ESTIMATING THE CURVATURE OF THE PHILLIPS CURVE: EVIDENCE FROM AGGREGATE DATA[1,2]

The RBA's Phillips curve models assume a negative convex relationship between inflation and unemployment. The degree of curvature in the Phillips curve could have material effects on our understanding of inflationary pressures in the economy, especially if the Australian unemployment rate falls as low as in other economies like the US, UK and New Zealand. To test whether our current method best fits the data, I estimate an encompassing Phillips curve model that nests both the linear model and the current Bank specification as special cases. The optimal estimates are close to the Bank's current specification, although some versions have differing curvature at very low unemployment rates.

The Bank's nonlinear Phillips curve

The current nonlinear specification comes from Debelle and Vickery (1997). The linear specification in that paper had a better fit than the nonlinear specification, but led to a wildly volatile NAIRU (see Graph 1). When the variance of the NAIRU in the linear model was restricted to match the nonlinear model, the nonlinear model fit better. The functional form of the nonlinearity is shown below in a simplified Phillips curve (PC):

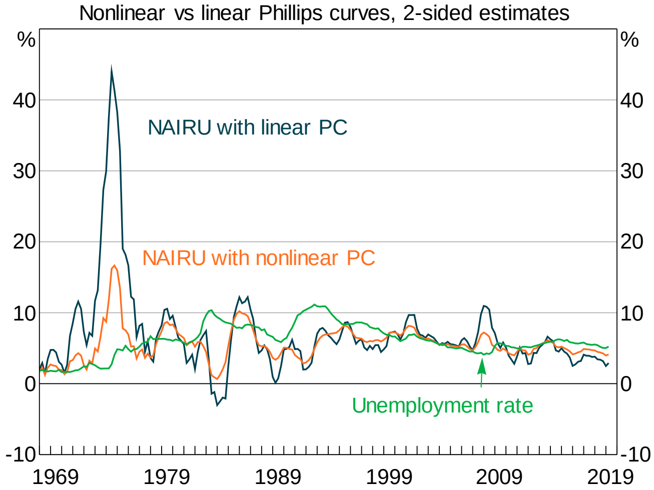
$$\pi_t = \phi\pi_t^e + (1-\phi)\pi_{t-1} - \gamma\frac{(U_t - U_t^*)}{U_t} - \beta\frac{\Delta U_{t-1}}{U_t} + \epsilon_t$$

This specification determines the curvature of the PC a priori, while the γ parameter determines the slope. The functional form was chosen based on tractability and theoretical concerns, with inflation approaching infinity as the unemployment rates approaches zero.[3]

Graph 1

NAIRU Estimates Using Debelle & Vickery (1997)*

Nonlinear vs linear Phillips curves, 2-sided estimates



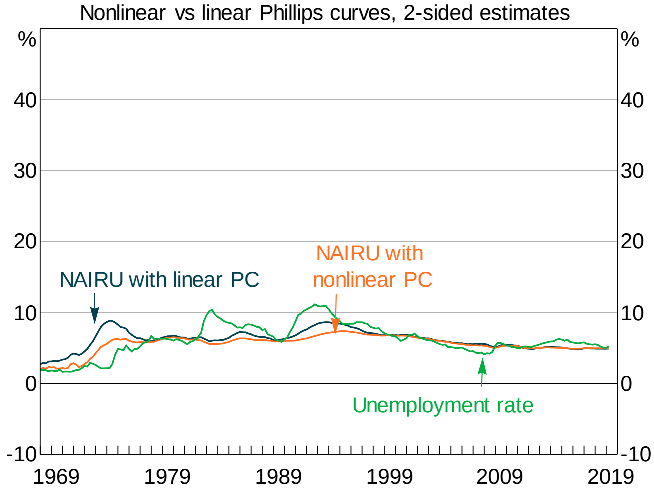
* Estimated using only inflation data

Sources: ABS; RBA

Graph 2

NAIRU Estimates Using Cusbert (2016)*

Nonlinear vs linear Phillips curves, 2-sided estimates



* Estimated using inflation and unit labour costs data

Sources: ABS; RBA

Is this the right form of nonlinearity?

More recent specifications of the NAIRU model use a richer lag structure, include imported supply shocks, and use inflation and ULC data simultaneously. These specifications give NAIRU estimates that are similar for both the linear and nonlinear versions, and that are less volatile than the NAIRUs resulting from the Debelle and Vickery specification (Graph 2). If there is little reason to prefer one model over the other based on the resulting NAIRU, I can use the fit of the models to select between these two options, and a wider class of models.

The current version of the NAIRU model in Read (2018) allows for structural breaks in parameter values. I want to identify changes in slope from the level of the unemployment rate, which may be correlated with

1 Thanks to Anthony Brassil for helpful discussions on the nonlinear specifications, as well as Adam Gorajek and Rochelle Guttmann.

2 See Bishop and Greenland (2019) for micro-data evidence based on variation across local labour markets.

3 See Box A in Bishop and Greenland (2019) for a broader discussion of the existing approach.

structural breaks. To simplify this issue I revert to the constant parameter version documented in Cusbert (2016). I can rewrite the simplified PC with an extra parameter, k, that determines the degree of curvature:

$$\pi_t = \phi\pi_t^e + (1-\phi)\pi_{t-1} - \gamma(U_t - U_t^*)(U_t)^k - \beta\Delta U_{t-1}(U_t)^k + \epsilon_t \tag{1}$$

This nests the linear and Debelle-Vickery cases as $k = 0$ and $k = -1$, respectively. I estimate the model including the curvature parameter to find the maximum likelihood estimates over the full class. Equation 1 can be written in terms of inflation, as shown, or unit labour costs (ULCs), and I examine the implications of choosing the same or different curvature parameters for both when estimating both equations as a system. Restricting the curvature to be the same for the inflation and ULC PCs gives an estimate of $\hat{k} = -0.99$, which is almost identical to the original nonlinear specification. Allowing different degrees of curvature in the inflation and ULC PCs, or removing the speed limit ΔU_t term gives somewhat different results. However, a null hypothesis of $k = -1$ is not rejected in any case (Table 1). Graph 3 and Graph 4 show the degree of curvature in the current nonlinear PC, compared to a linear version and one with greater convexity.

Table 1: Maximum likelihood estimates of Phillips Curvature

Variations on Equation 1, with specification from Cusbert (2016)

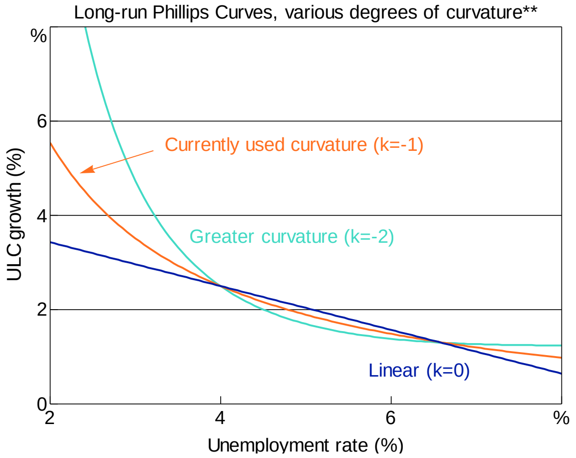
	Inflation Curvature (k) (Standard error)		**ULC curvature (k)** (Standard error)	**Test k=-1** Wald test P value	**Log Likelihood**
Same curvature, with speed limit		-0.99 (0.18)		0.97	-385
Same curvature, no speed limit		-1.2 (0.18)		0.21	-390
Different curvature, with speed limit	-1.4 (0.34)		-0.91 (0.21)	0.39	-384
Different curvature, no speed limit	-1.4 (0.26)		-1.18 (0.26)	0.29	-390

Sources: ABS; RBA

Graph 3

Unit Labour Costs Phillips Curves*

Long-run Phillips Curves, various degrees of curvature**



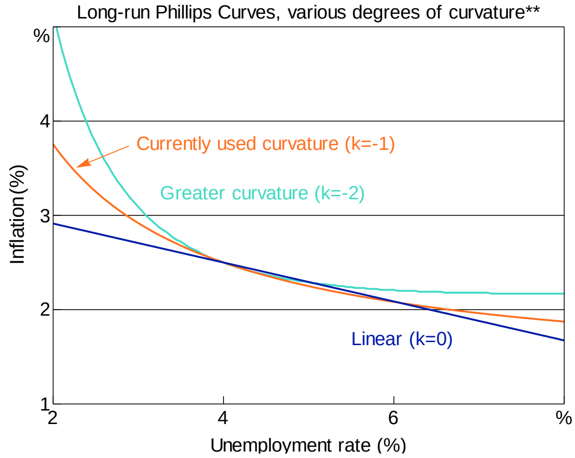
* NAIRU assumed to be 4.5% and inflation expectations 2.5%

** Curves generated by estimating Equation 1 with varying fixed values for k

Graph 4

Inflation Phillips Curves*

Long-run Phillips Curves, various degrees of curvature**



* NAIRU assumed to be 4.5% and inflation expectations 2.5%

** Curves generated by estimating Equation 1 with varying fixed values for k

A broader class of nonlinear Phillips curves

The estimated nonlinear PC in Equation 1 can be broadened by adding another parameter that shifts the PC left or right. Rather than restricting inflation to approach infinity as the unemployment rate approaches zero, the location of the asymptote (denoted λ) is estimated below. I omit the speed limit terms in these estimations for simplicity.

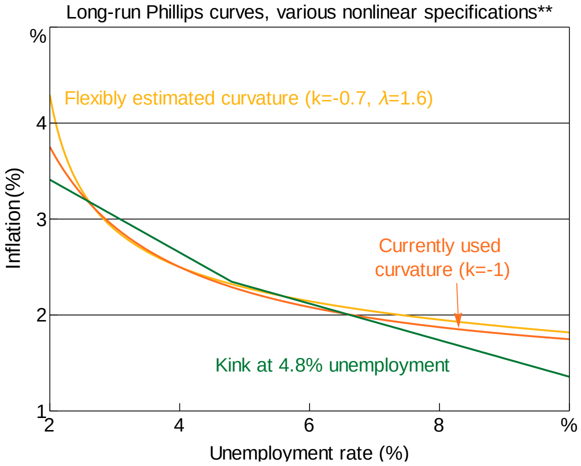
$$\pi_t = \phi\pi_t^e + (1-\phi)\pi_{t-1} - \gamma(U_t - U_t^*)(U_t - \lambda)^k + \epsilon_t \tag{2}$$

The PC estimate has a less convex exponent k, but a positive value for the asymptote (Table 2). Taken literally, this means inflation would asymptote to infinity as the unemployment rate approaches 1.6 per cent. In practice this is below the range of the unemployment rates used to estimate the model, so it can be interpreted as very convex curvature at low unemployment rates. Graph 5 compares this PC with the current version. The two curves only diverge noticeably where the unemployment rate is below 3 per cent.

Graph 5

Inflation Phillips Curves*

Long-run Phillips curves, various nonlinear specifications**



* NAIRU assumed to be 4.5% and inflation expectations 2.5%

** Estimated from equation 2

Table 2: Maximum likelihood estimates of Phillips curvature

Equation 2, with full specification from Cusbert (2016), no speed limit

	Unemployment rate asymptote (λ) (Standard error)	PC curvature (k) (Standard error)	Test k=-1 Wald test P value	Log Likelihood
Same curvature for π and ULC, no speed limit	1.6 (0.11)	-0.70 (0.13)	0.03	-387

Sources: ABS; RBA

Kinked linear Phillips curves

The Bank's existing PC is close to optimal within one class of models, but this is not exhaustive. Donayre and Panovska (2016) estimate kinked linear PCs for the United States, although they use exogenous CBO estimates of the NAIRU. This is less desirable because the estimation method of the NAIRU is likely to influence the nonlinearity detected. I estimated a kinked PC jointly with the NAIRU as follows.

$$\pi_t = \begin{cases} \phi\pi_t^e + (1-\phi)\pi_{t-1} - \gamma^{High}(U_t - U_t^*) & if\ \ U_t > U^{Kink} \\ \phi\pi_t^e + (1-\phi)\pi_{t-1} - \gamma^{Low}(U_t - U^{Kink}) - \gamma^{High}(U^{Kink} - U_t^*) & if\ U_t \leq U^{Kink} \end{cases} \quad (3)$$

The value of the threshold can be varied to search for the best fitting model. The likelihood is maximised with the kink at 4.8 per cent unemployment (this Phillips curve is shown in Graph 5). At lower levels for the kink, the estimated PC slope below the threshold is steeper (Graph 6). This is consistent with convex curvature of the nonlinear PC.

The more flexible nonlinear estimations indicate a high degree of curvature as the unemployment rate gets very low. A single kink may not be able to capture such curvature so I estimate PCs with two kinks by extending Equation 3 to include two thresholds. However, locating kinks at very low levels of the unemployment rate makes the estimates very imprecise (details in Appendix A). All these methods assume the PC is nonlinear in the unemployment rate, rather than in the unemployment gap (see Appendix).

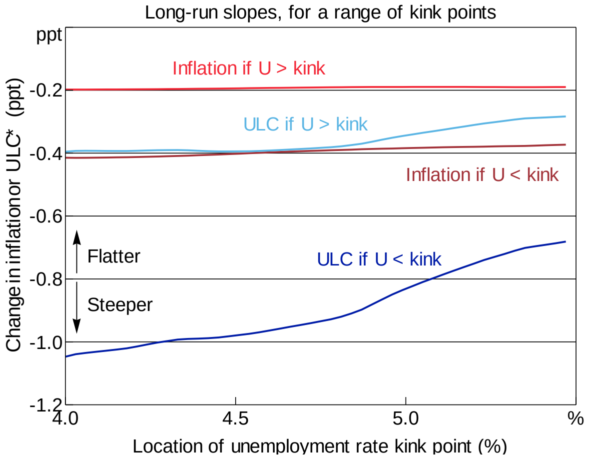
All estimated Phillips curves are convex, but some curves are more convex than others

The current curvature of the Phillips curve closely matches the data-driven estimate within a class of nonlinear curves. Adding one or more kinks to a linear PC can give either more convex or less convex estimates, but the slope is poorly identified at low unemployment rates. Graph 7 shows how the slopes of different PCs have varied over time due to variation in the level of the unemployment rate and the NAIRU. The kinked linear trend captures the steepening at low unemployment rates, but not the flattening at high unemployment rates. The current specification and the more flexible nonlinear specification give similar slopes over the past 40 years, but would diverge at much lower unemployment rates. The current specification of the curvature of the Phillips curve remains appropriate, but should be viewed with caution if the unemployment rate gets very low.

Graph 6

Slope of Kinked Phillips Curves

Long-run slopes, for a range of kink points

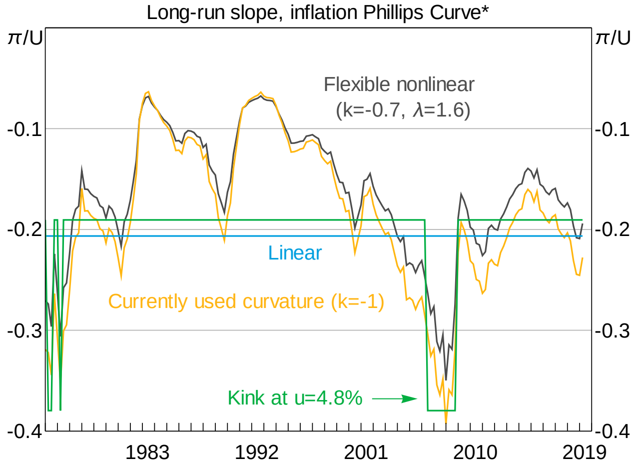


* Per percentage point change in the unemployment rate

Graph 7

Estimated Slopes of Phillips Curves Over Time

Long-run slope, inflation Phillips Curve*



* Unit labour curve slopes are the same shape but scaled up

Sources: ABS; RBA

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References

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Read (2018) NAIRoom for Improvement? Accounting for Structural Breaks in EA's NAIRU Estimation, EC Note

Appendix A – Double Kink Phillips Curves

Rather than optimising the kink locations I estimate three options with the kinks at the 5th and 95th, 10th and 90th, and 33rd and 67th percentiles of the unemployment distributions. The slope estimates with low kinks have high standard errors and are unstable (Table 3, Graph 8 and Graph 9).

Table 3: Linear Phillips curves with two kinks

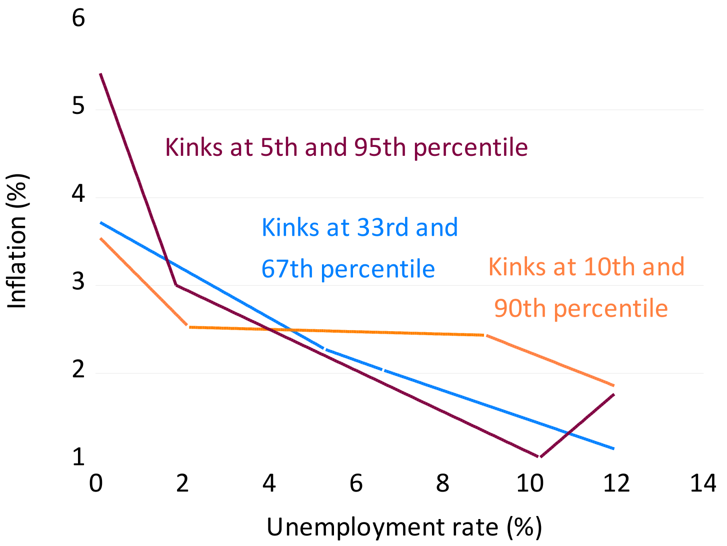
No speed limit; short-run slope coefficients shown(a)

Kink determination…	33rd and 67th percentile	10th and 90th percentile	5th and 95th percentile
Lower kink ($\underline{U}$)	5.3%	2.1%	1.8%
Higher kink ($\overline{U}$)	6.6%	9.0%	10.2%
π slope if $U_t < \underline{U}$	-0.13 (0.04)	-0.14 (0.17)	-0.62 (0.69)
π slope if $\underline{U} < U_t < \overline{U}$	-0.09 (0.07)	-0.004 (0.01)	-0.11 (0.026)
π slope if $U_t > \overline{U}$	-0.08 (0.04)	-0.06 (0.03)	0.19 (0.44)
ULC slope if $U_t < \underline{U}$	-0.63 (0.18)	0.16 (1.10)	-7.84 (3.29)
ULC slope if $\underline{U} < U_t < \overline{U}$	-0.29 (0.25)	-0.44 (0.15)	-0.47 (0.10)
ULC slope if $U_t > \overline{U}$	-0.40 (0.17)	-0.24 (0.31)	0.17 (1.49)

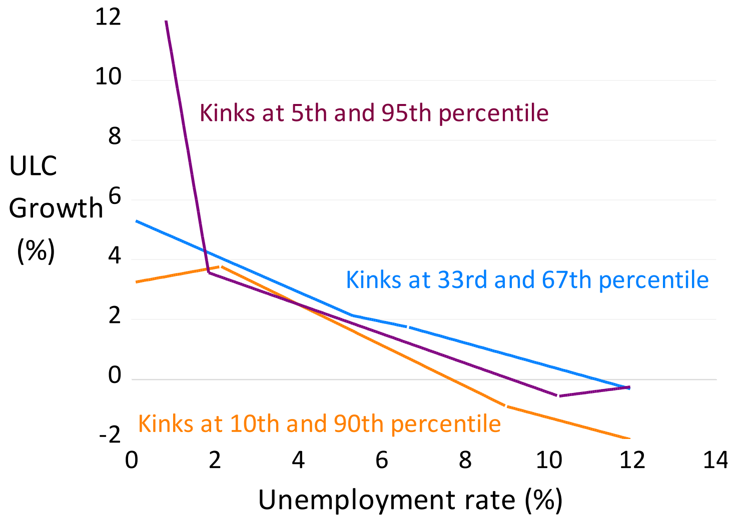
(a) Standard errors of slope coefficients in parentheses

Sources: ABS; RBA

Graph 8



Graph 9



Appendix B - What about nonlinearity in the unemployment gap?

All of the PCs in this note are nonlinear in the unemployment rate rather than the unemployment gap. It is not clear to me whether the nonlinearity should be with respect to the unemployment rate or the unemployment gap. If the NAIRU was 10 per cent rather than 5 per cent, would we expect to see the convexity kick in at a different unemployment rate? The difficulty with this approach is that it would require a nonlinear Kalman filter to estimate jointly with the NAIRU.

An intermediate solution for future work could be a kinked linear PC in the unemployment gap that is estimated through a conditionally linear Kalman filter using Markov Chain Monte Carlo estimation. The PC would be identical to Equation 3, but the threshold would be specified in terms of the gap rather than the unemployment rate.