased approximately 0.75% faster than indexes that do not, such as the Census's Composite Construction Cost index, on an annual basis. Davis and Heathcote (2004) compute a land price index based on this type of differential and find that land values have increased

[3]Davis and Heathcote (2005), however, argue that there are problems with the Flow of Funds data, particularly over long periods of time, and construct their own measures of housing wealth (though only going back to 1975) that exhibit a less clearcut trend.

at an average annual rate of approximately 3.5% (inflation-adjusted) over the period 1975-2005.

Of course there should be no surprise that there is a qualitative connection between economic growth and housing prices. The real issue is quantitative: Can a plausibly parameterized growth model account quantitatively for the boom and bust of housing markets over the past 35-40 years, or is there much left over that requires market failures, financial frictions, or bubbles to explain? Figures 5abc provide some more detailed facts about the connection between home price appreciation and real growth. They show a fairly consistent pattern in which variation in house price growth appears to be roughly double the differences in real consumption growth rates, which can be viewed as a proxy for changes in permanent income. The pattern is robust across different time periods (Figures 5a and 5b), or comparing changes within countries across the two periods (Figure 5c), which removes country fixed effects. It is magnitudes of this sort—the tendency for housing prices to move much more than one-for-one with broader real measures of growth—that this paper seeks to understand. Although the data do not cover a long span of time, the robustness of this "multiplier" phenomenon suggests that it is not merely an outgrowth of credit market innovations or other relatively recent phenomena. Rather, it suggests a deeper explanation not tied to particular institutions or transitory developments.

The fact that changes within a country over time appear to produce somewhat larger responses of housing prices than the cross-section (the slope is 2.6 versus 2) turns out also to be consistent with the story developed in this paper. When a country's trend growth rate changes, housing prices jump before settling into a more stable growth trend. This "news" effect is not present if one just compares two countries with persistently different growth trends.

2 Related Literature

Research on aggregate housing prices has emphasized demographics, income trends, and government policy as fundamental drivers. In one well-known study, Mankiw and Weil (1989) argued that population demographics were the prime determinant, and predicted that prices would fall in the subsequent two decades with the maturation of the baby boom generation and resulting decline in the growth rate of the prime home-owning age group. While their prediction proved inaccurate, Martin (2005) renewed the argument for an important role for demographics. Glaeser et al (2005) argues that price increases since 1970 largely reflect artificial supply restrictions. Gyourko et al (2006) also cite inelastically supplied land as a key driver of the phenomenon they call "superstar cities." Van Nieuwerbergh and Weill (2006), however, argue that so long as there are regional markets in which such restrictions are not present, the aggregate impact of restrictions in some local markets is likely to be modest—in other words, they primarily affect the cross-sectional distribution of housing prices as opposed to the aggregate. Iacoviello and Neri (2006) examine the role of monetary policy with credit market frictions.

Consistent with the approach in this proposal, Attanasio et al (2005) find, using data from the U.K, that "common causality" drives the comovement of house prices and consumption, as opposed to wealth or the collateral channels. Also consistent with the approach adopted here, Kiyotaki et al (2007) find that credit market frictions primarily affect own vs. rent decisions as opposed to prices. Piskorski and Tchistyi (2008) examine optimal mortgage lending in a setting where housing prices obey essentially the same type of regime-switching behavior assumed here, and find that "many features of subprime lending observed in practice are consistent with economic efficiency and rationality of both borrowers and lenders," though, as they point out, there may be negative externalities associated with massive defaults in a downturn.

Case and Shiller (2003) and Himmelberg et al (2005) investigate the bubble hypothesis, looking across a large number of cities, and both suggest that the phenomenon is limited to a few localities. As with the research above on inelastic land supplies, these papers emphasize the cross-sectional variation of house prices across metropolitan areas rather than aggregate time series variation.

One important innovation in this project is to allow for unbalanced sectoral growth. General equilibrium models with production have generally either assumed Cobb-Douglas preferences (e.g. Davis and Heathcote, 2005, Kiyotaki et al., 2007, Iacoviello and Neri, 2006) or have abstracted from longer-term growth issues (e.g. Van Nieuwerburgh and Weill, 2007). This is the first housing model (to my knowledge) with production that features balanced aggregate growth and systematically varying sectoral shares due to non-unit elastic preferences away from the balanced growth path. The importance of this is that it is more consistent with aggregate growth facts as well as with the evidence on substitution elasticities found by numerous authors (see the discussion below), and also enables the model to match the volatility of housing prices in a plausible and disciplined way. The model framework is based on recent work of Ngai and Pissarides (2007).

3 A Growth Model with Housing

For the sake of exposition, this section presents the closed economy general equilibrium growth model developed in Kahn (2008). The model has two sectors, a "manufacturing" sector that uses land, labor, and capital to produce non-housing related goods and services, as well as capital—including the capital that goes into housing services. A second sector uses capital, labor, and land to produce a flow of housing services. The model exhibits balanced aggregate growth, but unequal growth across sectors. The extension to a small open economy setting is then straightforward.

After describing the production technology of firms and the preferences and budgets of consumers, the model is solved as the solution to a planner's problem. I show that the aggregate behavior is isomorphic to the one-sector growth model, which is then extended to a small open economy in a standard way. Implicitly the assumption is that capital flows across borders (albeit with the interest rate dependent on the magnitude of net borrowing so as to keep net foreign assets stationary), while labor does not. Markets are incomplete because there is no direct risk-sharing across borders.

Within each country, the problem of resource allocation across sectors is a relatively straight-forward static problem. This is obviously unrealistic for understanding short-run dynamics, but reasonable for the kinds of medium-frequency questions that are the focus of this investigation.

3.1 Firms and Consumers

Competitive final goods firms produce two types of goods: A "manufactured" good Y_m, and housing services Y_h. Under perfect competition the final goods firms make zero profits and have perfectly elastic supplies of Y_m and Y_h at the above prices. The production functions for the two types of goods are

$$Y_j = A_j K_j^{\alpha} L_j^{\beta_j} \left(eN_j\right)^{1-\alpha-\beta_j}$$

for $j = m, h$, where K_j is capital allocated to j, L_j is land, and eN_j is labor input, with e representing work effort (i.e. hours) per person and N population. The goods producers rent inputs in competitive markets. In particular, capital is rented from final goods producers of Y_m.In the j sector, the representative firm's nominal profit in period t is given by

$$P_{jt}Y_{jt} - W_t e_t N_{mt} - R_{\ell t} L_{mt} - R_{kt} K_{mt} \tag{1}$$

where R_ℓ and R_k are nominal rental rates for land and capital respectively, and W_t is the nominal wage.

There are N_t representative agents at time t supplying $N_t e_t$ labor, where N is exogenous, growing exponentially at constant rate ν, and e endogenous, but, as usual in growth models, constant on a balanced growth path. Let C denote the aggregate non-housing consumption good, and H aggregate housing services. We let $c \equiv C/N$ and $h \equiv H/N$ denote per capita quantities. The representative consumer then cares about c and h, and dislikes working. He solves the problem

$$\max\ U = E_t \sum_{s=0}^{\infty} (1+\rho)^{-s} \left[\ln\left(\left[\omega_c c_{t+s}^{(\epsilon-1)/\epsilon} + \omega_h h_{t+s}^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} \right) - \psi\left(e_{t+s}\right) \right] \tag{2}$$

subject to

$$P_{m,t+s}\left(c_{t+s}+\iota_{t+s}\right)+P_{h,t+s}h_{t+s}+V_{t+s}\left[\left(1+\nu\right)\ell_{t+s}-\ell_{t+s-1}\right]+b_{t+s}/\left(1+R_{t+s}\right)$$

$$\leq \quad b_{t+s-1}+\left(1-\tau_{e}\right)W_{t+s}e_{t+s}+\left(1+\nu\right)R_{k,t+s}P_{m,t+s-1}k_{t+s-1} \tag{3}$$

$$+R_{\ell,t+s}V_{t+s-1}\ell_{t+s-1}+d_{m,t+s}+d_{h,t+s}+T_{t+s} \tag{4}$$

$$\left(1+\nu\right)k_{t+s} = \left(1-\delta\right)k_{t+s-1}+z\left(\iota_{t+s-1}/k_{t+s-1}\right)k_{t+s-1} \qquad \left(j=m,h\right) \tag{5}$$

where ι_t denotes total capital investment at date t, d_{jt} nominal dividends (for simplicity assumed to be distributed in a lump-sum fashion) from the profits of intermediate goods producers in sector j, b_t nominal one-period discount bonds, V_t the price of land at date t, W_t the wage, and k_t and ℓ_t per capita capital and land holdings at date t. Labor market distortions are captured by τ_e, and T_t is a lump-sum distribution (e.g. of tax revenues from τ_s). The constraints reflect the fact that population is growing, so that per capita stocks get deflated at rate ν. Both k_t and ℓ_t denote the sum of capital and land in both sectors. The function $z\left(x\right)$ represents adjustment costs, which will be discussed in more detail below.

3.2 Equilibrium Growth

Aggregating over producers in each sector, we have

$$\begin{aligned} C_t+I_t &= A_{mt}K_{mt}^{\alpha}L_{mt}^{\beta_m}\left(e_tN_{mt}\right)^{1-\alpha-\beta_m} \\ K_t-\left(1-\delta\right)K_{t-1} &= z\left(I_t/K_{t-1}\right)K_{t-1} \\ H_t &= A_{ht}K_{ht}^{\alpha}L_{ht}^{\beta_m}\left(e_tN_{ht}\right)^{1-\alpha-\beta_h} \end{aligned}$$

where

$$\begin{aligned} L_{mt}+L_{ht} &= \bar{L} \\ K_{mt}+K_{ht} &= K_{t-1} \\ N_{mt}+N_{ht} &= N_t \end{aligned}$$

The stocks of capital and land in the h sector would correspond to residential real estate. Labor in this sector would be partly non-market household labor, and partly service sector labor (particularly for apartment buildings). We assume (mainly for convenience) that capital's share is the same in both sectors, but labor's share is higher in manufacturing (implying of course that land's share is higher in the housing sector, i.e. $\beta_h > \beta_m$).

Let c and h denote per capita quantities of C and H, while k, ℓ, k_i, ℓ_h refer to per worker quantities in sector i (e.g. $k_{ht} \equiv K_{ht}/N_{ht}$, $k_t \equiv K_t/N_{t+1}$, i.e. no subscript refers to aggregates), while $n_{it} \equiv N_{it}/N_t$, $(i = m, h)$.[4] Given the assumption of perfect competition, we can assume the economy solves the following planner's problem:

$$\max\ U = E_0 \left\{ \sum_{t=0}^{\infty} (1+\rho)^{-t} \ln \left(\left[\omega_c c_t^{(\epsilon-1)/\epsilon} + \omega_h h_t^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} \right) - \psi\left(e_t\right) \right\} \tag{6}$$

subject to resource constraints (expressed in per capita units)

$$\begin{aligned}
c_t + i_t &= A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} n_{mt} && (7)\\
(1+\nu)\, k_t - (1-\delta)\, k_{t-1} &= z\left(i_t/k_{t-1}\right) k_{t-1} && (8)\\
h_t &= A_{ht} k_{ht}^{\alpha} \ell_{ht}^{\beta_h} e_t^{1-\alpha-\beta_h} n_{ht} && (9)\\
k_{mt} n_{mt} + k_{ht} n_{ht} &= k_{t-1} && (10)\\
\ell_{mt} n_{mt} + \ell_{ht} n_{ht} &= \ell_t && (11)\\
n_{mt} + n_{ht} &= 1. && (12)
\end{aligned}$$

Total land $\bar{L}$ is assumed fixed, so $\ell_t/\ell_{t-1} = (1+\nu)^{-1}$, and we can normalize $\bar{L} = 1$. Average technological progress in sector i, i.e. the average growth rate of A_i, is denoted γ_i $(i = m, h)$. We assume e is the same in the two sectors, and that $\beta_h \geq \beta_m$. Note that the timing assumptions in (10) and (11) are such that while aggregate capital k is chosen one period ahead of time, and total land and labor are exogenous, for simplicity the sectoral allocations are determined contemporaneously.

Note that technical progress in the h sector is unrelated to technological progress in construction. (In fact, home construction occurs in the m sector in this model.) Rather, it refers to an increase in the housing services from given stocks of K_h, L_h, and labor inputs eN_h. What this means in practice depends on exactly what the term "housing services" encompasses, and on how one measures K_h. In the model it is assumed for simplicity to be indistinguishable from K_m other than by its allocation to the h sector. In particular, it is assumed to have the same price as K_m and C. In principle it would include both residential structures and housing service-related consumer durables (home appliances). L_h would include both non-market and market labor involved in household production—time devoted to housework, food preparation, home and

[4] The derivations here draw on Ngai and Pissarides (2007), albeit in discrete time, and adding a fixed factor with heterogeneous technology.

yard maintenance, and the like.

The model obviously abstracts from a number of potentially important factors. First and foremost, the housing and construction sectors are heavily affected by government intervention, both via distortionary taxation and regulations. In particular, much land in the U.S. (and in most other countries as well) is neither residential nor commercial, and is either owned or heavily restricted in its use by the government. Second, there is tremendous heterogeneity in land and housing values. Land near navigable bodies of water, or ports, or along coastlines is much more valuable than land that does not have these features. Obviously this model will have nothing directly to say about the cross-sectional distribution of land values or housing prices (though many of the factors that affect them over time undoubtedly come into play in the cross-section as well). Nonetheless if all of these factors remain relatively constant over time, then ignoring them in a model such as this should not be too great a sin.

The dynamic first-order conditions for capital accumulation are as follows:

$$\lambda_t \ z'\left(i_t/k_{t-1}\right) = \mu_{mt} \tag{13}$$

$$\begin{aligned} \lambda_t\left(1+\nu\right)\left(1+\rho\right) = {} & E_t\left\{\mu_{mt+1}A_{mt+1}\alpha k_{mt+1}^{\alpha-1}\ell_{mt+1}^{\beta_m}e_t^{1-\alpha-\beta_m}+\right. \\ & \left.\lambda_{t+1}\left[z\left(i_{t+1}/k_t\right)-\left(i_{t+1}/k_t\right)z'\left(i_{t+1}/k_t\right)+1-\delta\right]\right\} \end{aligned} \tag{14}$$

where μ_{mt}, μ_{ht}, and λ_t are shadow prices on the resource constraints (7), (9), and (8). Note that in the absence of adjustment costs, i.e. when $z\left(x\right)=x$, we have $\lambda_t=\mu_{mt}$, and (72) becomes

$$\mu_{mt}\left(1+\nu\right)\left(1+\rho\right)=E_t\left\{\mu_{mt+1}\left[A_{mt+1}\alpha k_{mt+1}^{\alpha-1}\ell_{mt+1}^{\beta_m}e_t^{1-\alpha-\beta_m}+1-\delta\right]\right\} \tag{15}$$

which is just the familiar condition that the intertemporal marginal rate of substitution equals the marginal product of capital.

The static first-order conditions can be shown to imply that

$$\frac{k_m}{k_h} = \frac{1-\alpha-\beta_h}{1-\alpha-\beta_m} \tag{16}$$

$$\frac{\ell_m}{\ell_h} = \frac{\beta_m}{\beta_h}\frac{1-\alpha-\beta_h}{1-\alpha-\beta_m}, \tag{17}$$

Let p_t denote the relative price of housing services in terms of manufactured goods. We have

$$p_t = \frac{\mu_{ht}}{\mu_{mt}} = \frac{A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m}}{A_{ht}k_{ht}^{\alpha-1}\ell_{ht}^{\beta_h}e_t^{1-\alpha-\beta_h}} = \frac{A_{mt}}{A_{ht}}\left(\frac{\beta_m}{\beta_h}\right)^{\beta_m}\left(\frac{1-\alpha-\beta_h}{1-\alpha-\beta_m}\right)^{\alpha+\beta_m-1}\left(\frac{\ell_{ht}}{e_t}\right)^{-(\beta_h-\beta_m)} \tag{18}$$

Thus growth in the price of housing services reflects both relative productivity growth in manufacturing and the increasing scarcity of land.

Finally, the first-order condition for work effort, taking into account the distortion τ_e, can be expressed as

$$\psi'(e_t) = (1-\tau_e)\,\mu_{mt}(1-\alpha-\beta_h)\,A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{-\alpha-\beta_m}k_{t-1} \tag{19}$$

which equates the marginal rate of substitution between consumption and leisure with the marginal product of labor expressed in terms of m sector output. For now we will assume that $\tau_e = 0$, but will relax this later.

3.3 Aggregate Growth under Certainty

Let total expenditure $c + ph$ be denoted by x. It also turns out that $\mu_m = x^{-1}$ (see the proof in the Appendix), hence $\mu_{mt}/\mu_{mt-1} = x_{t-1}/x_t$. We can aggregate the two resource constraints as follows:For the dynamic equations describing the evolution of k_t, i_t, and x_t we then have

$$x_t + i_t = A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m}k_{t-1} \tag{20}$$

$$(1+\nu)\,k_t = z(i_t/k_{t-1})\,k_{t-1} + (1-\delta)\,k_{t-1} \tag{21}$$

$$q_t(1+\rho)(1+\nu) = E_t\Big\{(x_t/x_{t+1})\Big[A_{mt+1}\alpha k_{mt+1}^{\alpha-1}\ell_{mt+1}^{\beta_m}e_t^{1-\alpha-\beta_m} + q_{t+1}\left[z(i_{t+1}/k_t)+1-\delta\right] - i_{t+1}/k_t\Big]\Big\} \tag{22}$$

where $q_t \equiv \lambda_t/\mu_{mt} = [z'(i_t/k_{t-1})]^{-1}$, the shadow value of capital in terms of m output. Note that in the absence of adjustment costs, i.e. when $z(x) = x$, we have $\lambda_t = \mu_{mt}$, and (72) becomes

$$\mu_{mt}(1+\nu)(1+\rho) = E_t\left\{\mu_{mt+1}\left[A_{mt+1}\alpha k_{mt+1}^{\alpha-1} + 1 - \delta\right]\right\} \tag{23}$$

which is just the familiar condition that the intertemporal marginal rate of substitution equals the marginal product of capital. Apart from the inclusion of land as a factor of production, the only difference with the standard neoclassical growth model is the distinction between k_m and k, which is addressed below.

We will define aggregate balanced growth under certainty as an equilibrium path in which x and k both grow at a constant rate, and in which the interest rate (i.e. the marginal product of capital) is also constant. We will also assume that $z\left(i/k\right) = i/k$ and $z'\left(i/k\right) = 1$ at the steady state value of i/k, so that adjustment costs are zero on the balanced growth path. Balanced growth clearly requires that $A_{mt}k_{mt}^{\alpha-1}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m}$ be constant, which amounts to a linear restriction on the growth rates in the m sector of the capital-labor ratio, technological progress, and the land-labor ratio. Therefore, let

$$Z_t \equiv \left[A_{mt}\ell_{mt}^{\beta_m}e_t^{1-\alpha-\beta_m}\right]^{1/(1-\alpha)} \tag{24}$$

and define variables with "~" over them to be deflated by Z_t, e.g. $\tilde{k}_{mt} \equiv k_{mt}/Z_t$. We then have

$$x_t/k_{t-1} + i_t/k_{t-1} = \tilde{k}_{mt}^{\alpha-1} \tag{25}$$

$$(1+\nu)\, k_t/k_{t-1} = z\left(i_t/k_{t-1}\right) + 1 - \delta \tag{26}$$

$$\begin{aligned}\left(x_{t+1}/x_t\right)(1+\nu)(1+\rho)\, q_t = {} & \alpha\tilde{k}_{mt+1}^{\alpha-1} + \\ & q_{t+1}\left[z\left(i_{t+1}/k_t\right) + 1 - \delta\right] - \left(i_{t+1}/k_t\right)\end{aligned} \tag{27}$$

With $\tilde{k}_m$ constant under balanced growth, k and x both grow at the same constant rate.

From (10)-(12) and (16)-(17) we have

$$k_{mt}\left[\frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}n_{ht} + n_{mt}\right] = k_{t-1}. \tag{28}$$

Now let

$$Q_t \equiv \frac{1-\alpha-\beta_m}{1-\alpha-\beta_h}n_{ht} + n_{mt} \tag{29}$$

$$= 1 + \tau n_{ht} \tag{30}$$

where

$$\tau \equiv \frac{\beta_h - \beta_m}{1-\alpha-\beta_h}. \tag{31}$$

Then we have $k_{mt} = k_{t-1}/Q_t$, and we can define $\hat{k}_t \equiv k_t/\left(Z_tQ_t\right)$. This gives a normalization of k_t that is constant on the balanced growth path. Note that if $\beta_m = \beta_h$, then $Q = 1$ and we would have $k_{mt} = k_{ht} = k_{t-1}$. But with $\beta_m > \beta_h$, $Q > 1$ and n_h and n_m are changing over time (unless $\epsilon = 1$). In particular, if $\epsilon < 1$ and $\gamma_m \geq \gamma_h$, then n_h (and hence Q) grows over time.

Thus, strictly speaking, balanced growth requires one of these knife-edge conditions: $\epsilon = 1$, $\beta_m = \beta_h$, or

$$(1+\gamma_m)(1+\nu)^{\beta_h-\beta_m} = 1+\gamma_h \tag{32}$$

None of these is very palatable: Kahn (2008) provides evidence that ϵ is substantially less than one, and cites other studies with similar findings based on micro data. I also show that ϵ has important implications for housing price dynamics, so assuming $\epsilon = 1$ for convenience is not innocuous. Similar comments apply to the assumption $\beta_m = \beta_h$. To assume (32) is less problematic, as it is tantamount to assuming that p_t does not grow over time. It does imply that $\gamma_h > \gamma_m$, which is hard to believe but also hard to refute directly since γ_h is difficult to measure. If (32) fails to hold (say if $\gamma_m \geq \gamma_h$, so that p drifts higher over time), the dynamic response of the model will be a function of the level of p_t—in particular the aggregate growth rate varies over time and is only asymptotically constant.

It turns out, however, that the consequences of assuming this when it is false are in fact innocuous: Both the variation in the growth rate over time and the differences in dynamics are tiny (see Kahn, 2008). Thus when (32) does not hold, the model exhibits near-balanced aggregate growth and unbalanced sectoral dynamics, as in Ngai and Pissarides (2007). In other words, over a wide range of parameters, growth is so close to balanced even when p is growing over time that it is reasonable to treat it as balanced for computational purposes. Consequently in what follows, results that pertain to the balanced growth path also apply (approximately) in the case where γ_m and γ_h are constant but (32) does not hold. In particular, the benchmark assumption for the model simulations below will be that $(1+\gamma_m)(1+\nu)^{\beta_h-\beta_m} > 1+\gamma_h$, so that p grows over time.

On the balanced aggregate growth path, ZQ grows at a constant rate. In fact it is straightforward to show that its growth rate g satisfies

$$\left[(1+\gamma_m)(1+\nu)^{-\beta_m}\right]^{1/(1-\alpha)} \equiv 1+g^* \equiv G^* \tag{33}$$

We then have $\tilde{k}_{mt} = k_{mt}/Z_t = k_{t-1}/(Q_t Z_t) = \hat{k}_{t-1} Q_{t-1} Z_{t-1}/(Q_t Z_t) = \hat{k}_{t-1}/G_t$. Aggregate output per capita (in terms of manufactured goods), which we denote y_t, is $A_{mt} k_t^\alpha \ell_{mt}^{\beta_m} e_t n_{mt} + p_t A_{ht} k_{ht}^\alpha \ell_{ht}^{\beta_h} e_t n_{ht}$, or (after substituting for p_t and simplifying as before):

$$y_t = A_{mt} k_{mt}^\alpha \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} Q_t = \tilde{k}_{mt}^\alpha Z_t Q_t, \tag{34}$$

so we can also define $\hat{y}_t = y_t/(Z_t Q_t) = \tilde{k}_{mt}^\alpha = \left[\hat{k}_{t-1}/G_t\right]^\alpha$ and $\hat{x}_t = x_t/(Z_t Q_t)$.[5] There is also constant

[5] On the quasi-balanced growth path in which p grows over time, Kahn (2008) shows that ZQ grows at a rate that is virtually

work effort along a balanced growth path. From (19) we have

$$\psi'(e_t) = \mu_{mt}(1-\alpha-\beta_h) A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} e_t^{-\alpha-\beta_m} k_{t-1}$$

which after normalization with Z yields

$$\begin{aligned} \psi'(e_t) e_t &= (1-\alpha-\beta_h) \tilde{k}_{mt}^{\alpha-1} k_{t-1}/x_t \\ &= (1-\alpha-\beta_h) \tilde{k}_{mt}^{\alpha}/\hat{x}_t \end{aligned}$$

Since $\tilde{k}_{mt}$ and $\hat{x}_t$ are constant along the balanced growth path, e_t is constant as well.

We can now characterize the dynamics in terms of stationary variables:

$$\hat{x}_t + \hat{\imath}_t = \left[\hat{k}_{t-1}/G_t\right]^{\alpha} \tag{35}$$

$$G_t(1+\nu)\hat{k}_t = G_t\hat{\imath}_t + (1-\delta)\hat{k}_{t-1} \tag{36}$$

$$(\hat{x}_{t+1}/\hat{x}_t) G_t (1+\nu)(1+\rho) q_t = \alpha\left[\hat{k}_t/G_t\right]^{\alpha-1} + 1 - \delta \tag{37}$$

$$q_t = z'\left(G_t\hat{\imath}_t/\hat{k}_{t-1}\right)^{-1} \tag{38}$$

where $G_t = Q_t Z_t/(Q_{t-1}Z_{t-1})$. Since along the balanced growth path G_t is constant, $z(x) = x$, and $q = 1$, we have

$$\hat{x} + \hat{\imath} = \left[\hat{k}/G\right]^{\alpha} \tag{39}$$

$$(1+\nu) G = \left[\hat{k}/G\right]^{\alpha-1} - G\hat{x}/\hat{k} + 1 - \delta \tag{40}$$

$$(1+\nu)(1+\rho) G = \alpha\left[\hat{k}/G\right]^{\alpha-1} + 1 - \delta. \tag{41}$$

which is exactly as in the standard neoclassical growth model. The innovation in this paper is to simultaneously characterize the behavior of sectoral variables, and in particular housing prices and investment, within the aggregate steady state.

3.4 Open Economy

It is now relatively straightforward to extend the model to a small open economy framework in which interest rates are subject to exogenous shocks. Here the assumption is that land and labor are immobile

constant but (on average) slightly faster than that given by (33). This faster growth is the consequence of resources (capital, land, and labor) flowing into the h sector.

across countries, but capital flows across borders in a near-frictionless manner. Deviations from the closed economy equilibrium interest rates associated with changes in net foreign assets. Each country is assumed to have no impact on world interest rates, and the structural shocks that presumably are behind the "exogenous" changes in interest rates are not modeled.

Markets are incomplete, since the only way for a country to insure against idiosyncratic shocks is through capital flows. One example of such a model is Correia et al (1993). Letting B_t denote the stock of net foreign assets, and R_t the gross real interest rate (return from $t-1$ to t). The only friction is that to deal with the potential unit root problem for B_t, we assume R_t depends negatively on B_t, suitably normalized.[6] Individual decision-makers take R_t as given.

Without loss of generality we can assume that B_t only affects the resource constraint in the m sector. We then have, instead of (7),

$$c_t + i_t = A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} n_{mt} - b_t + R_t b_{t-1} / \left(1+\nu\right)$$

where $b_t \equiv B_t/N_t$. When we combine this with expenditures on h_t, we get

$$x_t + i_t = A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} k_{t-1} - b_t + R_t b_{t-1} / \left(1+\nu\right).$$

This implies an intertemporal first-order condition

$$\left(1+\nu\right)\left(1+\rho\right)\mu_{mt} = E_t\left\{\mu_{mt+1} R_{t+1}\right\}$$

where, again, $\mu_{mt} = 1/x_t$.

Let $\hat{b}_t \equiv B_t / \left(Q_t Z_t N_t\right)$, the normalized stock of net foreign assets, with a steady state value $\hat{b}$. The

[6] Schmitt-Grohe and Uribe (2003) discuss a variety of methods for models to deal with this problem, and conclude that it does not make much difference which method is used.

first-order conditions would then be:

$$G_t(1+\nu)\hat{k}_t = G_t\hat{\imath}_t + (1-\delta)\hat{k}_{t-1} \tag{42}$$

$$\hat{x}_t + \hat{\imath}_t = \left[\hat{k}_{t-1}/G_t\right]^{\alpha} - \hat{b}_t + R_t\hat{b}_{t-1}/\left[(1+\nu)G_t\right] \tag{43}$$

$$(1+\nu)(1+\rho) = E_t\left\{(\hat{x}_t/\hat{x}_{t+1})G_{t+1}^{-1}R_{t+1}\right\} \tag{44}$$

$$q_t(1+\rho)(1+\nu) = E_t\left\{(\hat{x}_t/\hat{x}_{t+1})G_{t+1}^{-1}\left[\alpha\left[\hat{k}_t/G_{t+1}\right]^{\alpha-1} + \right.\right. \tag{45}$$

$$\left.\left. q_{t+1}\left[z\left(G_{t+1}\hat{\imath}_{t+1}/\hat{k}_t\right) + 1 - \delta\right] - G_{t+1}\hat{\imath}_{t+1}/\hat{k}_t\right]\right\} \tag{46}$$

$$R_t = R_t^0\left(1 - \Phi\left(\hat{b}_t\right)\right) \tag{47}$$

where we assume that $\Phi\left(\hat{b}\right) = 0$, $\Phi'(b) \geq 0\ \forall b$, and R_t^0 is the exogenous "base rate" for $\hat{b}_t = \hat{b}$.

In the steady state we then have

$$\begin{aligned} G(1+\nu) &= G\hat{\imath}/\hat{k} + (1-\delta) \\ \hat{x} + \hat{\imath} &= \left[\hat{k}/G\right]^{\alpha} + \rho\hat{b} \\ (1+\rho)(1+\nu)G &= R^0 \\ \alpha\left[\hat{k}/G\right]^{\alpha-1} + 1 - \delta &= R^0 \end{aligned}$$

Since steady state $\hat{k}$, $\hat{\imath}$, and R must be as in the closed economy case (to avoid having B grow relative to the rest of the economy), the only difference is that the steady state level of $\hat{b}$ affects the steady state level of $\hat{x}$. The Appendix provides details on the solution of the model.

3.5 Sectoral Growth

The sectoral variables can be solved for directly as functions of the aggregates. We can write the relevant static first-order conditions for p, n_h, ℓ_m, h, k_m, and c as functions A_{mt}, A_{ht}, and the aggregates k_{t-1} and

x_t:

$$
\begin{aligned}
p_t h_t &= A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} (1+\tau) n_{ht} / (1+\tau n_{ht}) && (48)\\
1 &= \omega_c \phi(c_t, h_t)^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} x_t && (49)\\
h_t / c_t &= p_t^{-\epsilon} (\omega_h / \omega_c)^{\epsilon} && (50)\\
\ell_{mt} &= \frac{\bar{L}}{N_t} \frac{\beta_m/\beta_h}{n_{ht}(1+\tau-\beta_m/\beta_h)+\beta_m/\beta_h} && (51)\\
p_t &= \frac{A_{mt}}{A_{ht}} \left(\frac{\beta_m}{\beta_h}\right)^{\beta_h} (1+\tau)^{1-\alpha-\beta_h} \left(\frac{\ell_{mt}}{e_t}\right)^{-(\beta_h-\beta_m)}. && (52)\\
k_{mt} &= k_{t-1} / (1+\tau\, n_{ht}) && (53)\\
\psi'(e_t)\, x_t &= (1-\alpha-\beta_h) A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{-\alpha-\beta_m} (1+\tau\, n_{ht}). && (54)
\end{aligned}
$$

Only in the knife-edge cases of $\epsilon = 1$ or (32) will these variables exhibit balanced growth in the sense of either being constant or growing at the same rate as the aggregate economy.

Although land is not explicitly priced in the model, we can compute its shadow rental price v_t in terms of manufactured goods:

$$
v_t = \beta_m A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m - 1} e_t^{1-\alpha-\beta_m} \tag{55}
$$

To a first approximation we can say that the land rental price grows at rate $g + \nu$ on the balanced growth path—exactly $g + \nu$ if $\epsilon = 1$, a bit faster if $\epsilon < 1$ and p is growing.

3.6 Stochastic Growth

We suppose that the growth rate of A_h is fixed at γ_h, but that of A_m follows a Markov regime-switching process:

$$
A_{mt} / A_{mt-1} = (1 + \tilde{\gamma}_{mt}) \eta_t / \eta_{t-1} \tag{56}
$$

where

$$
\tilde{\gamma}_{mt} = \begin{cases} \gamma_m^1 & \text{if } \xi_t = 1 \\ \gamma_m^0 & \quad \xi_t = 0 \end{cases} \tag{57}
$$

η_t is a transitory disturbance, and ξ_t is a state variable with Markov transition matrix Θ, where $\Theta[i, j] = \Pr(\xi_t = j | \xi_{t-1} = i)$. Since the columns of Θ must sum to one, we write it as

$$
\Theta = \begin{bmatrix} \theta_1 & 1 - \theta_0 \\ 1 - \theta_1 & \theta_0 \end{bmatrix}. \tag{58}
$$

If the diagonal elements of Θ are close to one, the growth states will be highly persistent, and a shift from one state to the other will carry with it a sizeable adjustment in the long-term level of A_m. Since the stationary distribution of ξ is $\xi^* \equiv \begin{bmatrix} \frac{1-\theta_0}{2-\theta_1-\theta_0} & \frac{1-\theta_1}{2-\theta_1-\theta_0} \end{bmatrix}'$, the average growth rate of A_m is

$$\bar{\gamma}_m = \frac{1-\theta_0}{2-\theta_1-\theta_0}\gamma_m^1 + \frac{1-\theta_1}{2-\theta_1-\theta_0}\gamma_m^0. \tag{59}$$

For concreteness we will call $\xi = 1$ the "high-growth" regime, and $\xi = 0$ the "low-growth" regime, i.e. we assume $\gamma_m^1 > \gamma_m^0$.

Elaborating on Hamilton (1994), we can describe μ as an AR(1) process. We have

$$\xi_t = 1 - \theta_0 + (\theta_1 + \theta_0 - 1)\,\xi_{t-1} + v_t \tag{60}$$

where $E_{t-1}(v_t) = 0$, and is distributed as follows:

$$v_t = \begin{cases} 1-\theta_1 & \text{Prob } \theta_1 \\ -\theta_1 & \text{Prob } 1-\theta_1 \end{cases} \text{ if } \xi_{t-1} = 1 \qquad \begin{cases} -(1-\theta_0) & \text{Prob } \theta_0 \\ \theta_0 & \text{Prob } 1-\theta_0 \end{cases} \text{ if } \xi_{t-1} = 0 \,. \tag{61}$$

Note that while $E\left(v_t|\xi_{t-1}\right) = 0$, v_t is not identically distributed over time, as the conditional distribution depends on ξ_{t-1}.

The log deviation version of G_t can be written as

$$G_t = \frac{1}{1-\alpha}\left[\frac{\tilde{\gamma}_{mt} - \bar{\gamma}_m}{1+\bar{\gamma}_m} + \Delta\eta_t + \frac{\tau n_h}{1+\tau n_h}\Delta n_{ht} - \beta_m \Delta\ell_{mt} + (1-\alpha-\beta_m)\,\Delta e_t\right] \tag{62}$$

We suppose that $\eta_t = \phi_1\eta_{t-1} + \phi_2\eta_{t-1} + \upsilon_t$, where υ_t is i.i.d. with a zero mean. In what follows, we will first assume that economic agents observe both z_t and η_t before making their period t decisions.[7]

In principle the parameters of the growth process $\left(\gamma_m^1, \gamma_m^0, \theta_0, \theta_1\right)$ would be country-specific. In the simulations of the model we will choose them to correspond to observed growth rates over 10 to 15 year periods for each country. It would be more realistic to have spillovers across countries, but for the questions addressed here, and the country-by-country approach, it would be an unnecessary complication. The main point of the simulations is to gauge the relative medium-frequency volatility of house price and income

[7] Note that (62) means that, strictly speaking, the dynamics of the model are not independent of the initial position in levels, as n_h will will not be stationary. As indicated earlier, this variation is miniscule, because n_h evolves slowly and the coefficient on $(n_{ht} - n_{ht-1})$ above is sufficiently small and insensitive to n_h that growth is virtually constant. See Kahn (2008) for more details.

growth, which is likely to depend more on the persistence of growth variations rather than on its amplitude.

3.7 Asset Prices

Thus far we have only described the behavior of the price of housing services and rental prices for land. The term "housing prices" generally refers to asset values of homes, both the structures and the land. In this model we can calculate the value of what might be called "real estate wealth," which would be the total value of capital and land allocated to the housing services sector. The value of the capital is just $K_h = k_h n_h$. The asset value of the land $L_h = \ell_h n_h$ requires some computation, as described below. Given a land price, which we will denote by V_t (expressed in terms of m sector output), valuing a representative house requires constructing an index, because the composition of the representative house changes over time due to changes in the price of land. Given a path $\{K_{ht}, L_{ht}\}$ we will define a "constant-quality" house price index P_{ht} as a Laspeyres index by choosing a base year, say $t = 0$, and setting $P_{ht} = 100\,(V_t L_{h0} + K_{h0})\,/\,(V_0 L_{h0} + K_{h0})$.

We know that V_t is the present discounted value of the stream of rents $\{v_t\}$:

$$V_t = v_t + E_t\left\{\Phi_{t,1} V_{t+1}\right\} = E_t\left\{\sum_{\tau=0}^{\infty} \Phi_{t,\tau} v_{t+\tau}\right\} \tag{63}$$

where

$$\Phi_{t,\tau} = \frac{\mu_{m,t+\tau}}{\mu_{mt}\,(1+\nu)^{\tau}\,(1+\rho)^{\tau}} = \frac{x_t}{x_{t+\tau}\,(1+\nu)^{\tau}\,(1+\rho)^{\tau}} \tag{64}$$

is the stochastic discount factor. On the balanced growth path we have $\Phi^{-1} = (1+g)\,(1+\rho)\,(1+\nu)$, and $\hat{v}_t$, as mentioned previously, is (for plausible parameters) almost constant but technically a function of A_{mt}/A_{ht} and N_t (for $\epsilon < 1$ it is increasing in both arguments). Hence while the capital stock and the aggregate output grow at $g + \nu$, the price of land, and hence the price of "houses" (capital plus land in the h sector) grows at a rate (slightly) faster than $g + \nu$. We will examine the behavior of land prices off the steady state later after describing the model under stochastic growth.

3.8 Calibration

Most of the parameters take on standard values: $\alpha = 0.33$, $\nu = 0.01$, $\delta = 0.05$ (a compromise for structures and equipment). The parameters β_h and β_m should reflect the shares of land in the cost of housing services and non-housing output respectively. We set $\beta_h = 0.5$ and $\beta_m = 0.05$. Since housing services represent about 20 percent of overall consumer expenditures, we set $\omega_h = 0.2$, $\omega_c = 0.8$. We set the time preference rate ρ equal to 0.01. Finally, we choose the parameters of the regime-switching process for productivity to correspond roughly to the results in KR: $\left(\gamma_m^1 - \beta_m \nu\right) / (1-\alpha) = 0.029$, $\gamma_m^0 = 0.013$, $\theta_1 = 0.99$, $\theta_0 = 0.983$.

Thus high growth regimes are slightly more persistent than low-growth, and implied the overall mean growth rate of A_m, $\bar{\gamma}_m$, is 2.31 percent. Finally, based on the analysis in Kahn (2008), and consistent with broad range of estimates based on micro data, we set $\epsilon = 0.3$.

3.9 Model Simulations

On the balanced growth path, the model separates conveniently into its dynamic aggregate component, which is essentially the neoclassical growth model, and the sectoral variables, which are static functions of the aggregate state variables. With shocks that push the variables away from balanced growth, it is not quite as simple: The growth rate depends (very slightly) on the movements in resources from one sector to the other that occur because $\epsilon \neq 1$. This makes n_h a state variable, because the growth rate g_t depends on n_{ht} and n_{ht-1}. We can still use standard methods (e.g. Uhlig, 1997) to obtain a solution for the linearized aggregate model in the normalized variables.

The key to doing interesting simulations is to take the peculiar error structure of the disturbance process into account. Even though the conditional expectation of the errors in the z_t process (the regime states) is zero, actual realizations of zero are not possible, and in fact given the values of q_1 and q_0, a small error (of absolute value $1 - q_0$ or $1 - q_1$) that leaves z unchanged is highly likely in any given time period. So rather than consider a one-time shock to v, it makes sense to consider a single large shock (a regime-switch) followed by a sequence of identical small shocks that leave the regime unchanged for an extended period of time. Such a path is more like a modal outcome or scenario, and hence more interesting than the usual impulse response involving a shock at a single date.

Figure 6 gives an example of this type of simulation. The economy is in the low growth regime in periods 1 to 11, and then switches to the high growth regime, where it remains. The figure plots the behavior of the asset price of a house (a fixed-weight combination of capital and land—see the Appendix) against per capita income, for $\epsilon = 0.3$ and 0.9. House prices are clearly much more responsive, both at impact and during the regimes, in the $\epsilon = 0.3$ case. When the regime shift occurs, the price jumps about 5 percent if $\epsilon = 0.3$ versus around 2 percent if $\epsilon = 0.9$. During the high-growth regime the growth rate of the price is about 0.5 percent (annualized) faster if $\epsilon = 0.3$ versus $\epsilon = 0.9$. Prices actually accelerate as long as the economy remains in the high-growth regime, the more so the lower is ϵ.

A regime-switch simulation such as is depicted in Figure 6 is unrealistic in another important dimension, however. It assumes perfect information about the switch, which is implausible if applied to the data point by point at high frequencies. Kahn (2008) explores the idea of gradual learning about regime switches, which naturally has the effect of smoothing out the jumps. Since here the focus is on averages over decades or longer, the short-run dynamics and their interaction with imperfect information are a secondary issue.

4 Results

The goal of the simulation exercises is to gauge whether the model can match the stylized fact in the data that housing price growth responds in a roughly two-for-one fashion to differences in overall growth (as measured by consumption growth). There are two ways to do this, corresponding to the two types of comparisons from Figure 5. Corresponding to Figure 5c, we can simulate a regime shift and compare the difference in house price growth across the two regimes with the difference in consumption growth. Alternatively, corresponding to Figures 5a and 5b, we can compare across two separate simulations that result in different consumption growth rates—either because of being in different growth regimes, or spending more time in one regime versus the other.

Table 1 provides results to illustrate the responsiveness of housing prices to changes (or differences) in growth trends. These are shown for a range of values of the elasticity parameter ϵ. These comparisons show that while overall growth (as measured by consumption growth) is not substantially affected by ϵ, the responsiveness of housing price growth is. For the benchmark value of $\epsilon = 0.3$, a switch to a high-growth regime that results in the growth rate of consumption increasing by about 1.1 percent results in housing prices accelerating by more than 3 percent. This is consistent with the results in Figure 5c, where the slope is 2.6, meaning that a one percent increase in consumption growth would induce a 2.6 percent increase in housing prices. The case of $\epsilon = 0.9$ illustrates the fact that as preferences move close to Cobb-Douglass, housing price growth responds by roughly one-to-one with overall growth.

The other simulations in Table 1 correspond more to the cross-sectional results in Figures 5a and 5b. The results compare housing price growth and consumption growth for a country that is in a sustained low-growth regime with those for a country in a sustained high-growth regime. Here again housing prices are more responsive to changes in overall growth the smaller the value of ϵ, again broadly consistent with the figures. In the benchmark $\epsilon = 0.3$ case, for example, housing price growth increases by just over 2 percent versus a 1.4 percent increase in consumption growth. Whereas for the regime switch case the model predicts a slightly larger multiplier effect than is in the data, for this case the model somewhat underpredicts the multiplier. The model does have the prediction that the multiplier will be larger for the regime-switch case than for the sustained low vs. high growth comparison, which is borne out by Figure 5. So the model is successful qualitatively at producing an economically significant more than one-for-one response of housing prices to growth, and in particular obtains the qualitative result that the response is greater for within-country changes than across country differences.

One can also use this framework to gauge the impact of persistent changes in real interest rates on housing prices. Of course, given the small open economy assumption, interest rate movements play no role in the relative movements of housing prices across countries, only in the common component. For example, in Figures 5a and 5b we see that the regression line is approximately one percent (100 basis points) higher in the 1990-2005 period than in the 1980-90 period. This represents a common increase in house prices not explained by increases in consumption growth. (In fact, the mean growth rate of consumption was essentially the same in the two periods.) Interest rates (specifically lower real interest rates in the later time period) could be one candidate to explain this.

Figure 7 depicts the impact of a highly persistent (serial correlation parameter 0.99) decline in the "base" real interest rate R_t^0. The initial decline in the rate is large, about 150 basis points. The impact effect on both house prices and total expenditures is large (about 5 percent), but this is a one-time level effect that dissipates along with the interest rate shock. A sustained increase in the growth rate of housing prices of the sort we see in the data would appear to require a continued decline in interest rates. Moreover, such a decline would stimulate overall consumption expenditures, which is not evident in the data. Thus it would appear that while a real interest rate reduction may have contributed modestly to the international housing price boom, the model suggests it is a relatively small part of the overall story. It cannot explain the cross-sectional variation in housing price growth, and cannot (at least in this model) explain the fact that housing prices accelerated by more than overall economic activity of the advanced economies.

4.1 Permanent Income: Productivity or Hours?

The discussion thus far has used relative consumption growth as a proxy for relative changes in permanent income. For the United States, Kahn (2008) finds that productivity growth alone explains a considerable part of the fluctuations in housing prices around their long-term trend. Since income growth can be decomposed into productivity (output per hour) growth and growth in labor input (hours), one question is whether relative productivity growth is an important factor across countries in explaining relative housing price growth.

To answer this, I examine the relationship between the growth of housing prices, consumption, productivity, and hours across these countries. In contrast to what one might expect from the U.S. case, it turns out that much of the action across countries is in hours rather than productivity, especially with regard to the post-1990 boom. This is illustrated in Figures 8 and 9, which depict the cross-sectional relationship between consumption growth and, alternately, hours growth and productivity growth. It also shows up in variance decompositions. For example, in the 1990-2005 period, productivity growth and hours growth are almost orthogonal (correlation -0.09), and the R^2 in a univariate cross-section regression of consumption growth on hours growth is 0.67, whereas on productivity growth it is 0.12.

While it seems likely that a shock that results in persistent increases in hours of work would also increase housing prices, the quantitative impact is less clear, in particular with regard to the multiplier effect. If we look back at (18) we see that the flow price of housing services p_t is positively related to work effort e_t, but this channel lacks the direct impact of changes in relative productivity growth. Here we can reintroduce the labor market distortion τ_e and consider the impact of sustained changes to it, and the resulting changes in work effort e, corresponding to hours of work per capita in the data. The sustained changes in hours that are evident in Figure 8 clearly require similarly sustained changes in τ_e. To do so, however, in the context of the relatively simplistic labor supply behavior embodied in (19) above, requires growth regimes in $(1 - \tau_e)$. These can be incorporated into the model, and simulations suggest that the effects are similar in direction to those of productivity growth regime changes, but the relative impact on housing prices is more modest, and there is no multiplier. But even to the extent this can work mechanically, there is no empirical counterpart to such regimes. If we look more closely at labor market reforms in countries such as Ireland, we tend to see more or less one-time changes in distortions followed by a dynamic response in labor market activity that works itself out over many years, along with induced demographic changes such as immigration. Short-run dynamics in the response of other variables may also play an important role. This suggests further work is needed in sorting out the role of labor market changes versus productivity growth in gaining a more complete understanding of the dynamics of the housing sector.

5 Conclusions and Future Research

This paper has developed a small open economy growth model with land, housing services, and other goods and shown that it is capable both qualitatively and quantitatively of explaining a substantial portion of the roughly two-to-one responsiveness of housing prices to changes in overall economic growth seen in the major industrialized economies, and the even larger multiplier for within-country changes in growth. The paper also shows, however, that, in contrast to the United States, persistent changes (or differences across countries) in hours of work appear to be more important than changes (or differences) in productivity growth in accounting for changes in overall economic growth. While sustained labor market changes also affect house prices, the effects are likely to be more modest than those coming from changes in productivity growth. This suggests a need to incorporate labor market distortions in the analysis, but also a richer model of labor market activity that can mimic the dynamic response to changes in labor market distortions that can propagate over many years.

Other sources of short-run dynamics may prove important as well. One potentially important simplifying assumption in this regard is the flexible neoclassical technology for producing housing services. In particular, existing housing capital (i.e. the structures) can be freely recombined with land and labor in any ratio at each date. In reality, of course, most housing structures are fixed on particular plots of land for decades, with only modest changes in capital-land ratios from additions or other modifications. Thus a putty-clay framework would be a natural extension of the present model, and one that would likely result in substantially more price responsiveness, as most of the housing stock would be stuck with fixed capital-land ratios when the price of land changes.

6 Appendix

6.1 Solving the Closed Economy Model

Proof that $\mu_m = x^{-1}$: $\mu_m = u_c = \phi_c/\phi$. Since ϕ is homogeneous of degree one, $\phi = \phi_c c + \phi_h h$. Also $p = \phi_h/\phi_c$, so $\phi = \phi_c (c + ph) = \phi_c x$. Hence $\phi_c/\phi = x^{-1}$.

The first-order conditions for the planner are as follows: Letting $\phi(c,h) \equiv \left[\omega_c c^{(\epsilon-1)/\epsilon} + \omega_h h^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}$, we have:

$$\omega_c \phi^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} = \mu_{mt} \tag{65}$$

$$\omega_h \phi^{-(\epsilon-1)/\epsilon} h_t^{-1/\epsilon} = \mu_{ht} \tag{66}$$

$$\psi'(e_t) = \mu_{mt}(1-\alpha-\beta_m) A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{-\alpha-\beta_m} n_{mt} + \mu_{ht}(1-\alpha-\beta_h) A_{ht} k_{ht}^{\alpha} \ell_{ht}^{\beta_h} e_t^{-\alpha-\beta_h} n_{ht} \tag{67}$$

$$\mu_{mt}\beta_m A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m-1} e_t^{1-\alpha-\beta_m} = \mu_{ht}\beta_h A_{ht} k_{ht}^{\alpha} \ell_{ht}^{\beta_h-1} e_t^{1-\alpha-\beta_h} \tag{68}$$

$$\mu_{mt} A_{mt} k_{mt}^{\alpha-1} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} = \mu_{ht} A_{ht} k_{ht}^{\alpha-1} \ell_h^{\beta_h} e_t^{1-\alpha-\beta_h} \tag{69}$$

$$\mu_{mt} A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} = \mu_{ht} A_{ht} e_t^{1-\alpha-\beta_h} \times \tag{70}$$

$$\left[\alpha k_{ht}^{\alpha-1} \ell_{ht}^{\beta_h} e_t^{1-\alpha-\beta_h} k_{mt} + \beta_h k_{ht}^{\alpha} \ell_{ht}^{\beta_h-1} e_t^{1-\alpha-\beta_h} \ell_{mt} + (1-\alpha-\beta_h) k_{ht}^{\alpha} \ell_{ht}^{\beta_h} e_t^{1-\alpha-\beta_h}\right] \tag{71}$$

$$\mu_{mt}(1+\nu)(1+\rho) = E_t\left\{\mu_{mt+1} A_{mt+1} \alpha k_{mt+1}^{\alpha-1} \ell_{mt+1}^{\beta_m} e_t^{1-\alpha-\beta_m} + 1 - \delta\right\} \tag{72}$$

μ_{mt} and μ_{ht} are shadow prices on the resource constraints (7), (9), and (8). Note that in the absence of adjustment costs, i.e. when $z(x) = x$, we have $\lambda_t = \mu_{mt}$, and (72) becomes

$$\mu_{mt}(1+\nu)(1+\rho) = E_t\left\{\mu_{mt+1}\left[A_{mt+1} \alpha k_{mt+1}^{\alpha-1} \ell_{mt+1}^{\beta_m} e_t^{1-\alpha-\beta_m} + 1 - \delta\right]\right\} \tag{73}$$

which is just the familiar condition that the intertemporal marginal rate of substitution equals the marginal product of capital.

To solve the model, first we linearize the system

$$
\begin{aligned}
\hat{x}_t + \hat{\imath}_t &= \left[\hat{k}_{t-1}/G_t\right]^\alpha \\
G_t\left(1+\nu\right)\hat{k}_t &= z\left(G_t\hat{\imath}_t/\hat{k}_{t-1}\right)\hat{k}_{t-1} + \left(1-\delta\right)\hat{k}_{t-1} \\
\left(1+\nu\right)\left(1+\rho\right)q_t &= E_t\left\{\left(\hat{x}_t/\hat{x}_{t+1}\right)G_{t+1}^{-1}\left[\alpha\left[\hat{k}_t/G_{t+1}\right]^{\alpha-1} + \right.\right. \\
&\quad \left.\left. q_{t+1}\left[z\left(G_{t+1}\hat{\imath}_{t+1}/\hat{k}_t\right)+1-\delta\right] - G_{t+1}\hat{\imath}_{t+1}/\hat{k}_t\right]\right\} \\
q_t &= z'\left(G_t\hat{\imath}_t/\hat{k}_{t-1}\right)^{-1}
\end{aligned}
$$

around the quasi-steady state values $\hat{k}$, $\hat{x}$, and G.. After some rearranging, and letting $R \equiv \alpha\left[\hat{k}/G\right]^{\alpha-1} + 1-\delta = \left(1+\rho\right)\left(1+\nu\right)G$, the linearized versions of the four equations can be expressed as

$$
\begin{aligned}
G\left(1+\nu\right)\hat{k}_t &= R\left(\hat{k}_{t-1} - G_t\right) - \hat{x}_t\left[G\hat{x}/\hat{k}\right] \\
\hat{x}_t\left[G\hat{x}/\hat{k}\right] + \hat{\imath}_t\left[G\hat{\imath}/\hat{k}\right] &= \left[R-\left(1-\delta\right)\right]\left(\hat{k}_{t-1} - G_t\right) \\
Rq_t &= E_t\left\{\left[\hat{x}_t - \hat{x}_{t+1} - G_{t+1}\right]R\right. \\
&\quad \left. -\alpha\left(1-\alpha\right)\left(\frac{\hat{k}}{G}\right)^{\alpha-1}\left(\hat{k}_t - G_{t+1}\right) + q_{t+1}\left(G\hat{\imath}/\hat{k} + 1 - \delta\right)\right\} \\
0 &= q_t + z''\left(G\hat{\imath}/\hat{k}\right)\left(G_t + \hat{\imath}_t - \hat{k}_{t-1}\right)
\end{aligned}
$$

Note that

$$
\begin{aligned}
\alpha\left(\frac{\hat{k}}{G}\right)^{\alpha-1} + 1 - \delta &= \left(1+\rho\right)\left(1+\nu\right)G \\
G\hat{x}/\hat{k} &= \left(\frac{\hat{k}}{G}\right)^{\alpha-1} + 1 - \delta - \left(1+\nu\right)G.
\end{aligned}
$$

The log deviation version of G_t can be written as

$$
G_t = \frac{1}{1-\alpha}\left[\frac{\tilde{\gamma}_{mt} - \bar{\gamma}_m}{1+\bar{\gamma}_m} + \Delta\eta_t + \frac{\tau n_h}{1+\tau n_h}\Delta n_{ht} - \beta_m\Delta\ell_{mt} + \left(1-\alpha-\beta_m\right)\Delta e_t\right] \tag{74}
$$

where Δn_{ht}, $\Delta\ell_{mt}$, and Δe_t are deviations from their local means given growth at G. Thus G_t reflects the endogenous resource shift toward or away from the h sector in response to the shock.

The near-balanced growth behavior of the system means that we can approximate the growth rates of the sectoral variables by linearizing the system

$$
\begin{aligned}
p_t h_t &= A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{1-\alpha-\beta_m} (1+\tau) n_{ht} / (1+\tau n_{ht}) && (75) \\
1 &= \omega_c \phi (c_t, h_t)^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} x_t && (76) \\
h_t / c_t &= p_t^{-\epsilon} (\omega_h / \omega_c)^{\epsilon} && (77) \\
\ell_{mt} &= \frac{\bar{L}}{N_t} \frac{\beta_m / \beta_h}{n_{ht} (1 + \tau - \beta_m/\beta_h) + \beta_m / \beta_h} && (78) \\
p_t &= \frac{A_{mt}}{A_{ht}} \left(\frac{\beta_m}{\beta_h} \right)^{\beta_h} (1+\tau)^{1-\alpha-\beta_h} \ell_{mt}^{-(\beta_h - \beta_m)}. && (79) \\
k_{mt} &= k_{t-1} / (1 + \tau n_{ht}) && (80) \\
\psi'(e_t) x_t &= (1 - \alpha - \beta_h) A_{mt} k_{mt}^{\alpha} \ell_{mt}^{\beta_m} e_t^{-\alpha-\beta_m} (1 + \tau n_{ht}). && (81)
\end{aligned}
$$

in logarithmic first differences around the values that obtain under aggregate growth of G and population growth ν. Here we can assume that k_m and x grow at G, while h, c, n_h, and ℓ_m each grow at locally constant rates, which as described in Kahn (2008) is a good approximation over periods of many decades. This results in a system of the form:

$$\Phi \Delta s_t = \Omega \Delta m_t \tag{82}$$

where $s_t \equiv [p_t \; c_t \; h_t \; \ell_{mt} \; n_{ht} \; k_{mt} \; e_t]'$ and $\Delta m_t \equiv \begin{bmatrix} \gamma_{mt} & \Delta x_t & \Delta k_{t-1} & 0 & 0 \end{bmatrix}'$, all in deviations. Φ is 7×7 and Ω is 7×5.

The local constant growth rates can be found from

$$\Phi^{-1} \Omega \Delta \bar{m} \tag{83}$$

where $\Delta \bar{m} = \begin{bmatrix} \gamma_{mt} & g & g & \nu & \gamma_h \end{bmatrix}'$. We then have

$$
\begin{aligned}
\Delta s_t &= \Phi^{-1} \Omega \Delta m_t && (84) \\
G_t &= \zeta' \Delta m_t = \pi' \Phi^{-1} \Omega \Delta m_t + \frac{1}{1-\alpha} \gamma_{mt} && (85)
\end{aligned}
$$

where ζ is 5×1 and $\pi = \begin{bmatrix} 0 & 0 & 0 & \frac{\beta_m}{1-\alpha} & \tau n_h / (1 + \tau n_h) & 0 & \frac{1-\alpha-\beta_m}{1-\alpha} \end{bmatrix}'$ from (74) above. Thus the growth rate can be expressed in terms of the first differences of the aggregate exogenous and endogenous variables.

The result is a system is of the form:

$$
\begin{aligned}
0 &= \mathbf{A}\begin{bmatrix}\hat{k}_t\\ \hat{k}_{t-1}\\ \hat{x}_t\\ \hat{x}_{t-1}\end{bmatrix}+\mathbf{B}\begin{bmatrix}\hat{k}_{t-1}\\ \hat{k}_{t-2}\\ \hat{x}_{t-1}\\ \hat{x}_{t-2}\end{bmatrix}+\mathbf{C}\begin{bmatrix}G_t\\ \hat{\imath}_t\\ q_t\end{bmatrix}+\mathbf{D}\Lambda_t\\
0 &= E_t\left\{\mathbf{F}\begin{bmatrix}\hat{k}_{t+1}\\ \hat{k}_t\\ \hat{x}_{t+1}\\ \hat{x}_t\end{bmatrix}+\mathbf{G}\begin{bmatrix}\hat{k}_t\\ \hat{k}_{t-1}\\ \hat{x}_t\\ \hat{x}_{t-1}\end{bmatrix}+\mathbf{H}\begin{bmatrix}\hat{k}_{t-1}\\ \hat{k}_{t-2}\\ \hat{x}_{t-1}\\ \hat{x}_{t-2}\end{bmatrix}+\mathbf{J}\begin{bmatrix}G_{t+1}\\ \hat{\imath}_{t+1}\\ q_{t+1}\end{bmatrix}+\mathbf{K}\begin{bmatrix}G_t\\ \hat{\imath}_t\\ q_t\end{bmatrix}+\mathbf{L}\Lambda_{t+1}+\mathbf{M}\Lambda_t\right\}\\
\Lambda_{t+1} &= \mathbf{N}\Lambda_t+\Xi_{t+1}
\end{aligned}
$$

where

$$\Lambda_t=\begin{bmatrix}\hat{\xi}_t & \eta_t & \eta_{t-1}\end{bmatrix}',$$

$\hat{\xi}_t \equiv \xi_t - \bar{\xi}$, and

$$
\mathbf{A} = \begin{bmatrix}(1+\nu)G & 0 & G\hat{x}/\hat{k} & 0\\ 0 & -\zeta_3 & -\zeta_2 & 0\\ 0 & 0 & G\hat{x}/\hat{k} & 0\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}-R & 0 & 0 & 0\\ 0 & \zeta_3 & \zeta_2 & 0\\ -[R-(1-\delta)] & 0 & 0 & 0\\ -z'' & 0 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\end{bmatrix},
$$

$$
\mathbf{C} = \begin{bmatrix}R & 0 & 0\\ 1 & 0 & 0\\ R-(1-\delta) & G\hat{x}/\hat{k} & 0\\ z'' & z'' & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \mathbf{D}=\begin{bmatrix}0 & 0 & 0\\ -\left(\zeta_1+\frac{1}{1-\alpha}\right)\frac{\gamma_m^1-\gamma_m^0}{1+\bar{\gamma}_m} & -1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}
$$

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & -R & 0 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} -(1-\alpha)\left[R-(1-\delta)\right] & 0 & R & 0 \end{bmatrix}$$
$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{J} = \begin{bmatrix} (1-\alpha)\left[R-(1-\delta)\right]-R & 0 & G\hat{\imath}/\hat{k}+1-\delta \end{bmatrix}$$
$$\mathbf{K} = \begin{bmatrix} 0 & 0 & -R \end{bmatrix}, \ \mathbf{L}=0, \ \mathbf{M}=0$$

$$\mathbf{N} = \begin{bmatrix} \theta_1+\theta_0-1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix}$$

and

$$\Xi_t = \begin{bmatrix} v_t \\ u_t \\ 0 \end{bmatrix}$$

where v_{1t} and v_{2t} are as defined earlier. We can then use the method of undetermined coefficients outlined by Uhlig (1997) to find the solution of the model in the form

$$\begin{bmatrix} \hat{k}_t \\ \hat{k}_{t-1} \\ \hat{x}_t \\ \hat{x}_{t-1} \end{bmatrix} = \mathbf{P} \begin{bmatrix} \hat{k}_{t-1} \\ \hat{k}_{t-2} \\ \hat{x}_{t-1} \\ \hat{x}_{t-2} \end{bmatrix} + \mathbf{Q}\Lambda_t, \quad \begin{bmatrix} G_t \\ \hat{\imath}_t \\ q_t \end{bmatrix} = \mathbf{R} \begin{bmatrix} \hat{k}_{t-1} \\ \hat{k}_{t-2} \\ \hat{x}_{t-1} \\ \hat{x}_{t-2} \end{bmatrix} + \mathbf{S}\Lambda_t$$
$$\Lambda_t = \mathbf{N}\Lambda_{t-1} + \Xi_t.$$

where in this we know that the $(2,1)$ and $(4,3)$ elements of $\mathbf{P}$ are 1 and the remaining elements of the two rows are zeros. Given paths for $\hat{k}_t$, $\hat{x}_t$, and A_{mt} we can compute the path of s_t, i.e. the levels of the sectoral variables and work effort, using the system (75) – (81) The rates of change of these variables will be very similar to those computed from (84), the linearized system.

6.2 Small Open Economy

The linearized aggregate equations can be expressed as

$$G(1+\nu)\hat{k}_t = R\left(\hat{k}_{t-1} - G_t\right) - \hat{x}_t\left[G\hat{x}/\hat{k}\right] \tag{86}$$

$$\hat{x}_t\left[\hat{x}G/\hat{k}\right] + \hat{\imath}_t\left[\hat{\imath}G/\hat{k}\right] = \alpha\left[\hat{k}/G\right]^{\alpha-1}\left(\hat{k}_{t-1} - G_t\right) \tag{87}$$

$$+(1+\rho)\left(\hat{b}G/\hat{k}\right)\left(\hat{b}_{t-1} + \hat{R}_t - G_t\right) - \left(\hat{b}G/\hat{k}\right)\hat{b}_t \tag{88}$$

$$0 = E_t\left\{\hat{x}_t - \hat{x}_{t+1} - G_{t+1} + R_{t+1}\right\} \tag{89}$$

$$Rq_t = E_t\left\{\left[\hat{x}_t - \hat{x}_{t+1} - G_{t+1}\right]R\right. \tag{90}$$

$$\left. -\alpha(1-\alpha)\left(\frac{\hat{k}}{G}\right)^{\alpha-1}\left(\hat{k}_t - G_{t+1}\right) + q_{t+1}\left(G\hat{\imath}/\hat{k} + 1 - \delta\right)\right\} \tag{91}$$

$$0 = q_t + z''\left(G\hat{\imath}/\hat{k}\right)\left(G_t + \hat{\imath}_t - \hat{k}_{t-1}\right) \tag{92}$$

$$R_t = R_t^0 - \Phi'\left(\hat{b}\right)\hat{b}\hat{b}_t \tag{93}$$

We have one additional state variable now, $\hat{b}_t$, an additional endogenous variable R_t, and an additional exogenous stochastic variable R_t^0.

The result is a system is of the form:

$$0 = \mathbf{A}\begin{bmatrix}\hat{k}_t\\ \hat{k}_{t-1}\\ \hat{x}_t\\ \hat{x}_{t-1}\\ \hat{b}_t\end{bmatrix} + \mathbf{B}\begin{bmatrix}\hat{k}_{t-1}\\ \hat{k}_{t-2}\\ \hat{x}_{t-1}\\ \hat{x}_{t-2}\\ \hat{b}_{t-1}\end{bmatrix} + \mathbf{C}\begin{bmatrix}G_t\\ \hat{\imath}_t\\ q_t\\ R_t\end{bmatrix} + \mathbf{D}\Lambda_t$$

$$0 = E_t\left\{\mathbf{F}\begin{bmatrix}\hat{k}_{t+1}\\ \hat{k}_t\\ \hat{x}_{t+1}\\ \hat{x}_t\\ \hat{b}_{t+1}\end{bmatrix} + \mathbf{G}\begin{bmatrix}\hat{k}_t\\ \hat{k}_{t-1}\\ \hat{x}_t\\ \hat{x}_{t-1}\\ \hat{b}_t\end{bmatrix} + \mathbf{H}\begin{bmatrix}\hat{k}_{t-1}\\ \hat{k}_{t-2}\\ \hat{x}_{t-1}\\ \hat{x}_{t-2}\\ \hat{b}_{t-1}\end{bmatrix} + \mathbf{J}\begin{bmatrix}G_{t+1}\\ \hat{\imath}_{t+1}\\ q_{t+1}\\ R_{t+1}\end{bmatrix} + \mathbf{K}\begin{bmatrix}G_t\\ \hat{\imath}_t\\ q_t\\ R_t\end{bmatrix} + \mathbf{L}\Lambda_{t+1} + \mathbf{M}\Lambda_t\right\}$$

$$\Lambda_{t+1} = \mathbf{N}\Lambda_t + \Xi_{t+1}$$

where

$$\Lambda_t = \begin{bmatrix}\hat{\xi}_t & \eta_t & \eta_{t-1} & R_t^0\end{bmatrix}',$$

$\hat{\xi}_t \equiv \xi_t - \bar{\xi}$, and

$$\mathbf{A} = \begin{bmatrix} (1+\nu)\,G & 0 & G\hat{x}/\hat{k} & 0 & 0 \\ 0 & -\zeta_1 & -\zeta_2 & 0 & 0 \\ 0 & 0 & G\hat{x}/\hat{k} & 0 & \hat{b}G/\hat{k} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \Phi'\left(\hat{b}\right)\hat{b} \end{bmatrix} \mathbf{B} = \begin{bmatrix} -R & 0 & 0 & 0 & 0 \\ 0 & \zeta_1 & \zeta_2 & 0 & 0 \\ -\left[R-(1-\delta)\right] & 0 & 0 & 0 & -(1+\rho)\left(\hat{b}G/\hat{k}\right) \\ -z'' & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} R & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ R-(1-\delta)+(1+\rho)\left(\hat{b}G/\hat{k}\right) & G\hat{\imath}/\hat{k} & 0 & -(1+\rho)\left(\hat{b}G/\hat{k}\right) \\ z'' & z'' & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\zeta_1 - \frac{1}{1-\alpha}\frac{\gamma_m^1-\gamma_m^0}{1+\bar{\gamma}_m} & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & -R & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} -(1-\alpha)\left[R-(1-\delta)\right] & 0 & R & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} (1-\alpha)\left[R-(1-\delta)\right]-R & 0 & G\hat{\imath}/\hat{k}+1-\delta & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & -R & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{L} = 0, \ \mathbf{M} = 0$$

$$\mathbf{N} = \begin{bmatrix} \theta_1+\theta_0-1 & 0 & 0 & 0 \\ 0 & \phi_1 & \phi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \mu \end{bmatrix}$$

and

$$\Xi_t = \begin{bmatrix} v_t \\ u_t \\ 0 \\ \epsilon_t \end{bmatrix}$$

where v_{1t} and v_{2t} are as defined earlier. We can then use the method of undetermined coefficients outlined by Uhlig (1997) to find the solution of the model in the form

$$\begin{aligned}\begin{bmatrix} \hat{k}_t \\ \hat{k}_{t-1} \\ \hat{x}_t \\ \hat{x}_{t-1} \\ \hat{b}_t \end{bmatrix} &= \mathbf{P} \begin{bmatrix} \hat{k}_{t-1} \\ \hat{k}_{t-2} \\ \hat{x}_{t-1} \\ \hat{x}_{t-2} \\ \hat{b}_{t-1} \end{bmatrix} + \mathbf{Q}\Lambda_t, \quad \begin{bmatrix} G_t \\ \hat{\imath}_t \\ q_t \\ R_t \end{bmatrix} = \mathbf{R} \begin{bmatrix} \hat{k}_{t-1} \\ \hat{k}_{t-2} \\ \hat{x}_{t-1} \\ \hat{x}_{t-2} \\ \hat{b}_{t-1} \end{bmatrix} + \mathbf{S}\Lambda_t \\ \Lambda_t &= \mathbf{N}\Lambda_{t-1} + \Xi_t.\end{aligned}$$

where in this we know that the $(2,1)$ and $(4,3)$ elements of $\mathbf{P}$ are 1 and the remaining elements of the two rows are zeros. Given paths for $\hat{k}_t$, $\hat{x}_t$, and A_{mt} we can compute the path of s_t, i.e. the levels of the sectoral variables and work effort, using the system (75) – (81) The rates of change of these variables will be very similar to those computed from (84), the linearized system.

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Table 1: Simulated Effects of Trend Growth Difference on Housing Price Growth

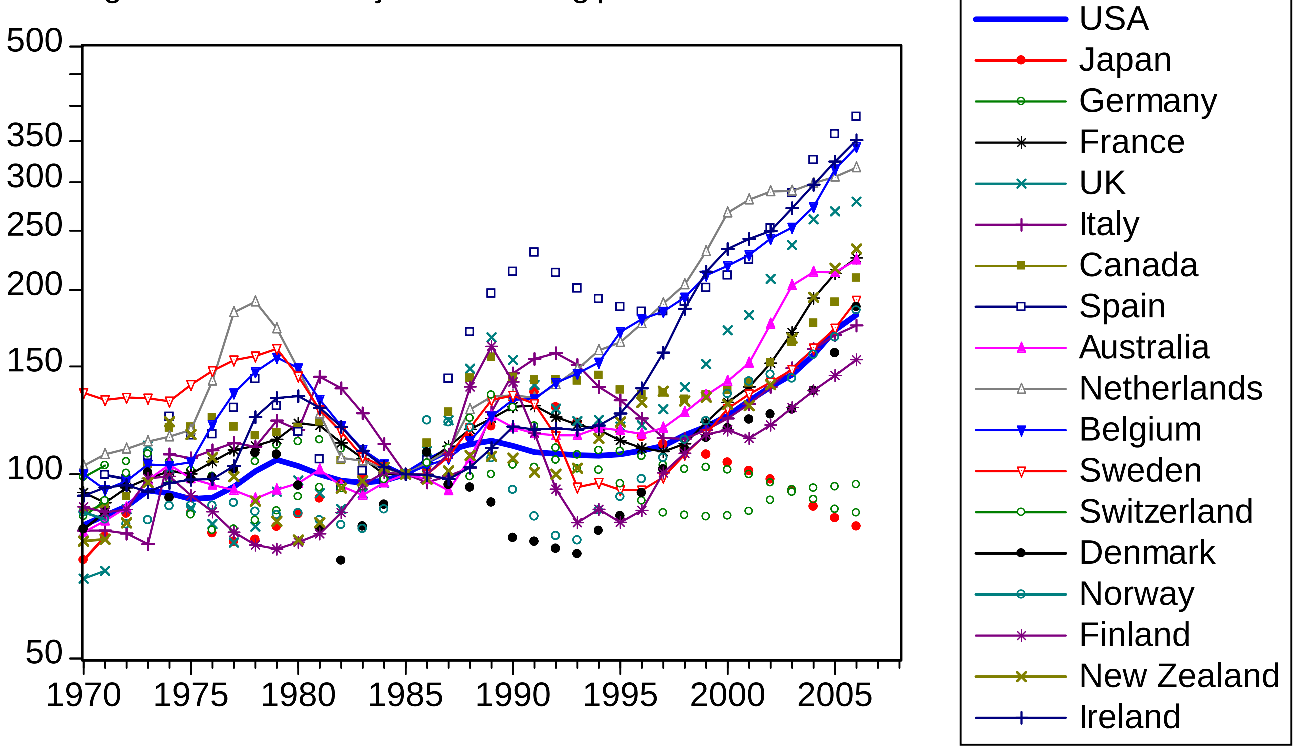
Changes in Consumption (x) Growth vs. Housing Price (P_h) Growth

	$\epsilon = 0.15$		$\epsilon = 0.3$		$\epsilon = 0.5$		$\epsilon = 0.9$	
Change in % growth rate of	x	P_h	x	P_h	x	P_h	x	P_h
Low-to-High Switch*	1.10	4.01	1.09	3.13	1.07	2.29	1.04	1.27
High vs. Low**	1.39	2.55	1.38	2.04	1.36	1.73	1.34	1.24

*10-year simulation, switch after 5 years.

**10-years in high-growth regime vs. 10 years in low-growth regime

Figure 1: Inflation-adjusted housing prices in 18 countries



Note: Normalized 1985=100

Figure 2: Ratio of Housing Wealth to Consumption

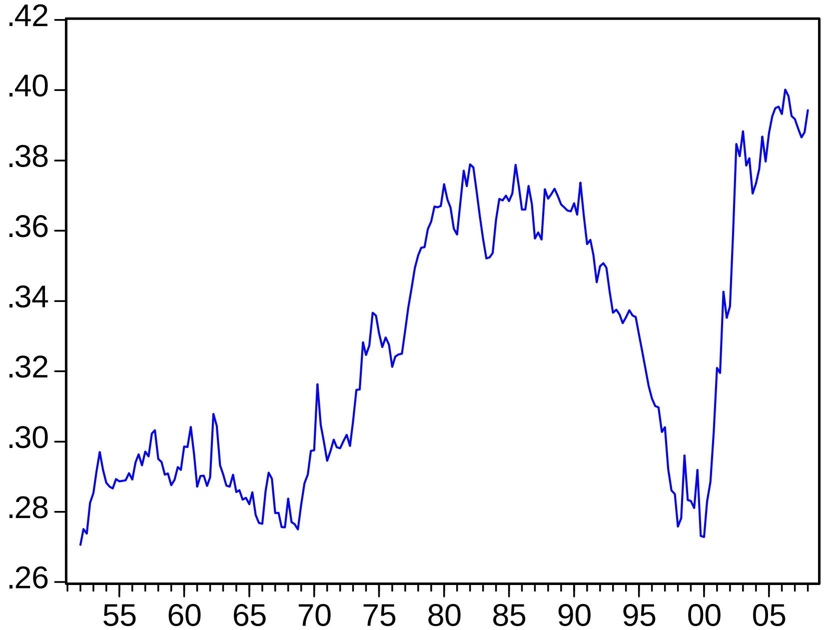
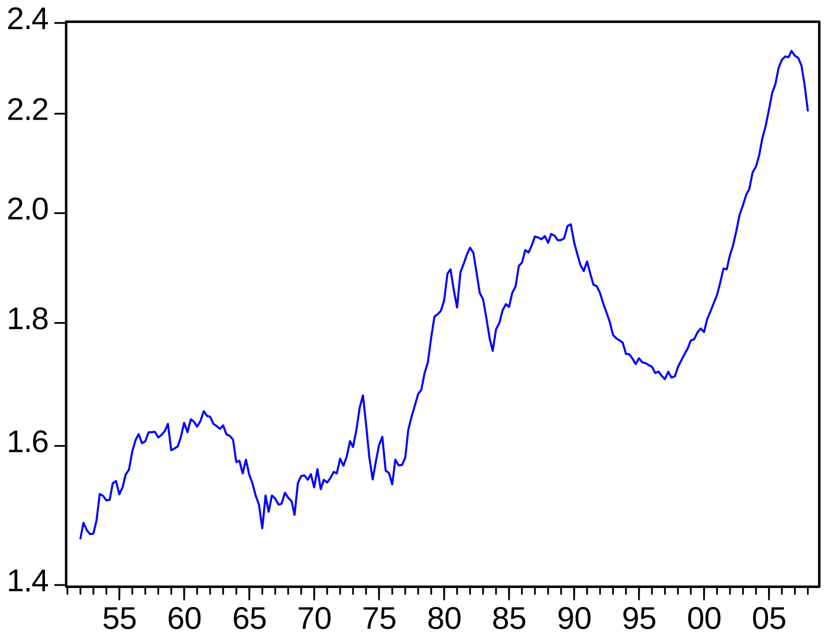


Figure 3: Ratio of Housing Wealth to Total Net Worth



Source: Flow of Funds

Figure 4: Housing Services: Expenditures and Prices

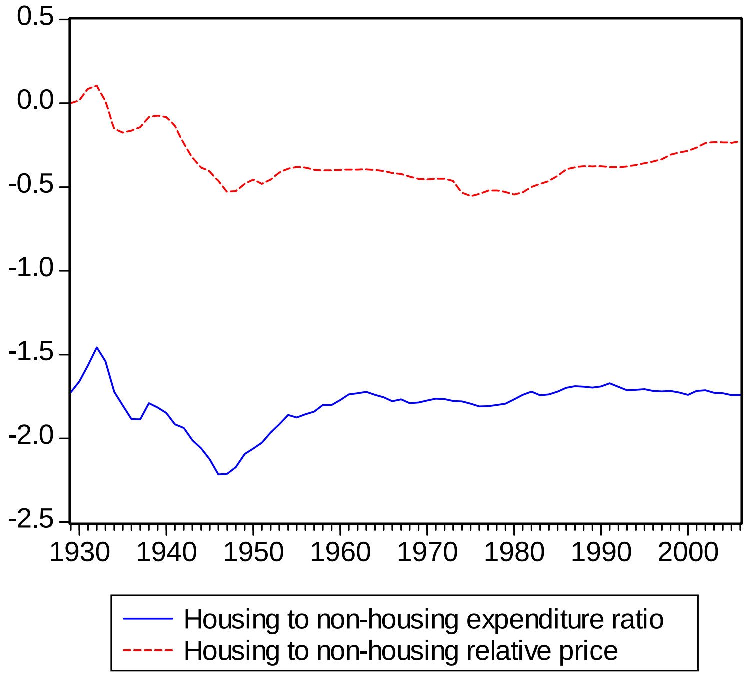


Figure 5: House Price Appreciation vs. Consumption Growth

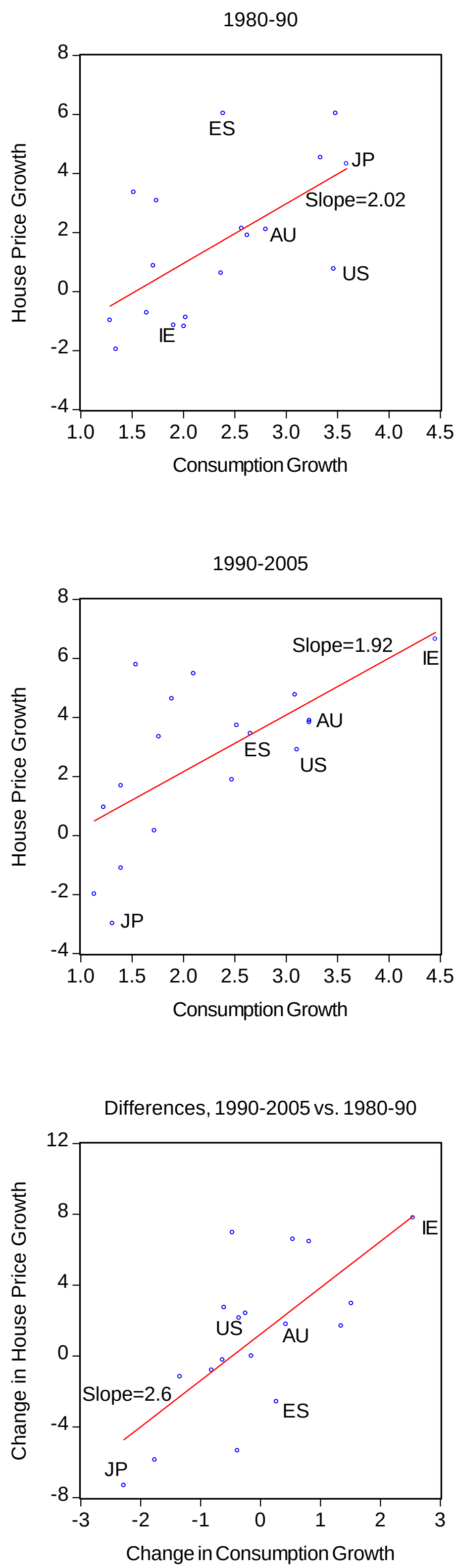


Figure 6: Housing Price Response to
Low-to-High Growth Regime Switch

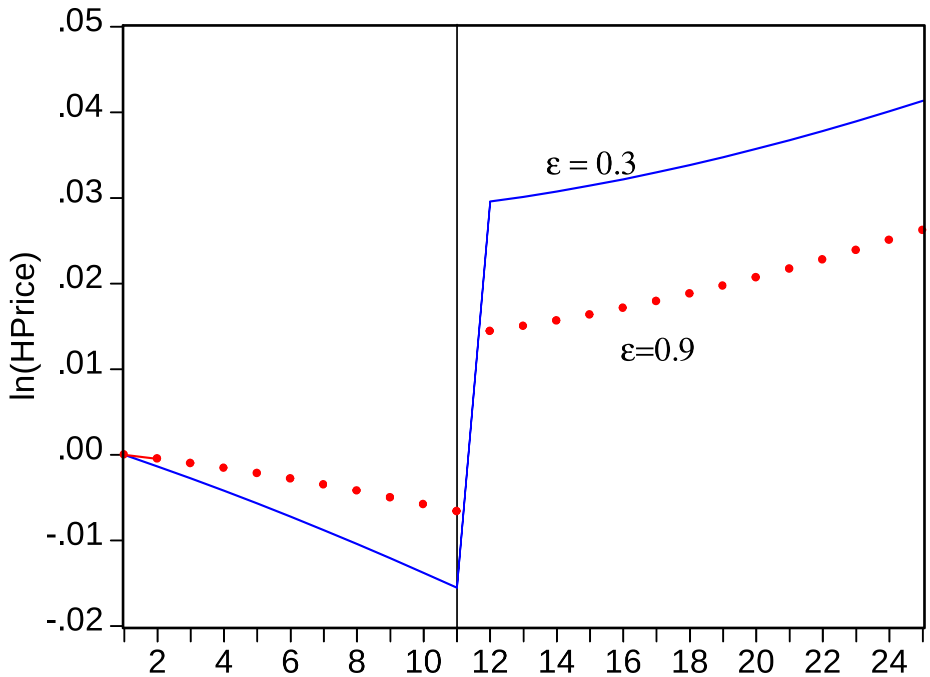
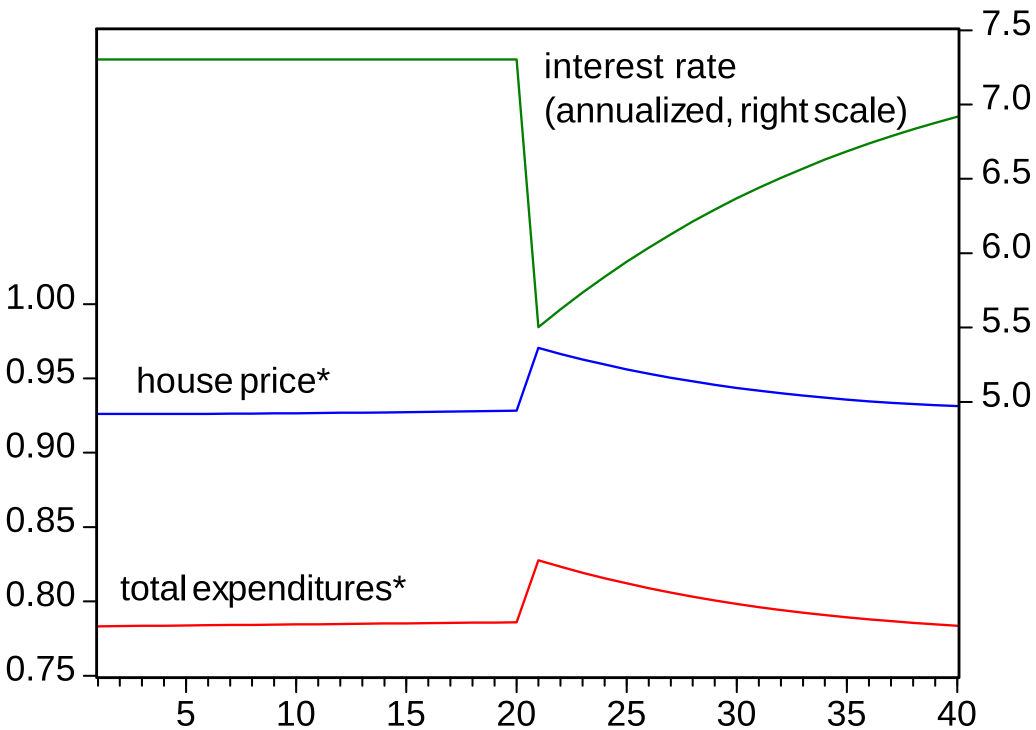


Figure 7: Effect of Interest Rate Shock



*logarithmic, detrended

Figure 8: Consumption Growth vs. Hours Growth

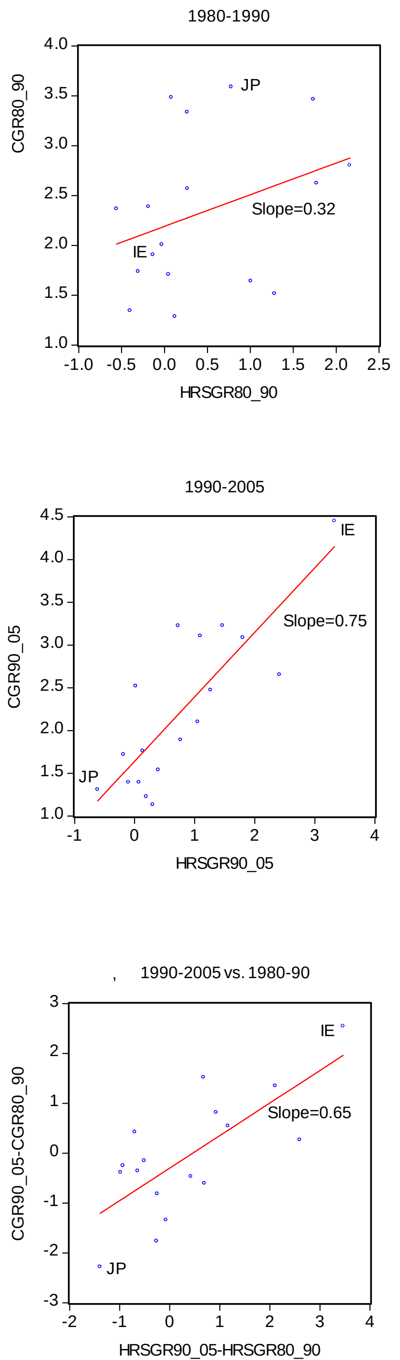


Figure 9: Consumption Growth vs. Productivity Growth