RESERVE BANK
OF AUSTRALIA

Research Discussion Paper

Issues in Estimating New-Keynesian Phillips Curves in the Presence of Unknown Structural Change

Mariano Kulish and Adrian Pagan

RDP 2013-11

ISSN 1320-7229 (Print)

ISSN 1448-5109 (Online)

Issues in Estimating New-Keynesian Phillips Curves in the Presence of Unknown Structural Change

Mariano Kulish* and Adrian Pagan**

Research Discussion Paper
2013-11

September 2013

*University of New South Wales
**University of Sydney

We thank Adam Cagliarini, Alex Heath and Christopher Kent for useful suggestions. We also thank Sophocles Mavroeidis for comments on an earlier draft. Responsibility for any remaining errors rests with us. Much of this work was done when Mariano Kulish was an RBA employee. The views expressed here are those of the authors and do not necessarily reflect those of the Reserve Bank of Australia.

Authors: m.kulish at domain unsw.edu.au and adrian.pagan at domain sydney.edu.au

Media Office: rbainfo@rba.gov.au

Abstract

Structural change has been conjectured to lead to an upward bias in the estimated forward expectations coefficient in New-Keynesian Phillips curves. We present a simple New-Keynesian model that enables us to assess this proposition. In particular, we investigate the issue of upward bias in the estimated coefficients of the expectations variable in the New-Keynesian Phillips curve based on a model where we can see what causes the structural breaks and how to control for them. We find that structural breaks in the means of the series can often change the properties of instruments a great deal, and may well be a bigger source of small-sample bias than that due to specification error. Moreover, we also find that the direction of the specification bias is not predictable. It is necessary to check for weak instruments before deciding that the magnitude of any estimator bias reflects specification errors coming from structural change.

JEL Classification Numbers: C13, C32, C63, E52
Keywords: expectations, structural change, regime change, weak instruments, IV estimation, Phillips curves

Table of Contents

Issues in Estimating New-Keynesian Phillips Curves in the Presence of Unknown Structural Change

Mariano Kulish and Adrian Pagan

1. Introduction

Many papers that estimate models with forward-looking expectations report that the magnitude of the coefficient attached to the forward expectations term is very large compared with that attached to the term which captures past dynamics. This has often been regarded as implausible, leading to the conjecture that the estimator of the former is biased upwards. Exactly why there should be an upward bias is less clear. One possibility is that weak instruments can result in an estimator bias in small samples (e.g. Mavroeidis 2004), although there is no reason to think that it is an upwards bias. Another possibility is specification error in the structural equation containing the expectations. While there is little one can say about this in general, as the nature of the specification error will be crucial, a specific argument has been that the bias could be due to changes in the means of the variables entering the structural equation. Russell *et al* (2010) give this explanation. Using Bai-Perron tests they find that there were eight breaks in the mean of the inflation rate in the United States over 1960–2007. Assuming that the timing of these breaks coincides with changes in the intercept of the New-Keynesian Phillips curve (NKPC), they then augment the NKPC with dummy variables to capture intercept breaks. Re-estimating with such dummies, they show that the coefficient of the expectations term is greatly reduced. This leads them to conclude that 'Once the shifts in the mean rate of inflation have been accounted for in the estimation of the United States Phillips curves we find that ... there is no significant role for expected inflation ... in the NK and hybrid models of inflation' (p 5).

Castle *et al* (2010) provide an explanation of what Russell *et al* (2010) found. It revolves around the fact that a standard way of estimating an equation with forward-looking expectations, like the NKPC, involves replacing the expectations term with future inflation and then applying an instrumental variables estimator to the resulting equation in observable variables. If the intercept breaks are unaccounted for when estimating the NKPC, a specification error exists and this will be correlated with future values of inflation. Thus the argument in

Castle *et al* (2010) is that the explanatory power of the future observable variables is due to the breaks and not to forward-looking expectations. Technically, one gets an inconsistency in the instrumental variables (IV) estimator of the coefficient of the future expectations term. In some simple experiments they show that this effect can be substantial.

In this paper we investigate the issue of upward bias in the estimated coefficients of the expectations variable based on a model where we can see what causes the breaks and how to control for them. Since many of the applications involve a NKPC, we work with that as the structural equation, embedding it in a standard New-Keynesian (NK) model that also has equations for real marginal cost and an interest rate rule. In each case the agent may know of the breaks but the econometrician is assumed to be ignorant of when they occur. We discuss how to solve this model in the presence of breaks, both when agents know exactly where the breaks occur and also when they get the timing of the break wrong. The method of solution does not depend on the simple model we use for experiments but can be used for any model with forward-looking expectations. The method is set out in detail in Kulish and Pagan (2012).

Because the model is simple, we are able to perform an experiment in which there are breaks in the means of the target inflation rate and real marginal cost that offset one another exactly so as to produce no breaks in the intercept of the NKPC. This experiment just makes the point that breaks in the means of variables such as inflation may not cause breaks in the intercepts of the NKPC. Yet in this experiment we find a bias in some commonly used estimators. Since the equation is correctly specified, due to the intercept being constant (and in this experiment we assume that agents know exactly the timing of the mean shifts), the only reason that this bias can arise is the presence of weak instruments. This leads us to make a distinction between large-sample biases due to specification errors and those arising in smaller samples owing to weak instruments. The latter can sometimes be resolved by using different estimators whereas the former cannot. We find that breaks in the means of the series can often change the properties of instruments a great deal, and may well be a bigger source of small-sample bias than that coming from specification error. Moreover, we also find that the direction of the specification bias is not predictable. With some estimators and breaks the coefficient of the expectations variable is over-estimated, but with others it is under-estimated. This leads to the conclusion that it is necessary to check for factors such as the presence of weak instruments before deciding that the magnitude of any estimator bias reflects specification errors coming from breaking means.

The next section sets out our simple model and distinguishes three estimators of the NKPC. One of these cannot be implemented in practice but gives a useful benchmark. Section 3 then provides a simplified account of how the NK model can be solved in the presence of structural change. Section 4 looks at a range of simulations, beginning with no breaks, moving on to breaks in the reduced form but not the structure, and finishing with breaks in both the structure and reduced form. Breaks in the reduced form correspond to breaks in the means of the inflation rate while breaks in the structure come from changes in the intercept in the NKPC. The estimators introduced in Section 2 are examined, and we assess which one performs best in the presence of breaks. In this section we also investigate the robustness of our results to agents not knowing the timing of breaks precisely when they form expectations.

Section 5 looks at the empirical work in Castle *et al* (2010) on the NKPC in the euro area. They argue that the structural equation requires the addition of a number of indicator variables which, when added, reduce the estimated expectations coefficient by a large amount. Of course indicators are only very short-lived breaks, whereas the breaks we look at in this paper are rather longer-lived. Nevertheless, even short-lived breaks can cause specification bias, and their presence in the reduced form can lead to weak instruments. We assess whether the smaller coefficient on expectations that Castle *et al* note when dummy variables are present is due to weak instruments or to specification issues. Our finding suggests that it is probably a consequence of weak instruments.

2. The Model and Estimators

We will be working with the simple three-equation system that is a basic construct of the NK framework. These equations are:

$$\begin{aligned}
\pi_t &= (1-s)\bar{\pi}_t - \delta\bar{x}_t + \alpha\pi_{t-1} + \gamma\mathbb{E}_t\pi_{t+1} + \delta x_t + \varepsilon_{1t} && (1)\\
x_t &= (1-\rho_1)\bar{x}_t - d(\bar{r} - \bar{\pi}_t) + \rho_1 x_{t-1} + d(r_{t-1} - \pi_{t-1}) + \varepsilon_{2t} && (2)\\
r_t &= (1-\lambda_1)\bar{r} - \lambda_2\bar{x}_t - \lambda_3\bar{\pi}_t + \lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3\pi_t + \varepsilon_{3t} && (3)
\end{aligned}$$

where π_t is inflation, x_t is the log of output (or real marginal cost) and r_t is the nominal interest rate. The parameter $s = \alpha + \gamma$ is the sum of the coefficients on π_{t-1} and $\mathbb{E}_t \pi_{t+1}$ in the Phillips curve. Over-bars mean equilibrium solutions.

The only two equilibrium values that are assumed to change are the inflation target, $\bar{\pi}_t$, and the log of potential output, $\bar{x}_t$. One can re-express the equations above in terms of the deviations of variables from their equilibrium values, which is how the NK system is generally presented. It is known that when $\bar{\pi}_t$ and $\bar{x}_t$ vary one might expect the parameters of the NKPC, Equation (1), to be dependent on these quantities, and so be changing as well – see Ascari (2004) and Cogley and Sbordone (2008). Although our solution algorithm also allows for the parameters to be indexed by time, we want to focus on how changing equilibrium values affect the estimators of the NKPC coefficients, as that has been the focus in the literature.

We can think of two possible IV estimators of the Phillips curve in Equation (1). The first – called the restricted IV estimator (RE) – works with the re-parameterised equation (where the term $\{\gamma(\mathbb{E}_t \pi_{t+1} - \pi_{t+1}) + \varepsilon_{1t}\}$ is the error term):

$$\pi_t - s\pi_{t-1} = c_1 + \gamma(\pi_{t+1} - \pi_{t-1}) + \delta x_t + \{\gamma(\mathbb{E}_t \pi_{t+1} - \pi_{t+1}) + \varepsilon_{1t}\}$$

As there are three coefficients to be estimated, c_1, γ and δ, we need at least three instruments. Variables that are uncorrelated with ε_{1t} provide instruments for the regressors and these are π_{t-1}, x_t, x_{t-1} and r_{t-1}. x_t qualifies as an instrument because only lagged values determine it and ε_{2t} is uncorrelated with ε_{1t}. Appendix A shows that π_{t+1} depends on x_t, π_{t-1} and r_{t-1} but not x_{t-1}. Hence x_{t-1} is not a relevant instrument for π_{t+1}, a fact noted by Pesaran (1987), when d in Equation (2) is zero.

For the second IV estimator, called the unrestricted estimator (UE), the model estimated is:

$$\pi_t = c_1 + \alpha\pi_{t-1} + \gamma\pi_{t+1} + \delta x_t + \{\gamma(\mathbb{E}_t \pi_{t+1} - \pi_{t+1}) + \varepsilon_{1t}\}$$

in this case, π_{t-1}, x_t and r_{t-1} provide exactly the right number of instruments for the three variables π_{t-1}, π_{t+1} and x_t. In much empirical work, a broader set of instruments is assumed without specifying a model, but it is useful to have a small model from which to generate the instruments.

3. A Simple Variant of the Solution Algorithm

Kulish and Pagan (2012) set out an algorithm that can be used to compute solutions to a system in which there are changes in structural parameters that are potentially foreseen. It has more general application than the context we are working with here, so we provide a simplified discussion of its workings to highlight some of its significant features.

The system we consider has the format:

$$z_t = c_t + Az_{t-1} + B\mathbb{E}_t z_{t+1} + \varepsilon_t \tag{4}$$

where z_t is a vector of n (=3) variables (π_t, x_t, r_t), c_t are the intercepts in the equations, and ε_t are identically and independently distributed (iid) $(0, \sigma^2)$ shocks. It is necessary to allow agents to have different beliefs about the timing of any breaks than is the case in reality. To study the effects of mean breaks we assume that agents always know the true shocks and the parameters A and B of the system, and that their beliefs may only be incorrect about c_t. Agents will be assumed to believe that the intercepts of the three equations at time t have the values c_t^a rather than c_t. Then agents solve the system in Equation (4) (with c_t^a replacing c_t) to form their expectations $\mathbb{E}_t^a z_{t+1}$, where the 'a' indexes the agent's beliefs. Those expectations will then determine the actual outcomes of the system, i.e. the observed variables will be consistent with:

$$z_t = c_t + Az_{t-1} + B\mathbb{E}_t^a z_{t+1} + \varepsilon_t$$

We adopt the Binder and Pesaran (1995) solution method to solve the system. Briefly, this involves converting Equation (4) into a purely forward-looking form, solving that, and then recovering the solution to the original system. In the case that agents believe that the economy is described by Equation (4), with $c_t = c_t^a$ the solution method comes down to solving

$$Z_t = (I - BP)^{-1} c_t^a + S\mathbb{E}_t(Z_{t+1}) + Q\varepsilon_t$$

where $Z_t = z_t - Pz_{t-1}$, $S = (I - BP)^{-1}B$, $Q = (I - BP)^{-1}T$ and P is chosen such that $(A + BP^2 - P) = 0$. Because shocks are iid, the solution will be

$$Z_t = \mathbb{E}_t \sum_{j=0}^{\infty} S^j (I - BP)^{-1} c_t^a + Q\varepsilon_t$$

Now we can write the complete solution in one of two forms – either

$$z_t^a = Pz_{t-1} + \sum_{j=0}^{\infty} S^j (I - BP)^{-1} c_t^a + Q\varepsilon_t$$

or

$$z_t^a = Pz_{t-1}^a + \sum_{j=0}^{\infty} S^j (I - BP)^{-1} c_t^a + Q\varepsilon_t$$

In the first case, agents compute the weights to be applied to z_{t-1} from what they believe the data-generating process (DGP) for the macroeconomic variables is, i.e. *actual* past outcomes are used in forming expectations. In the second, they solve for the complete path that they would have expected if the DGP was from the model compatible with their beliefs. The former seems more reasonable as the latter implies that agents persist in believing in a path even when they can observe persistent departures from it. Up until the break there will be no difference, but thereafter, unless the agents know exactly when the break occurs, there will be a systematic difference between z_{t-1} and z_{t-1}^a, since z_t will adjust to the structural changes when they happen. In 'control' jargon the path z_{t-1}^a is often called the 'open loop controller' while that based on z_{t-1} is the 'closed loop controller'. Our algorithm will allow for expectations formation based on either z_{t-1} or z_{t-1}^a.

In the first case, expectations formation would follow

$$\mathbb{E}_t^a z_{t+1} = Pz_t + \sum_{j=1}^{\infty} S^j (I - BP)^{-1} c_t^a$$

and the values of z_t that are generated by the model economy will then be

$$z_t = c_t + Az_{t-1} + B\mathbb{E}_t^a z_{t+1} + \varepsilon_t$$

The solution for the second case is analogous to this.

4. Simulation Experiments

4.1 Parameter Values

In the experiments we report on later, some parameters are assumed to be constant and are thought to apply to the model at a quarterly frequency. These are summarised in Table 1 below.

Table 1: Parameter Values

Parameter	Description	Value
α	NKPC parameter on lagged inflation	0.70
γ	NKPC parameter on expected inflation	0.29
$s = \alpha + \gamma$	Sum of lagged and expected inflation parameter	0.99
δ	Slope of NKPC	0.10
ρ	Persistence of the output gap	0.70
d	Interest rate sensitivity of aggregate demand	–0.10
λ_1	Policy rule parameter: persistence of the interest rate	0.70
λ_2	Policy rule parameter: response to output gap	0.50
λ_3	Policy rule parameter: response to inflation	1.50
$\bar{\pi}$	Inflation target	0.00
$\bar{x}$	Long-run equilibrium of the output gap	0.00

The structural changes we allow for are in either $\bar{\pi}$ or $\bar{x}$. If there are no breaks (Experiment 1) these remain zero throughout. In the remaining experiments there is a break in either the inflation target or potential output. When it is the former, $\bar{\pi}$ becomes 0.02 after 40 per cent of the sample while, if it is the latter, $\bar{x}$ becomes –0.015 at that point in the sample. Experiment 2 is an exception. Here a break in the inflation target occurs as above but we allow potential output to move from zero to $\bar{x} = \frac{(1-s)\bar{\pi}_t}{\delta}$. This choice means that the intercept in the NKPC, c_1, remains at zero, i.e. it does not show any breaks. Experiments 3 and 4 look at changing either the mean inflation rate, $\bar{\pi}$, or the mean of potential output, $\bar{x}$. Finally, Experiments 5 and 6 look at whether our conclusions depend on agents correctly knowing the timing of any breaks. To assess this we consider the case where agents believe that the means $\bar{x}$ and $\bar{\pi}$ remain constant until 60 per cent of the sample is completed, whereas the actual shift is at 40 per cent of the sample. As mentioned in Section 3 there are two ways of representing the decision rules of agents used in forming expectations. One is to use the assumed (incorrect over part of the sample) model to produce weights that can be applied to observables when forming expectations. This is the ‘feedback’ form. The other is for agents to compute the path that they believe the model variables would follow, and then use that when forming expectations.

The parameter values above are standard. s was chosen to be 0.99 owing to the fact that in NKPCs s must equal or exceed the discount rate. In the model of Galí and Gertler (1999), when the discount rate is 0.99, the value of s varies between 0.99 and 0.996 – the variation comes from changing the probability of firms resetting prices, and the fraction of firms setting optimal prices, which can range from zero to one. This suggests that one might assume that $s = 1$ and then estimate a restricted estimator as per the unrestricted estimator but in which the true value of s is replaced by unity. We refer to this estimator as RES. This estimator will be biased in large samples since the error term will be augmented by the term $(1-s)\pi_{t-1}$.

The six experiments were chosen to elucidate many of the issues mentioned in the introduction to the paper. When examining the results our focus will be on two things. First, even if there is no structural change, we can have a small-sample bias in the estimator of γ, simply because of weak instruments.[1] Second, when the intercept does shift, and no allowance is made for that, we have a specification error, which can cause a large-sample bias. One way to distinguish these two effects is to have breaks in means but none in the NKPC intercept, i.e. the breaks are offsetting as in Experiment 2. Then any bias must be due to weak instruments since the equation is correctly specified.

One way to detect weak instruments (infeasible in practice) is to compare the median and the mean of the estimators, since weak instruments mostly show up in the IV estimator not being normally distributed. Another way is to extend the sample and see if the bias goes away. Of course, in the case of the unrestricted estimator we only have one 'free' instrument, so that examining the F-test relating to the coefficient of r_{t-1} in the regression of π_{t+1} against x_t, π_{t-1} and r_{t-1} will give good (feasible) information about weak instruments. The popular rule of thumb that $F > 10$ does in fact provide a reliable guide to those experiments where there are weak instruments. In interpreting the results below, it should be noted that structural change not only affects intercepts in equations like the Phillips curve but can also make instruments stronger or weaker due to changes in the reduced form. As mentioned earlier, we consider breaks that occur at 40 per cent of the sample

1 In fact the bias can exist for very large samples, even though it disappears asymptotically.

size. But when agents have the timing of breaks wrong, they think breaks occur at 60 per cent of the sample. We use percentages as this enables us to increase the sample size to study the 'asymptotic' properties as well as the small-sample ones.

4.2 The Experiments

4.2.1 Experiment 1: no breaks

This experiment keeps all parameters constant while $\bar{x}$ and $\bar{\pi}$ are both set to zero. Two sample sizes are used, $T = 100$ and $T = 1\ 000$, and 500 replications are performed to assess estimator bias. The estimators are the unrestricted estimator (UE), the restricted estimator that uses the correct sum of the forward and backward parameters (RE), and the restricted estimator that just sets this sum to unity (RES). We also estimate the sum of the coefficients with the unrestricted estimator. Table 2 contains the results.

Table 2: Estimators of the NKPC – No Breaks

	***T* = 100**				***T* = 1 000**			
	UE		RE	RES	UE		RE	RES
	γ	$\gamma+\alpha$	γ	γ	γ	$\gamma+\alpha$	γ	γ
Median	0.59	1.05	0.35	0.39	0.36	1.00	0.30	0.35
Mean	0.84	1.10	0.33	0.38	0.41	1.01	0.30	0.35

Note: True value of γ is 0.29

The weak instrument bias shows up strongly in the unrestricted estimators. It is still present even when there are 1 000 observations; by 10 000 observations (results not shown) it has gone. The RES estimator always has a bias since, as we noted earlier, there is a missing regressor $(1-s)\pi_{t-1}$ that enters the error term. When there are 10 000 observations, the mean and median of the RES estimator of γ are 0.34 which seems to be the asymptotic bias. The general impression from Table 2 is that for typical sample sizes (i.e. 25 years of quarterly data) it would be worth using RES rather than UE, despite its large-sample bias. It is close to being normally distributed and the bias is reasonably small. In practice, many people have imposed the restriction that $s = 1$ in estimation, and the bias could be lower than shown here as we have replaced the true $s = 0.99$ with an assumed value of $s = 1$, whereas a discount factor of 0.99 would imply that the true value of s lies much closer to unity than we have assumed in this experiment. Since the bias depends directly on the difference between the assumed and true values it can be much smaller if these are close.

4.2.2 Experiment 2: breaks in means but not in the intercept of the NKPC

Here we allow $\bar{\pi}$ to break, going to 0.02 per quarter from zero, but choose $\bar{x}$ so that the intercept in the Phillips curve remains constant. Hence there is no specification bias in the UE and RE estimators. All other parameters are as in Experiment 1. As the standard deviation of inflation in the model is 0.003 this is an enormous break and is chosen to maximise the effects. Table 3 shows that the UE estimator now has a relatively small bias and is close to being normally distributed.[2] This is due to the instruments becoming far better due to the break in the inflation process. Indeed, this is confirmed when we allow for a much smaller break of 0.005. Then the median and mean of γ in the UE case become 0.39 and –0.08 when $T = 100$, while the RES estimator does not change. By $T = 1\ 000$, however, the UE estimator seems reasonably well behaved. If the evidence is that the instruments are good, it would suggest that one use UE, but otherwise the RES estimator does not seem to be badly biased, even in small samples, and is probably preferred.

Table 3: Estimators of the NKPC – Breaks in Means but Not Intercepts

	***T* = 100**				***T* = 1 000**			
	UE		RE	RES	UE		RE	RES
	γ	$\gamma+\alpha$	γ	γ	γ	$\gamma+\alpha$	γ	γ
Median	0.32	0.99	0.34	0.37	0.30	0.99	0.31	0.34
Mean	0.37	0.98	0.32	0.36	0.30	0.99	0.31	0.36

Note: True value of γ is 0.29

4.2.3 Experiment 3: $\bar{\pi}$ breaks, intercept in NKPC shifts

This experiment is like the preceding one but $\bar{x}$ is assumed not to change. It should be noted that even though the break in $\bar{\pi}$ is large (0.02) this does not lead to a large break in the NKPC intercept, as the latter is $(1-s)\bar{\pi}_t$. Table 4 gives the results. There do seem to be some weak instrument issues with the UE estimator but the striking result is that the bias in γ is *downward*, showing that breaks in the NKPC

2 The distribution of the IV estimator is often non-normal in the presence of weak instruments. As non-normality implies a difference between the mean and median of the estimator a simple way to detect weak instruments (not available in practice) is to compare these two moments.

intercept cannot always be assumed to lead to an upward bias. The restricted estimators are now more biased than before, but RES still seems a reasonable estimator to employ when the alternative is the UE estimator. It is noticeable that $s = \gamma + \alpha$ is quite well estimated here, so the downward bias in the estimator of γ is offset by an upward bias in the estimator of α.

Table 4: Estimators of the NKPC – Break in Inflation Mean

	***T* = 100**				***T* = 1 000**			
	UE		RE	RES	UE		RE	RES
	γ	$\gamma+\alpha$	γ	γ	γ	$\gamma+\alpha$	γ	γ
Median	0.26	0.98	0.34	0.38	0.20	0.97	0.33	0.38
Mean	0.35	0.99	0.34	0.39	0.16	0.97	0.33	0.38

Note: True value of γ is 0.29

4.2.4 Experiment 4: $\bar{x}$ breaks, intercept in NKPC shifts

The size of the break in $\bar{x}$ for this experiment is –0.015. Again this is some five times the standard deviation and so is very large. Because it is not multiplied by the term (1–s) in the NKPC intercept (as $\bar{\pi}$ was) it produces large changes in the latter. Table 5 gives the results. Instruments are obviously much better so the distributions look normal, although there is a major specification bias. The specification error does result in an upward bias to the UE estimator of γ, as was observed by Castle *et al* (2010) and Russell *et al* (2010), although it is not due to shifting means in the inflation process. Even if the shift is just –0.005, the UE estimates of the mean and median of γ remain at 0.5 ($T = 100$), while the RES estimates are 0.43, making them much more robust against moderate breaks, and one retains the impression from previous experiments that it would be a preferred estimator.

Table 5: Estimators of the NKPC – Breaks in Mean of Potential Output

	***T* = 100**				***T* = 1 000**			
	UE		RE	RES	UE		RE	RES
	γ	$\gamma+\alpha$	γ	γ	γ	$\gamma+\alpha$	γ	γ
Median	0.51	1.03	0.47	0.47	0.51	1.03	0.47	0.47
Mean	0.51	1.13	0.46	0.47	0.51	1.03	0.46	0.47

Note: True value of γ is 0.29

4.2.5 *Experiment 5: as Experiment 3 but agents think the break is at 60 per cent of the sample*

By comparing Tables 4 and 6 it is clear that the properties of the estimators are basically the same, even though agents get the timing of the structural break incorrect.

Table 6: **Estimators of the NKPC – Agents Mistime the Break but Use Feedback Form of Expectations**

	***T* = 100**				***T* = 1 000**			
	UE		RE	RES	UE		RE	RES
	γ	$\gamma+\alpha$	γ	γ	γ	$\gamma+\alpha$	γ	γ
Median	0.26	0.98	0.34	0.38	0.21	0.98	0.33	0.37
Mean	0.22	0.97	0.34	0.38	0.19	0.97	0.32	0.37

Note: True value of γ is 0.29

4.2.6 *Experiment 6: as Experiment 3, agents still think the break is at 60 per cent of the sample but expectations are based on path computation*

We only show results for a large sample (Table 7). Compared to either Tables 4 or 6, the pathway of forming expectations produces a very large *downward* bias in the estimates of γ for the UE. With 10 000 observations, the median of the UE is –1.27 and the mean –1.67, while the two restricted estimators stay at 0.41 and 0.47. By 100 observations, the UE median and mean become –1.24 and –1.30, so the generation of expectations increases the bias a great deal in the UE but not for the RE. Again the sum is much better estimated.

This experiment makes the point that, if agents ignore observable evidence that their past expectations were wrong, one could see major biases in the NKPC. In the previous experiment, agents did not ignore this evidence. Their expectations were model consistent but to the wrong model (at least until 60 per cent of the sample was complete), but the feedback form they use for expectations does reflect the structural changes that took place, i.e. it shows up in z_{t-1}. Hence, agents in Experiment 5 are effectively learning about the structural change from the viewpoint of expectations formation.

Table 7: **Estimators of the NKPC – Agents Mistime the Break but Use Path Computation Form of Expectations**

	T = 1 000			
	UE		RE	RES
	γ	$\gamma+\alpha$	γ	γ
Median	–0.25	0.97	0.41	0.46
Mean	–1.29	0.93	0.41	0.46

Note: True value of γ is 0.29

5. The Euro Area New-Keynesian Phillips Curve and Breaks

Castle *et al* (2010) use the euro area NKPC to illustrate the effects of breaks, arguing that the large coefficient on forward expectations is due to breaks. Our argument is that this could be true but one should always check whether it is a consequence of weak instruments. Consequently, we re-do their analysis here to look at that question.

First, we estimate the hybrid NKPC over the period 1972:Q2 to 1998:Q1:[3]

$$\pi_t = c + \alpha\pi_{t-1} + \delta\mathbb{E}_t\pi_{t+1} + \delta x_t + \varepsilon_{1t}$$

where x_t is the labour share (s_t in their equations). Their paper states that the instruments used were $\{\pi_{t-j}\}_{j=1}^{5}, x_{t-1}, x_{t-2}, \Delta w_{t-1}, \Delta w_{t-2}, gap_{t-1}, gap_{t-2}$ where Δw_t is wage inflation and gap_{t-1} is the euro area output gap. The estimated hybrid curve is:

$$\begin{array}{rcl} \pi_t & = & \underset{(0.7)}{0.007} + \underset{(2.3)}{0.26}\pi_{t-1} + \underset{(5.4)}{0.69}\mathbb{E}_t\pi_{t+1} + \underset{(0.7)}{0.01}x_t + \varepsilon_{1t} \end{array} \tag{5}$$

which differs from the estimates they present. To reconcile them we re-run the regression with Δw_{t-1}, Δw_{t-2} deleted from the instrument set and then we get *exactly* what they report. It appears that all the results they report delete these instruments. As there is no reason to delete the lagged wage inflation variables as instruments we add them back in as we proceed.

3 We are grateful to Ragner Nymoen for providing the data and corresponding with us over its use.

The inclusion of dummy variables to account for breaks does not seem to have much impact upon the forward expectation coefficient.

$$\begin{aligned} \pi_t &= \underset{(1.9)}{0.02} + \underset{(2.9)}{0.28}\pi_{t-1} + \underset{(4.2)}{0.54}\mathbb{E}_t\pi_{t+1} + \underset{(1.8)}{0.03}x_t + dummies + \varepsilon_{1t} \end{aligned} \quad (6)$$

In order to get closer to their specification we add gap_{t-1} into the regression:

$$\pi_t = \underset{(3.6)}{0.01} + \underset{(3.9)}{0.37}\pi_{t-1} + \underset{(0.6)}{0.12}\mathbb{E}_t\pi_{t+1} + \underset{(3.5)}{0.07}x_t + \underset{(2.9)}{0.009}gap_{t-1} + dummies + \varepsilon_{1t}$$

One interpretation of this is that gap_{t-1} is a good instrument that cannot be used when the variable appears in the regression. Indeed if we regress π_{t+1} against all the instruments, gap_{t-1} has a t ratio of 2.6.

The results above do show a significant decline in the forward coefficient estimate when gap_{t-1} is added to the regression and are compatible with what Castle, Doornik, Hendry and Nymoen (2010) report, except that they get a negative value for the forward coefficient. To get that result we had to drop Δw_{t-1}, Δw_{t-2} from the instrument set. Doing so gave us $\hat{\alpha} = 0.48$ and $\hat{\delta} = -0.21$, so a negative forward estimate seems to come from dropping some instruments, although they report $\hat{\alpha} = 0.51$ and $\hat{\delta} = -0.30$.

Finally, we put the lagged wage inflation variables back into the instrument set and run the regression with gap_{t-1} included, but now impose the adding up constraint $\alpha + \delta = 1$. Our argument is that this is often a way of dealing with weak instruments.

$$\Delta\pi_t = \underset{(0.9)}{-0.007} + \underset{(6.6)}{0.72}(E_t\pi_{t+1} - \pi_{t-1}) - \underset{(0.1)}{0.0008}x_t - \underset{(0.1)}{0.0003}gap_{t-1} + dummies + \varepsilon_{1t}$$

So the forward coefficient is largely back to what it was in Equation (5) and supports the idea that we are dealing with weak instruments. The t ratios in the instrument regression for Δw_{t-1} and Δw_{t-2} are 2.6 and 1.5 respectively. Because gap_{t-1} is insignificant in this regression, deleting it would take us back to Equation (6), with much the same conclusion about the forward coefficient. The specification in the equation above is not particularly appealing as the marginal cost variable has a negative coefficient, although it is clearly ill determined.

However, our objective is not to produce a well-defined Phillips curve but to explore the influences on the forward expectations estimated coefficient. The sequence of regressions above suggests that the changes in the forward coefficient come more from changing the instrument list than from the break dummies.

At the end of their paper Castle *et al* (2010) look at the NKPC estimated over a period that they think has no breaks – 1983:Q2–1998:Q1. The idea is to see if one gets a high estimate for the forward coefficient then. They report that the coefficient on $E_t\pi_{t+1}$ is now 0.08 and so conclude that the high value found in the initial hybrid curve regressions was due to breaks. However to get $\hat{\delta} = 0.08$ one needs to delete Δw_{t-1} and Δw_{t-2} from the instrument set. If one does use them one gets:

$$\begin{aligned} \pi_t &= \underset{(0.6)}{0.015} + \underset{(1.9)}{0.23}\pi_{t-1} + \underset{(1.9)}{0.61}E_t\pi_{t+1} + \underset{(0.6)}{0.019}x_t + \underset{(0.4)}{0.002}gap_{t-1} + \varepsilon_{1t} \end{aligned} \tag{7}$$

which is very close to the full period regression which includes the dummies in Equation (6). Deleting gap_{t-1} from this equation results in:

$$\begin{aligned} \pi_t &= \underset{(0.4)}{0.007} + \underset{(1.7)}{0.21}\pi_{t-1} + \underset{(3.1)}{0.70}E_t\pi_{t+1} + \underset{(0.4)}{0.01}x_t + \varepsilon_{1t} \end{aligned} \tag{8}$$

This is virtually the same as the long sample estimates given in Equation (5). Thus our assessment would be that the high forward coefficient does not come from breaks in the inflation process. It may reflect misspecification of the curve, which is what breaks are about; gap_{t-1}, which looked like it could be a possible augmenting variable, was not sustained in the shorter sample.

6. Conclusion

Structural change has been conjectured to lead to an upward bias in the estimated forward expectations coefficient in NKPCs. We have presented a simple New-Keynesian model that enables us to assess this proposition. The model enables us to distinguish the effects of specification error caused by structural change from small-sample biases that simply arise due to weak instruments. Experiments suggest that the latter dominates the former.

Imposing the restriction that the forward and backward coefficients sum to unity seems to be a useful thing to do, as it generally produces better instruments at the expense of a small specification bias. We also find that biases are relatively small when the coefficient on the forward-looking inflation variable is high and that structural change can improve the quality of instruments, so it may actually aid estimation.

We looked at an empirical study of the euro area Phillips curve by Castle *et al* (2010), who concluded that the large coefficient on the forward-looking inflation variable was due to structural change. Our analysis suggests that this is not true. It may be that the large coefficient reflects some misspecification, but it is not due to structural change. It seems that the estimators used by Castle *et al* (2010) were probably subject to weak instrument bias.

Appendix A: Relevant Instruments

Setting means to zero, the system used is:

$$
\begin{aligned}
\pi_t &= \alpha\pi_{t-1} + \gamma E_t \pi_{t+1} + \delta x_t + \varepsilon_{1t} && \text{(A1)}\\
x_t &= \rho_1 x_{t-1} + d(r_{t-1} - \pi_{t-1}) + \varepsilon_{2t} && \text{(A2)}\\
r_t &= \lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3 \pi_t + \varepsilon_{3t} && \text{(A3)}
\end{aligned}
$$

where the shocks ε_{jt} are uncorrelated with one another and not autocorrelated. It is known that the solution to this system would have the form (for inflation and potential output):

$$
\begin{aligned}
\pi_t &= \phi_1\pi_{t-1} + \phi_2 x_{t-1} + \phi_3 r_{t-1} + v_{1t} && \text{(A4)}\\
x_t &= \phi_4\pi_{t-1} + \phi_5 x_{t-1} + \phi_6 r_{t-1} + v_{2t} && \text{(A5)}
\end{aligned}
$$

Substituting Equation (A4) into Equation (A1) and taking the expectation we get:

$$\pi_t = \alpha\pi_{t-1} + \gamma(\phi_1\pi_t + \phi_2 x_t + \phi_3 r_t) + \delta x_t + \varepsilon_{1t} \qquad \text{(A6)}$$

Gathering terms in Equation (A6) produces:

$$\pi_t = \psi_1\pi_{t-1} + \psi_2 x_t + \psi_3 r_t + v_t$$

where $\psi_1 = \frac{\alpha}{1-\gamma\phi_1}$, $\psi_2 = \frac{\gamma\phi_2+\delta}{1-\gamma\phi_1}$, $\psi_3 = \frac{\gamma\phi_3}{1-\gamma\phi_1}$, $v_t = \frac{\varepsilon_{1t}}{1-\gamma\phi_1}$. Substituting for r_t from Equation (A3) we get:

$$
\begin{aligned}
\pi_t &= \psi_1\pi_{t-1} + \psi_2 x_t + \psi_3(\lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3\pi_t + \varepsilon_{3t}) + v_t && \text{(A7)}\\
&= a_1\pi_{t-1} + a_2 x_t + a_3 r_{t-1} + \eta_t && \text{(A8)}
\end{aligned}
$$

where $a_1 = \frac{\psi_1}{1-\psi_3\lambda_3}$, $a_2 = \frac{\psi_2+\psi_3 d_2}{1-\psi_3\lambda_3}$, $a_3 = \frac{\psi_3\lambda_3}{1-\psi_3\lambda_3}$, $\eta_t = \frac{v_t+\psi_3\varepsilon_{3t}}{1-\psi_3\lambda_3}$.

Leading π_t from Equation (A8) and substituting for x_{t+1} from Equation (A5) gives

$$\pi_{t+1} = a_1\pi_t + a_2 x_{t+1} + a_3 r_t + \eta_{t+1}$$

Substituting from Equation (A5) again we get:

$$
\begin{aligned}
\pi_{t+1} &= a_1\pi_t + a_2(\phi_5 x_t + \phi_4\pi_t + \phi_6 r_t + v_{2t+1}) + a_3 r_t + \eta_{t+1}\\
&= b_1\pi_t + b_2 x_t + b_3 r_t + \xi_{t+1}
\end{aligned} \qquad \text{(A9)}
$$

where $b_1 = a_1 + a_2\phi_4$, $b_2 = a_2\phi_5$, $b_3 = a_3 + a_2\phi_6$, $\xi_{t+1} = \eta_{t+1} + a_2 v_{2t+1}$. Now using Equation (A3) for r_t in Equation (A9) we get:

$$\begin{aligned}\pi_{t+1} &= b_1\pi_t + b_2 x_t + b_3(\lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3 \pi_t + \varepsilon_{3t}) + \xi_{t+1} \\ &= d_1\pi_t + d_2 x_t + d_3 r_{t-1} + \zeta_{t+1}\end{aligned}$$

where $d_1 = b_1 + b_3\lambda_3$, $d_2 = b_2 + b_3\lambda_2$, $d_3 = b_3\lambda_1$, $\zeta_{t+1} = \xi_{t+1} + b_3\varepsilon_{3t}$. Now replace π_t by Equation (A8):

$$\begin{aligned}\pi_{t+1} &= d_1(a_1\pi_{t-1} + a_2 x_t + a_3 r_{t-1} + \eta_t) + d_2 x_t + d_3 r_{t-1} + \zeta_{t+1} \\ &= f_1\pi_{t-1} + f_2 x_t + f_3 r_{t-1} + e_{t+1}\end{aligned}$$

where $f_1 = d_1 a_1$, $f_2 = d_1 a_2 + d_2$, $f_3 = d_1 a_3 + d_3$ and $e_{t+1} = d_1\eta_t + \zeta_{t+1}$. Hence π_{t+1} does not depend on x_{t-1} and so the latter is not an instrument.

References

Ascari G (2004), 'Staggered Prices and Trend Inflation: Some Nuisances', *Review of Economic Dynamics*, 7(3), pp 642–667.

Binder M and MH Pesaran (1995), 'Multivariate Rational Expectations Models and Macroeconometric Modelling: A Review and Some New Results', in MH Pesaran and MR Wickens (eds), *Handbook of Applied Econometrics: Volume 1: Macroeconomics*, Blackwell Handbooks in Economics, Blackwell, Oxford, pp 139–187.

Castle JL, JA Doornik, DF Hendry and R Nymoen (2010), 'Testing the Invariance of Expectations Models of Inflation', University of Oxford, Department of Economics Discussion Papers Series No 510.

Cogley T and AM Sbordone (2008), 'Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve', *The American Economic Review*, 98(5), pp 2101–2126.

Galí J and M Gertler (1999), 'Inflation Dynamics: A Structural Econometric Analysis', *Journal of Monetary Economics*, 44(2), pp 195–222.

Kulish M and A Pagan (2012), 'Estimation and Solution of Models with Expectations and Structural Changes', RBA Research Discussion Paper No 2012-08.

Mavroeidis S (2004), 'Weak Identification of Forward-Looking Models in Monetary Economics', *Oxford Bulletin of Economics and Statistics*, 66(Supplement s1), pp 609–635.

Pesaran MH (1987), *The Limits to Rational Expectations*, Blackwell Publishers, Oxford.

Russell B, A Banerjee, I Malki and N Ponomareva (2010), 'A Multiple Break Panel Approach to Estimating United States Phillips Curves', University of Dundee, Dundee Discussion Papers in Economics No 232.

RESEARCH DISCUSSION PAPERS

These papers can be downloaded from the Bank's website or a hard copy may be obtained by writing to:

Mail Room Supervisor
Information Department
Reserve Bank of Australia
GPO Box 3947
SYDNEY NSW 2001

Enquiries:

Phone: +61 2 9551 9830
Facsimile: +61 2 9551 8033
Email: rbainfo@rba.gov.au
Website: http://www.rba.gov.au

2013-01	Currency Demand during the Global Financial Crisis: Evidence from Australia	Tom Cusbert Thomas Rohling
2013-02	Industry Dimensions of the Resource Boom: An Input-Output Analysis	Vanessa Rayner James Bishop
2013-03	Implications for the Australian Economy of Strong Growth in Asia	Michael Plumb Christopher Kent James Bishop
2013-04	Home Prices and Household Spending	Callan Windsor Jarkko Jääskelä Richard Finlay
2013-05	Liquidity Shocks and the US Housing Credit Crisis of 2007–2008	Gianni La Cava
2013-06	Estimating and Identifying Empirical BVAR-DSGE Models for Small Open Economies	Tim Robinson
2013-07	An Empirical BVAR-DSGE Model of the Australian Economy	Sean Langcake Tim Robinson
2013-08	International Business Cycles with Complete Markets	Alexandre Dmitriev Ivan Roberts
2013-09	Terms of Trade Shocks and Incomplete Information	Daniel Rees
2013-10	Stochastic Terms of Trade Volatility in Small Open Economies	Patricia Gómez-González Daniel Rees

RESERVE BANK OF AUSTRALIA