Monetary Transmission and the Yield Curve in a Small Open Economy

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Abstract

Long-term nominal interest rates in small open economies that have adopted inflation targeting tend to be highly correlated with those of the United States. This observation has recently led support to the view that the long-end of the yield curve is determined abroad. We set up and estimate a micro-founded small-large-country model to study the co-movement of long-term nominal interest rates of different currencies. The expectations hypothesis together with uncovered interest rate parity, which both hold in our model, can account for much of the co-movement observed in the data.

1 Introduction

Long-term nominal interest rates in inflation-targeting small-open economies, like Australia, Canada, New Zealand, Norway, Sweden and the United Kingdom, have moved very closely with those of the United States over the past fifteen years or so. Figure 1 shows the pattern of interest rate correlations at different points on the yield curve for each country with the US. The pattern is stark: long-term nominal rates are highly correlated with their US counterparts, generally more so than rates at shorter maturities, and more so than with their respective short-term rates (not shown in Figure 1). This pattern – a result of correlated high frequency movements – has led to the view that long-term nominal interest rates are somehow determined abroad.

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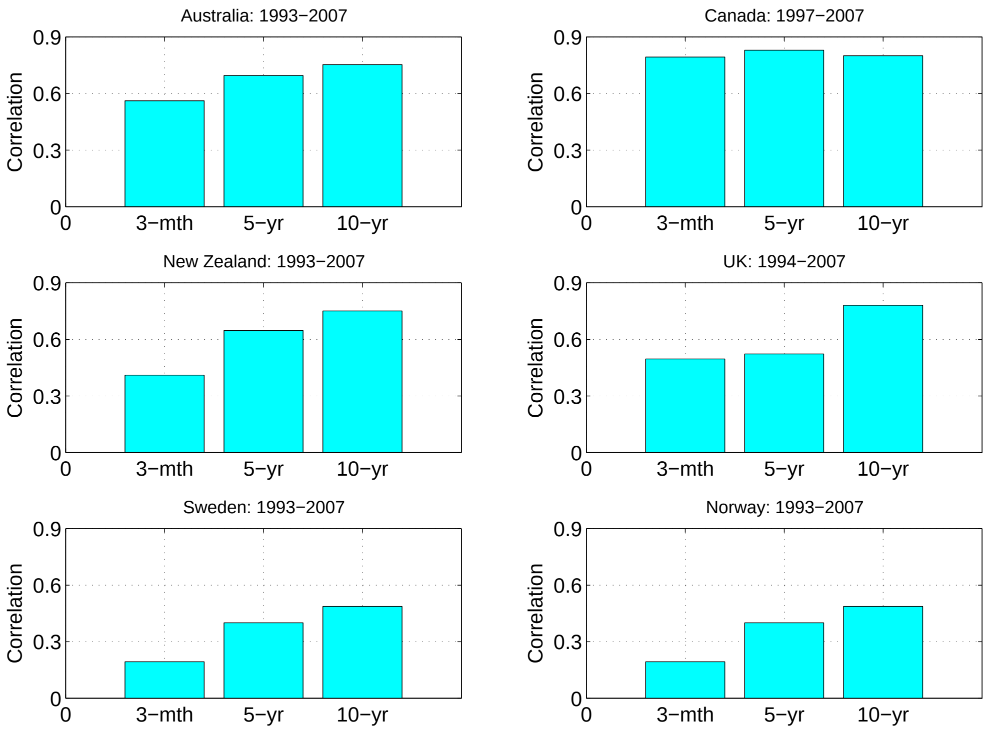

Figure 1: Cross correlations with US interest rates: HP-filtered quarterly data.

Traditionally, the transmission mechanism of monetary policy is understood (among other things) as linking a short-term nominal interest rate to a long-term real interest rate, which in turn influences aggregate demand. Of course, for a short-term *nominal* rate to influence a long-term *real* rate, one thing must 'happen first': the short-term *nominal* rate must influence a long-term *nominal* rate.

But if long-term nominal rates were to be determined abroad, one would be left to wonder how monetary policy would transmit, in the first place, to the rest of the economy. Are these reduced-form patterns of correlations consistent with theory? Are these reduced-form correlations evidence of a weaker transmission mechanism? What are the forces at work in the determination of a small open economy's yield curve? In this paper we address these questions and find that the expectations hypothesis together with uncovered interest rate parity can account for much of the observed co-movement in interest rates of different currencies. Indeed, the main contribution of our paper is to uncover the mechanism that gives rise to these observed reduced-form correlations in optimising general equilibrium. As we discuss below, differences in the persistence of domestic and foreign disturbances can bring about the pattern of correlations of Figure 1.

Other papers have tackled the related question of how much influence domestic and foreign factors have on domestic interest rates. For example, Campbell and Lewis [1998] use an event study to examine how Australian bond yields respond to new information and find that US economic news has a larger effect than domestic news on Australian yields. Tarditi [1996] estimates a reduced-form model of the Australian 10-year bond yield and finds that a one percentage point increase in the US 10-year bond yield is associated with around half a percentage point increase in Australian long-term yields.

There is also a large literature that analyses the yield curve with affine term structure models. These studies typically assume that bond yields are affine functions of unobservable factors and incorporate cross-equation restrictions that eliminate arbitrage opportunities. (See Backus et al. [2001], Knez et al. [1994], Duffie and Kan [1996], Dai and Singleton [2000] and the references therein.) But while factor models have been relatively successful in matching key statistical properties of the yield curve, factor models are not structural. Recent work addresses this issue by fitting the term structure to macroeconomic factors, either by combining them within unobserved factors, as in Ang and Piazzesi [2003] and Bernanke et al. [2004], or by incorporating a no-arbitrage model of the term structure within a macroeconomic model as in Rudebusch and Wu [2004], Bekaert et al. [2006] and Hordahl et al. [2006].

Our focus here is different. We set up a micro-founded two-block model consisting of a small open economy and a large (closed) economy and extend the set of equilibrium conditions in both the large and small economies to allow for an explicit consideration of the co-movement of foreign and domestic interest rates. In our model, the expectations hypothesis links interest rates of different maturities and uncovered interest rate parity links interest rates of different currencies. Short-term nominal rates are set by the monetary authorities on the basis of the fundamentals of their economies. In this respect our analysis resembles that of Evans and Marshall [1998], but unlike them, we study the behaviour of a *small open* economy's yield curve and pay particular attention to its relation to the *large* economy's yield curve. We then estimate the model's parameters and examine its ability to match the co-movement of interest rates of different currencies.

Elsewhere in the literature, monetary policy shocks have been identified within the context of structural vector autoregressions (SVARs). To complement our analysis we estimate impulse responses to a monetary policy shock from identified SVARs. We find that on impact a 25 basis point policy shock raises interest rates of all maturities, that it raises shorter-term rates by more than longer-term rates and that this effect is statistically significant on impact for all rates.

The rest of the paper is organised as follows: Section 2 describes the model, its estimation, and the role of the yield curve in the transmission mechanism of a small open economy; Section 3 contrasts the model's moments with their empirical counterparts; Section 4 discusses SVARs; and Section 5 concludes.

2 The Model

We extend the Galí and Monacelli [2005] small open economy model in two ways: (1) we increase the set of equilibrium conditions in both the large and small economies to incorporate interest rates of longer maturities and; (2) we add foreign and domestic demand shocks. Instead of working through the details of the derivation, which are in Galí and Monacelli [2005], we discuss the log-linear aggregate equations and the role of the yield curve in the transmission mechanism.

2.1 The Large Economy

Variables with a *star superscript* correspond to the large economy, which obeys a standard set of New Keynesian closed economy equations.[1]

The aggregate demand schedule links the current level of the foreign output gap, x_t^*, to its expected future level, the ex-ante short-term real interest rate, foreign total factor productivity, a_t^*, and a foreign aggregate demand disturbance, g_t^*, as follows

$$x_t^* = E_t x_{t+1}^* - \sigma^{-1}\left(R_{1,t}^* - E_t \pi_{t+1}^*\right) - \phi_1(1-\rho_a^*)a_t^* + \sigma^{-1}\left(1-\rho_g^*\right) g_t^* \tag{1}$$

where: $R_{1,t}^*$ is the foreign short-term nominal interest rate; π_t^* is the foreign inflation rate; σ is strictly positive and governs intertemporal substitution; ρ_a^* is the persistence of a_t^*; ρ_g^* is the persistence of g_t^*; and ϕ_1, defined for convenience, is $\frac{1+\varphi}{\sigma+\varphi}$, where the parameter $\varphi > 0$ captures the elasticity of labour supply.

It can be shown that in this model the theory of the term structure implied by optimising behaviour is the expectations hypothesis. Thus, the nominal interest rate at period t associated with a bond that promises to pay one unit of foreign currency at the end of period $t+m-1$ is determined by

$$R_{m,t}^* = \frac{1}{m} E_t \sum_{j=1}^{m} R_{1,t+j-1}^* \qquad m = 2,3,4,... \tag{2}$$

Firms operate in a monopolistically competitive goods market and are subject to Calvo-price stickiness. Factor markets are competitive and goods are produced with a constant returns to scale technology. These assumptions yield the New Phillips curve:

$$\pi_t^* = \kappa x_t^* + \beta E_t \pi_{t+1}^*. \tag{3}$$

where: $\kappa \equiv \lambda\left(\sigma+\varphi\right)$; $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$; and θ governs the degree of price stickiness.

[1] Ireland [2004] and Woodford [2003] contain detailed discussions of the New Keynesian closed economy model.

The foreign monetary authority follows a Taylor-type rule of the form

$$R^*_{1,t} = \rho^*_r R^*_{1,t-1} + \alpha^*_\pi \pi^*_t + \alpha^*_x x^*_t + \varepsilon^*_{r,t} \tag{4}$$

where $\varepsilon^*_{r,t}$ is an independent and identically distributed (iid) foreign monetary disturbance with zero mean and standard deviation $\sigma_{\varepsilon^*_r}$.

The potential level of foreign output, $\bar{y}^*_t$, is the level that would prevail in the absence of nominal rigidities: $\phi_1 a^*_t$. So, in the large economy, the actual level of output, y^*_t, and the output gap, x^*_t, obey the equation below

$$x^*_t = y^*_t - \phi_1 a^*_t. \tag{5}$$

The technology shock, a^*_t, and the demand shock, g^*_t, follow autoregressive processes of the form

$$\begin{aligned} a^*_t &= \rho^*_a a^*_{t-1} + \varepsilon^*_{a,t} && (6)\\ g^*_t &= \rho^*_g g^*_{t-1} + \varepsilon^*_{g,t} && (7) \end{aligned}$$

where the persistence parameters, ρ^*_a and ρ^*_g, are less than unity in absolute value and the shocks, $\varepsilon^*_{a,t}$ and $\varepsilon^*_{g,t}$, are zero mean iid disturbances with standard deviations $\sigma_{\varepsilon^*_a}$ and $\sigma_{\varepsilon^*_g}$.

2.2 The Small Open Economy

The small economy's IS-curve links the output gap, x_t, to its expected future value, the one-period nominal interest rate, $R_{1,t}$, the expected rate of *domestically-produced goods* inflation, $E_t\pi_{h,t+1}$, the expected growth rate of foreign output, foreign and domestic aggregate demand shocks, and total factor productivity, a_t. Following Galí and Monacelli [2005] the small open economy's IS-curve can be shown to take the form

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma_\alpha^{-1}\left(R_{1,t} - E_t\pi_{h,t+1}\right) + \phi_3 E_t \Delta y^*_{t+1} + \\ &\quad \sigma^{-1}\left(1-\rho_g\right)(1-\phi_2)g_t + \sigma^{-1}\left(1-\rho^*_g\right)\phi_3 g^*_t - \phi_4(1-\rho_a)a_t \end{aligned} \tag{8}$$

where ρ_a and ρ_g are the persistence parameters of a_t and g_t. The parameters $\sigma_\alpha, \phi_2, \phi_3,$ and ϕ_4 are, in turn, functions of deeper parameters. In particular,

$$\begin{aligned} \sigma_\alpha &\equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} \\ \omega &\equiv \sigma\tau + (1-\alpha)(\sigma\iota - 1) \\ \phi_2 &\equiv \frac{\sigma_\alpha - \sigma}{\sigma_\alpha + \varphi} \\ \phi_3 &\equiv \alpha(\omega-1) + \phi_2 \\ \phi_4 &\equiv \frac{1+\varphi}{\sigma_\alpha+\varphi} \end{aligned}$$

where $\alpha \in [0,1]$ is the share of foreign goods in the consumption basket, and therefore serves as a measure of openness. It is worth noting that for $\alpha = 0$, the small economy's equations reduce to the standard set of closed economy equations discussed above. Thus, the small economy shares all structural features of the large economy, except, of course, for openness. Indeed, as discussed in Galí and Monacelli [2005], the linearised equations hold around a symmetric steady state. τ is the intratemporal elasticity of substitution between foreign and domestically produced goods, and ι is the elasticity of substitution across varieties of foreign goods.

In equilibrium, the nominal interest rate at t associated with a bond that promises to pay one unit of *domestic* currency at the end of period $t+m-1$ is determined by

$$R_{m,t} = \frac{1}{m} E_t \sum_{j=1}^{m} R_{1,t+j-1} \qquad m = 2,3,4,... \tag{9}$$

The dynamics of *domestically-produced*goods price inflation, $\pi_{h,t}$, are governed by an analogous New Phillips curve equation

$$\pi_{h,t} = \kappa_\alpha x_t + \beta E_t \pi_{h,t+1} \tag{10}$$

where $\kappa_\alpha \equiv \lambda\left(\sigma_\alpha + \varphi\right)$.

Monetary policy in the small economy is assumed to follow a Taylor-type rule that sets the nominal interest rate, $R_{1,t}$, in response to its own lagged value, deviations of *consumer price* inflation, π_t, and the output gap, x_t, from their steady state values. The rule is given by

$$R_{1,t} = \rho_r R_{1,t-1} + \alpha_\pi \pi_t + \alpha_x x_t + \varepsilon_{r,t} \tag{11}$$

where $\varepsilon_{r,t}$ is an iid monetary policy shock with zero mean and standard deviation σ_{ε_r}.

The terms of trade, s_t, are defined (from the perspective of the large economy) as the price of foreign goods, $p_{f,t}$, in terms of the price of home goods, $p_{h,t}$. That is, $s_t = p_{f,t} - p_{h,t}$. Around a symmetric steady state the consumer price index is a weighted average of the form $p_t = (1-\alpha)p_{h,t} + \alpha p_{f,t}$. It is straightforward to show that $p_t = p_{h,t} + \alpha s_t$, which implies that consumer price inflation and domestically-produced goods inflation are linked by the expression below.

$$\pi_t = \pi_{h,t} + \alpha \Delta s_t \tag{12}$$

The nominal exchange rate, e_t, is defined as the price of foreign currency in terms of the domestic currency. The real exchange rate, q_t, in turn, is defined as $q_t \equiv e_t + p_t^* - p_t$. It follows that changes in the nominal exchange rate, Δe_t, can be decomposed into changes in the real exchange rate and the differential in consumer price inflation.

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^* \tag{13}$$

Positive values of Δe_t indicate a nominal depreciation of the domestic currency, as the price of the foreign currency increases. Because the law of one price is assumed to hold,

$p_{f,t} = e_t + p_t^*$, which implies that the terms of trade can alternatively be written as $s_t = e_t + p_t^* - p_{H,t}$. Combining these expressions, it is easy to show that the real exchange rate is proportional to the terms of trade as follows

$$\Delta q_t = (1-\alpha)\Delta s_t \tag{14}$$

Complete international securities markets together with market clearing imply the following relationship between the terms of trade, s_t, and output differentials and demand shock differentials[2]

$$s_t = \sigma_\alpha (y_t - y_t^*) - \frac{\sigma_\alpha}{\sigma}\left(g_t - g_t^*\right) \tag{15}$$

The presence of the aggregate demand shocks differential in Equation (15), $g_t - g_t^*$, alters the small economy's flexible price level of output, relative to Galí and Monacelli [2005]. The relationship between the actual level of output, y_t, and the output gap, x_t, satisfies[3]

$$x_t = y_t - \phi_2 y_t^* - \frac{\phi_2}{\sigma}\left(g_t - g_t^*\right) - \phi_4 a_t \tag{16}$$

Finally, exogenous domestic processes evolve according to

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{17}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \tag{18}$$

where the shocks, $\varepsilon_{a,t}$, and $\varepsilon_{g,t}$, are iid with zero-mean and standard deviations σ_{ε_a} and σ_{ε_g}. The persistence parameters, ρ_a and ρ_g, are, as before, less than unity in absolute value.

2.3 The Transmission Mechanism

The linearised dynamics of the model, as we mentioned above, are valid around a symmetric steady state in which the condition of uncovered interest rate parity holds:

$$R_{1,t} = R_{1,t}^* + E_t \Delta e_{t+1} \tag{19}$$

Equation (19), however, is not an independent equilibrium condition since it can be recovered from the Euler equations for consumption and the international risk-sharing condition, Equation (15). Equations (2) and (9), combined with Equation (19) relate foreign and

[2]Because the demand shock, g_t, enters the household's lifetime expected utility as in $E_0 \sum_{t=0}^{\infty} \beta^t e^{g_t}\left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)$, it follows that demand shocks enter the international risk-sharing condition as in Equation (15).

[3]One could show that the level of potential output in the small economy is given by $\phi_2 y_t^* + \frac{\phi_2}{\sigma}(g_t - g_t^*) + \phi_4 a_t$. If aggregate demand shocks were absent from our model, the expression for the output gap collapses back to that of Galí and Monacelli [2005].

domestic interest rates of equivalent maturities as follows,

$$R_{m,t} = R^*_{m,t} + \frac{1}{m} E_t \sum_{j=1}^{m} \Delta e_{t+j} \tag{20}$$

Equation (20) highlights that the expected path of the nominal exchange rate plays a central role to the extent that it governs the degree of 'pass through' from movements in foreign rates to movements in domestic rates. For example, in an extreme case in which the small economy becomes closed, $\alpha \to 0$, movements in foreign rates would not translate into movements in domestic rates.

While Equations (2) and (9) are independent equilibrium conditions, they are nevertheless redundant for the determination of the equilibrium; that is, the equilibrium has a representation without reference to the equations that determine the yield curve. But this is not to say that long-term nominal rates are not central for the transmission of monetary policy, nor that they do not contain important information.

As emphasised by Rotemberg and Woodford [1997], in sticky-price models it is the *ex-ante* long-term real interest rate that matters for aggregate demand. In the small open economy version of the model, it also happens to be the *ex-ante* long-term real interest rate that matters for aggregate demand, although the economy's openness alters the relevant measure of the long-term real rate as well as the interest rate sensitivity of aggregate demand. To see this, take the IS curve for the small economy, Equation (8), set all shocks to zero for simplicity, and set the large economy to its steady state. This implies that

$$x_t = E_t x_{t+1} - \sigma_\alpha^{-1} \left(R_{1,t} - E_t \pi_{h,t+1} \right)$$

Advance the equation one period and substitute back the resulting expression to obtain,

$$x_t = -\sigma_\alpha^{-1} \left(R_{1,t} - E_t \pi_{h,t+1} \right) - \sigma_\alpha^{-1} \left(E_t R_{1,t+1} - E_t \pi_{h,t+2} \right) + E_t x_{t+2}$$

Repeating this operation m times and using Equation (9) we may write

$$x_t = -\sigma_\alpha^{-1} m \left(R_{m,t} - \frac{1}{m} E_t \sum_{j=1}^{m} \pi_{h,t+j} \right) \tag{21}$$

since in a stationary equilibrium $E_t x_{t+m}$ is approximately zero for large m. Equation (21) asserts that the current level of the output gap depends on an *ex-ante* long-term real interest rate, measured in domestically-produced goods price inflation, magnified by maturity and scaled by the economy's intertemporal substitution. If the economy were closed, Equation (21) becomes $x_t = -\sigma^{-1} m \left(R_{m,t} - \frac{1}{m} E_t \sum_{j=1}^{m} \pi_{t+j} \right)$, because for $\alpha = 0$, we have that $\sigma_\alpha = \sigma$ and $\pi_{h,t} = \pi_t$. Thus, the sticky-price small open economy model puts long-term nominal interest rates at the very heart of the transmission mechanism in very much the same way the closed economy sticky-price model does: the expectations hypothesis implies that monetary policy influences long-term nominal interest rates and nominal rigidities mean that policy will therefore influence real activity.

2.4 Estimation

For estimation purposes, the discount factor, β, is set at 0.99 and the degree of openness, α, is set at 0.2.[4] At a quarterly frequency, the discount factor corresponds to a steady-state real rate of interest of 4.1 per cent and the degree of openness is consistent with the value of the share of foreign goods in the Australian consumption basket.

The rest of the model's parameters are estimated with Bayesian techniques, as discussed in An and Schorfheide [2006], Lubik and Schorfheide [2005] and Griffoli [2007].[5] We do so in two steps: in the first, we estimate the large economy's parameters, and in the second, we estimate the remaining small economy's parameters, taking the posterior mean values of the common parameters as given from the first step.[6]

Since our focus is on the cross correlations of domestic interest rates with their US counterparts, we take the US to be the large economy. We use quarterly HP-filtered data on real US GDP per capita, US CPI-inflation and a US 3-month nominal interest for the sample period 1983Q1-2007Q2. Table 1 summarises results for this first step of the estimation.

Since the large economy is exogenous to the small economy, we take the smoothed estimates for g_t^* and $E_t y_{t+1}^*$ from the first step of estimation and use these as additional series in the estimation of the small economy's parameters. This differs from much of the relevant literature on small open economies which typically adopts an unrestricted reduced-form VAR process for foreign variables. This reduced-form process may or may not be consistent with the theory at hand. But, to the extent that an arbitrarily imposed reduced-form specification for the dynamics of foreign variables differs from that of the theory, the structural equations for the small economy are invalid. As noted by Justiniano and Preston [2004], the structural equations of the domestic economy depend on the assumption that the large economy is populated with households and firms with identical preferences and technology. Therefore, the assumption of an arbitrary reduced-form process would not generally be consistent with the structural equations of the small economy.

We take the estimated posterior mean parameter values as given from the first step and estimate the remaining small open economy parameters on Australian data. For the small economy we use quarterly HP-filtered data on real GDP per capita, CPI-inflation and a 3-month nominal interest rate for the sample period 1993Q1-2007Q2. [7] Table 2 below sum-

[4]In preliminary attempts to estimate the model, we found that α would invariably tend towards zero for a range of prior distributions. Nimark [2007] follows the similar strategy of calibrating these two parameters.

[5]We used the MATLAB package Dynare for the estimation of the model. Files that replicate our results are available upon request.

[6]As we noted before, because the small economy is open the definition of potential output and that of CPI inflation is different from that of the large economy. So, the choice of prior distributions for the two economies need not be the same.

[7]We take the inflation-targeting period for the estimation of the small economy's parameters because the assumption of a symmetric steady state, which entails relatively similar rates of steady-state inflation, does

Table 1: **Large Economy**

Parameters	Prior Mean	Post. Mean	90 percent C.I.		Prior Dist.	Prior Std.
σ^{-1}	0.50	0.78	[0.46	1.08]	Gamma	0.20
ϕ_1	0.90	0.93	[0.62	1.23]	Gamma	0.20
κ	0.35	0.48	[0.31	0.65]	Gamma	0.20
ρ_r^*	0.90	0.89	[0.87	0.92]	Beta	0.02
α_π^*	0.25	0.35	[0.19	0.50]	Normal	0.10
α_x^*	0.25	0.36	[0.23	0.49]	Normal	0.10
ρ_g^*	0.90	0.89	[0.86	0.92]	Beta	0.02
ρ_a^*	0.90	0.92	[0.90	0.95]	Beta	0.02
Standard Deviations						
$\sigma_{\varepsilon_a^*}.$	0.008	0.006	[0.004	0.008]	Inv. Gamma	∞
$\sigma_{\varepsilon_g^*}.$	0.015	0.016	[0.012	0.020]	Inv. Gamma	∞
$\sigma_{\varepsilon_r^*}.$	0.003	0.001	[0.001	0.002]	Inv. Gamma	∞

Table 2: **Small Economy**

Parameters	Prior Mean	Post. Mean	90 percent C.I.		Prior Dist.	Prior Std.
ω	2.00	2.25	[1.42	3.04]	Gamma	0.50
ρ_r	0.85	0.84	[0.81	0.88]	Beta	0.02
α_π	0.60	0.76	[0.63	0.89]	Normal	0.10
α_x	0.10	0.03	[-0.09	0.14]	Normal	0.10
ρ_a	0.90	0.91	[0.88	0.93]	Beta	0.02
ρ_g	0.90	0.88	[0.85	0.91]	Beta	0.02
Standard Deviations						
σ_{ε_a}	0.007	0.008	[0.007	0.009]	Inv. Gamma	∞
σ_{ε_g}	0.007	0.007	[0.005	0.008]	Inv. Gamma	∞
σ_{ε_r}	0.001	0.002	[0.001	0.002]	Inv. Gamma	∞

marises results for this second step of the estimation.

2.5 The Dynamics of the Yield Curve

Figure 2 shows the impulse responses of the yield curve for two different parameterisations of the persistence of the domestic technology shock: $\rho_a = 0.9$ and $\rho_a = 0.65$.[8] The less persistent the shock, the less the impact on longer-term rates. The intuition for this is straightforward: more persistent shocks have a larger effect on long-term rates on impact

not appear valid before then.

[8] All other parameters are set at the estimated posterior mean values of Tables 1 and 2.

than do less persistent shocks because they induce more persistent expected movements of the short-term rate. Perhaps less obvious is that a less persistent shock moves shorter-term rates on impact by more than a more persistent shock would. Notice that as a result of consumption smoothing and rational expectations, the coefficient on the technology shock in Equation (8) is $(1-\rho_a)$. And so, in the extreme case in which $\rho_a = 0$, the contemporaneous impact of the shock would be the highest possible.

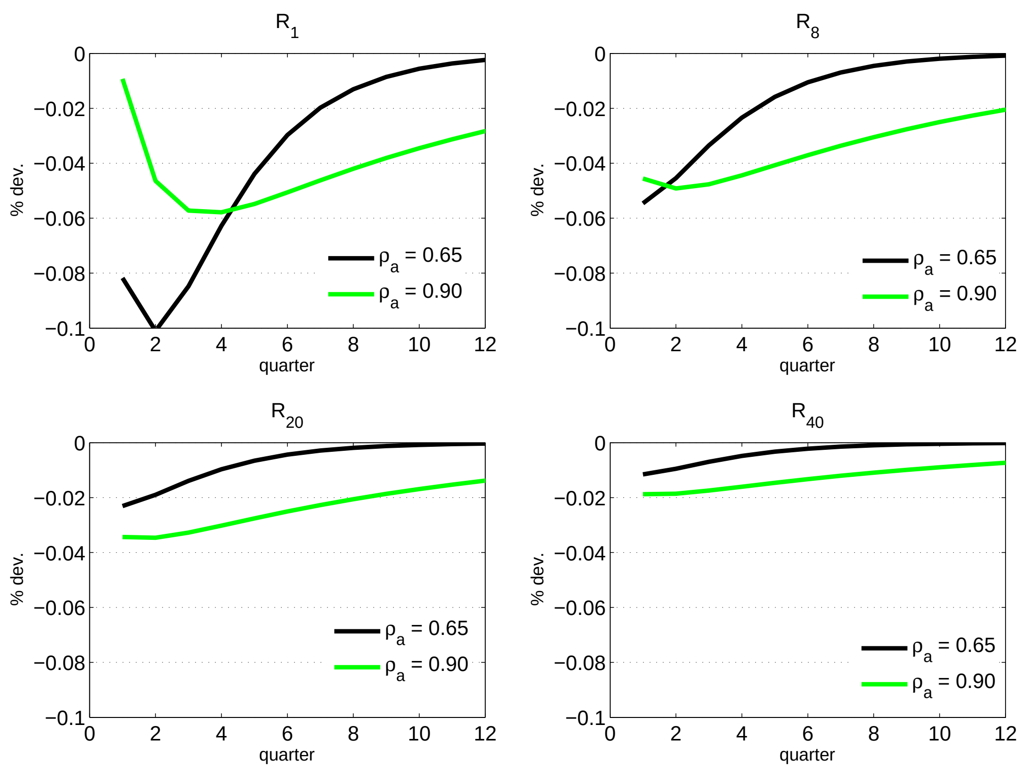

Figure 2: Impulse response of the small country's yield curve: more versus less persistent technology shocks.

The key to understanding the model's ability to reproduce the pattern of correlations of Figure 1 is that less persistent domestic shocks produce a source of variation in the yield curve that is relatively stronger at the short-end of the yield curve than at the long-end of the yield curve. Other things equal, if the persistence of a domestic shock decreases, the correlation between the short-term rates of the two economies would decrease, while the correlation between their longer-term rates would increase. The correlation at the long-end increases because, if the persistence of domestic shocks decreases, foreign shocks – the cause of variability of foreign rates – become a relatively more important source of variation for domestic long-term rates. Also note that domestic shocks, regardless of their persistence, are a source of variation for the domestic yield curve and not, of course, one for the foreign yield curve.[9] Thus, the larger is the variance of domestic shocks relative to that of foreign shocks, the smaller (in absolute value) is the correlation between domestic and foreign interest rates.

3 Unconditional Moments

Table 3 compares the theoretical standard deviations of output, inflation, the nominal exchange rate and nominal interest rates, all computed at the posterior mean values of the parameters with their empirical counterparts. The model over-estimates the volatility of output and the short-term interest rate and under-estimates the volatility of inflation, the change in the nominal exchange rate and long-term interest rates. The model's inability to capture the variability of interest rates across the yield curve echoes Shiller [1979]'s finding of 'excess volatility'. While the volatility of the short-term interest rate in the model is four times that of the data, the volatility of the 10-year interest rate in the data is three times that of the model.

Table 3: **Standard Deviations**

	Model (posterior mean)	Data (1993:1 - 2007:2)
y_t	0.0178	0.0066
π_t	0.0020	0.0055
Δe_t	0.0116	0.0338
$R_{1,t}$	0.0074	0.0017
$R_{8,t}$	0.0016	0.0020
$R_{20,t}$	0.0010	0.0019
$R_{40,t}$	0.0006	0.0018

Table 4 compares the actual correlations of foreign interest rates, domestic interest rates, output, the change in the nominal exchange rate and CPI inflation with their theoretical counterparts at the estimated posterior mean parameter values of Tables 1 and 2. The model has mixed success in confronting these dimensions of the data. On the one hand, it fails to capture the pro-cyclical behaviour of the yield curve and the correlations between changes in the nominal exchange rate with output, inflation and the short-term interest rate. On the other hand, the signs of the correlations between interest rates of various maturities and currencies, as well as the signs of the correlations between inflation and interest rates are correctly predicted by the model.

[9] We confine our attention to unique rational expectations solutions of this model in which the large economy is exogenous to the small one. See Jääskelä and Kulish [2007] for an analysis of the determinants of size of this model.

Table 4: **Contemporaneous Correlations**

Data (1993q1–2007q2)									
	y_t	π_t	Δe_t	$R_{1,t}$	$R_{20,t}$	$R_{40,t}$	$R^*_{1,t}$	$R^*_{20,t}$	$R^*_{40,t}$
y_t	1.00								
π_t	0.14	1.00							
Δe_t	0.19	-0.01	1.00						
$R_{1,t}$	0.08	0.48	-0.01	1.00					
$R_{20,t}$	0.19	0.36	0.17	0.75	1.00				
$R_{40,t}$	0.16	0.34	0.18	0.69	0.99	1.00			
$R^*_{1,t}$	0.00	0.40	-0.37	0.56	0.34	0.28	1.00		
$R^*_{20,t}$	0.19	0.30	-0.19	0.49	0.70	0.66	0.66	1.00	
$R^*_{40,t}$	0.18	0.25	-0.07	0.46	0.77	0.75	0.48	0.96	1.00
Model (posterior mean)									
	y_t	π_t	Δe_t	$R_{1,t}$	$R_{20,t}$	$R_{40,t}$	$R^*_{1,t}$	$R^*_{20,t}$	$R^*_{40,t}$
y_t	1.00								
π_t	-0.13	1.00							
Δe_t	-0.01	0.66	1.00						
$R_{1,t}$	-0.52	0.24	0.26	1.00					
$R_{20,t}$	-0.65	0.32	0.06	0.93	1.00				
$R_{40,t}$	-0.66	0.32	0.06	0.93	1.00	1.00			
$R^*_{1,t}$	0.07	0.19	0.08	0.65	0.57	0.56	1.00		
$R^*_{20,t}$	0.06	0.26	0.17	0.67	0.59	0.58	0.98	1.00	
$R^*_{40,t}$	0.06	0.26	0.17	0.67	0.59	0.58	0.97	1.00	1.00

The model's benchmark parametrization (at the posterior mean) does not generate the hump-shaped pattern of correlations between interest rates of equivalent maturities shown in Figure 1. In particular, the model over-estimates the correlation at the short-end of the yield curve and under-estimates it at the long-end of the yield curve.

However, a set of plausible parameters values capable of producing the hump-shaped pattern of interest rates correlations of Figure 1 may well exist. In light of the earlier discussion about the role that the persistence of shocks might play in accounting for the moments of Figure 1, we set each of the domestic autoregressive coefficients to the lower bound of their 90 per cent confidence intervals and the foreign autoregressive coefficients to their upper bounds. Then, we adjust the standard deviations of the *iid* disturbances as follows: $\sigma_{\varepsilon_a} = .0095$, $\sigma_{\varepsilon_g} = .013$, $\sigma_{\varepsilon_r} = .004$, $\sigma_{\varepsilon^*_a} = .013$ and $\sigma_{\varepsilon^*_g} = .02$.[10] Notice that this alternative calibration changes only the parameters that govern the exogenous processes. Figure 3 shows the cross correlations for the two parameterisations of the model. The top panel con-

[10]The logic behind these changes is as follows. The larger variances of foreign persistent shocks increases the correlations of all rates but it increases those at the long-end of the yield curve relatively more. The larger variance of the domestic monetary policy shock reduces the correlations at the short-end, while the larger variances of the persistent domestic shocks reduce the correlations in the middle of the term structure.

tains the correlations for the parameters at the posterior mean values of Tables 1 and 2 and the bottom panel the correlations for this alternative parametrisation.

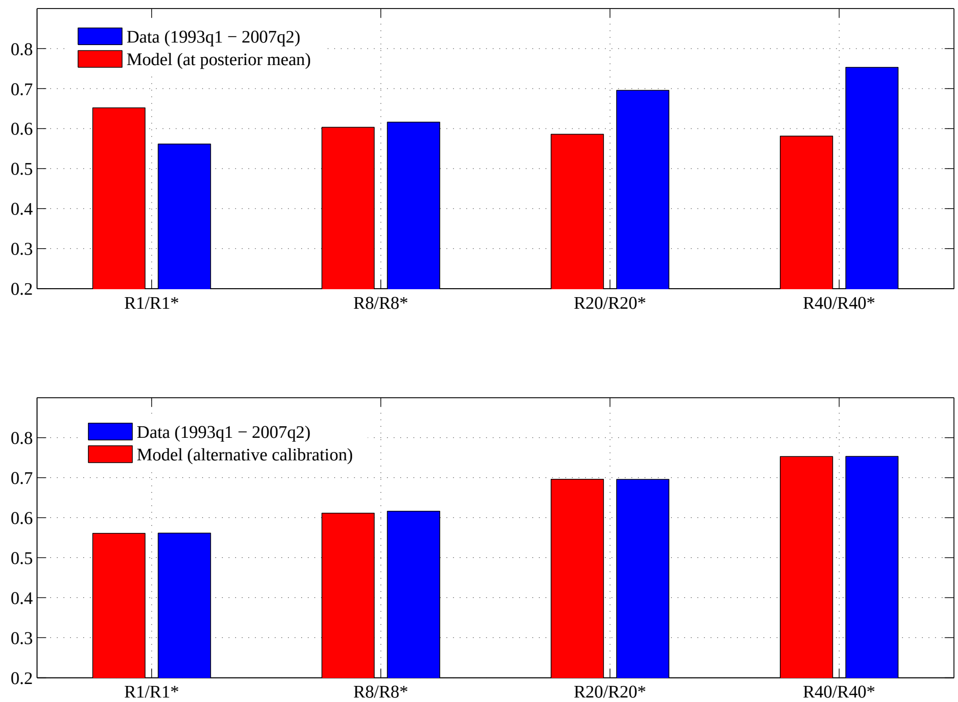

Figure 3: Cross Correlations with US interest rates.

For these alternative plausible parameter values the model matches remarkably well the pattern of interest rate correlations at different points in the yield curve. It is worth emphasizing that none of the deep parameters were changed, but only those that govern the exogenous processes. Moreover, the theory of interest rate determination that holds for these different parameter values – the expectations hypothesis – is, of course, unchanged.

The actual pattern of interest rate correlations does not necessarily constitute evidence of interest rate being 'determined abroad'. Interest rates in the model are always linked to the expected future path of the domestic short-term nominal interest rate. The subtle but important difference is this: in the presence of domestic disturbances which are relatively less persistent, the short-term interest rate adjusts, but this variation does not translate contemporaneously into large variations in longer-term interest rates if the shock's persistence is relatively low. As our impulse response analysis reveals, in the initial periods after a less persistent shock, theory implies that the correlation between short-term interest rates and

Table 5: **Contemporaneous Correlations**

	Model (alternative calibration)								
	y_t	π_t	Δe_t	$R_{1,t}$	$R_{20,t}$	$R_{40,t}$	$R^*_{1,t}$	$R^*_{20,t}$	$R^*_{40,t}$
y_t	1.00								
π_t	0.06	1.00							
Δe_t	-0.03	0.59	1.00						
$R_{1,t}$	-0.43	-0.04	0.14	1.00					
$R_{20,t}$	-0.41	0.19	0.04	0.92	1.00				
$R_{40,t}$	-0.37	0.19	0.05	0.90	1.00	1.00			
$R^*_{1,t}$	0.15	0.11	0.04	0.56	0.68	0.73	1.00		
$R^*_{20,t}$	0.13	0.15	0.12	0.58	0.70	0.75	0.98	1.00	
$R^*_{40,t}$	0.13	0.15	0.12	0.58	0.69	0.75	0.98	1.00	1.00

long-term interest rates is relatively low. This fact, however, has nothing to do with the failure of the expectations hypothesis.

Table 5 shows the theoretical moments associated with this alternative calibration. The most notable differences with respect to the bottom panel of Table 4 appear between the correlations of inflation with interest rates and of inflation with output. This parameterisation makes domestic interest rates less counter-cyclical, inflation and output become positively correlated as in the data, but the correlations between inflation and the yield curve, which are reasonably well matched to the data in the bottom panel of Table 4, deteriorate.

4 Structural VARs

There is a large literature where monetary policy shocks have been identified within structural vector autoregression models (see for example Bernanke [1986], Christiano et al. [1996], Dungey and Pagan [2000], Sims [1986] and Sims and Zha [2006]). To complement our analysis we estimate an SVAR model for Australia which includes data on bond yields.

4.1 Identification

Our SVAR is as follows: $\mathbf{Z}^*_t$ is an $n^* \times 1$ vector of foreign macroeconomic variables at time t, $\mathbf{Z}_t$ is an $n \times 1$ vector of domestic macroeconomic variables and $\mathbf{R}_t$ is a $r \times 1$ vector of bond yields of different maturities. We estimate the following structural VAR:

$$\begin{bmatrix} \mathbf{a} & \mathbf{0} & \mathbf{0} \\ \mathbf{d} & \mathbf{e} & \mathbf{0} \\ \mathbf{g} & \mathbf{h} & \mathbf{j} \end{bmatrix} \begin{bmatrix} \mathbf{Z}^*_t \\ \mathbf{Z}_t \\ \mathbf{R}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}(L) & \mathbf{0} & \mathbf{0} \\ \mathbf{D}(L) & \mathbf{E}(L) & \mathbf{0} \\ \mathbf{G}(L) & \mathbf{H}(L) & \mathbf{J}(L) \end{bmatrix} \begin{bmatrix} \mathbf{Z}^*_t \\ \mathbf{Z}_t \\ \mathbf{R}_t \end{bmatrix} + \begin{bmatrix} \mathbf{u}^*_t \\ \mathbf{u}_t \\ \mathbf{u}_{R,t} \end{bmatrix} \tag{22}$$

where: **a**, **e** and **j** are square matrices with 1s on the diagonal; **d**, **g** and **h** are matrices of conformable dimensions and L is the lag operator. The process $\begin{bmatrix} \mathbf{u}_t^* & \mathbf{u}_t & \mathbf{u}_{R,t} \end{bmatrix}'$ is an iid vector of mutually and serially uncorrelated structural shocks. Our SVAR contains eleven variables. The vector of foreign variables is given by $\mathbf{Z}^* \equiv (PCOM, Y^*, P^*, FF^*)$, where $PCOM$ is an index of commodity prices, Y^* is US real per capita GDP, P^* is the US consumer price index and FF^* is a US short term interest rate.[11] The vector of domestic macroeconomic data is given by $\mathbf{Z} \equiv (Y, P, CR, e)$, where Y is Australian real GDP per capita, P is the Australian consumer price index excluding interest and taxes, CR is an Australian short term interest rate and e is the nominal Australian dollar-US dollar exchange rate. The Australian long-term yields are the 2-year, 5-year and 10-year Australian Commonwealth Government Securities (CGS) bond yields. The data are quarterly, beginning in March 1985 through to June 2007. All variables are in log-levels except for interest rates, which are expressed in percentage point terms on a quarterly basis. The SVAR incorporates 3 lags.

We identify monetary policy shocks by placing restrictions on the contemporaneous relationships between the variables using the recursive identification scheme of Christiano et al. [1996].[12] This identification assumes that the monetary authority observes current prices and economic activity when setting the short-term interest rate; prices and activity measures, however, only respond to monetary policy with a one-quarter lag. These assumptions yield a domestic monetary policy reaction function of the form:

$$CR_t = \mathbf{D}(L)\mathbf{Z}_t^* + \mathbf{E}(L)\mathbf{Z}_t - \mathbf{d}\mathbf{Z}_t^* - e_{1,3}GDP_t - e_{2,3}\pi_t + u_{3,t} \tag{23}$$

We also apply restrictions to ensure that Australia is a small open economy. So while foreign variables can affect domestic variables contemporaneously, domestic variables cannot affect foreign variables, neither contemporaneously nor with a lag. As in Evans and Marshall [1998], we assume that long-term interest rates cannot affect macroeconomic variables either contemporaneously or with a lag.[13] This assumption has the implication of preserving the dynamic behaviour of the vectors of macroeconomic variables and it is consistent with the theoretical idea that long-term interest rates play a redundant role in the determination of the equilibrium.

[11] The three-month bond yield is used as the monetary policy instrument. As we estimate the SVAR at a quarterly frequency, this rate is arguably a better match for the theoretical short rate than the conventional overnight rate – the Federal Funds rate in the US and the cash rate in Australia. The impulse responses of the SVAR are quantitatively similar when alternative short-rates are used.

[12] Although the recursiveness assumption is not sufficient to identify all the elements of the contemporaneous matrix, it is sufficient to identify monetary policy shocks. As Christiano et al. [1996] discuss, there is a set of contemporaneous matrices that satisfy the recursiveness assumption and one element of this set has the property of being lower triangular with positive terms on the main diagonal. By adopting the normalization of selecting this lower triangular matrix, the dynamic responses of the variables are invariant to the ordering of variables that are not influenced contemporaneously by the policy shock as well as to the ordering of the variables that are influenced contemporaneously. The dynamic effects to the non-policy shocks, however, are sensitive to the ordering. Without further identifying restrictions, the non-monetary policy shocks and their implied dynamic responses simply reflect normalisations undertaken for convenience.

[13] We restrict the lag structure so that long-term rates do not affect other long-term rates. This is equivalent to the strategy of estimating the SVARs sequentially for each bond yield adopted by Evans and Marshall [1998].

4.2 Impulse Responses

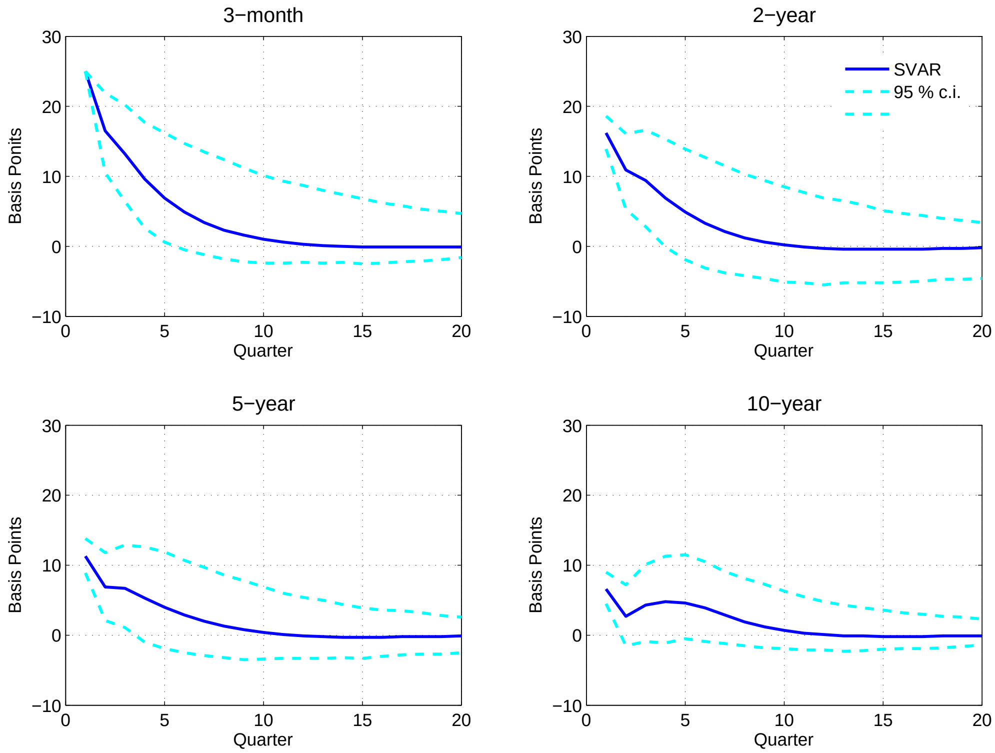

Figure 4: Response of Bond Yields to a Domestic Monetary Policy Shock.

Figure (4) shows the responses of bond yields to a monetary policy shock, $u_{3,t}$ in the SVAR. Dashed lines represent 95 per cent confidence intervals, calculated using the bootstrap procedure from Kilian [1998]. The responses are normalised so that a shock causes a 25 basis point increase in the domestic short-term rate. Three aspects of the responses are notable. First, the monetary policy shock raises interest rates across the yield curve; the initial responses are statistically significant, even at the long-end of the yield curve. Second, the magnitude of the response declines as the maturity of the bond yield increases. And third, recalling that the monetary policy shock in the SVAR is serially uncorrelated, and so has no persistence, the responses corresponds neatly to the implication of our theoretical model that domestic shocks with little persistence have a larger effect on short-term yields than they do on long-term yields.

5 Conclusion

Recently, long-term nominal interest rates in inflation-targeting small open economies, like Australia, Canada, New Zealand, Norway, Sweden and the United Kingdom, have moved very closely with those of the United States. This observation has led many to the view that the long-end of the domestic yield curve is determined abroad, and with it, to a concern that the transmission mechanism of small open inflation-targeting economies may be weaker than it otherwise might be.

In this paper we have set up a fully micro-founded two-block small open economy model and incorporated long-term interest rates to study the co-movement of interest rates of different currencies. We have shown that the reduced-form correlations at the short- and long-end of the domestic and foreign yield curves can be explained by the model, in which the expectations hypothesis and uncovered interest rate parity hold. In particular, if foreign shocks are more persistent than domestic shocks then it makes sense that long-term nominal interest rates in the small and large economies are highly correlated. Hence, these correlations are not evidence that long-term interest rates are determined abroad, nor are they evidence of a weaker transmission mechanism of monetary policy. Another way of concluding is to say that the reduced-form correlations that the theory generates depend on the properties of the exogenous processes.

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