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# Stochastic Terms of Trade Volatility in Small Open Economies

Patricia Gómez-González and Daniel Rees

RDP 2013-10

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## **Abstract**

The terms of trade of commodity-producing small open economies are subject to large shocks that can be an important source of economic fluctuations. Alongside times of high volatility, however, these economies also experience periods in which their terms of trade are comparatively stable. We estimate the empirical process for the terms of trade for six small open economies and examine the responses of output, the current account and prices to changes in terms of trade volatility using a vector autoregression (VAR). We find that increased terms of trade volatility, by itself, is associated with a contraction in domestic demand and an increase in the current account. We then set up a small open economy real business cycle model and show that it can broadly replicate the responses to a volatility shock estimated in the VAR. We use this model to explore the sectoral implications of terms of trade volatility shocks and to quantify the importance of these shocks as a source of business cycle fluctuations. Our results suggest that the direct effects of terms of trade volatility shocks on output, consumption and investment are generally small. But, interacted with shocks to the level of the terms of trade, volatility shocks account for around one-quarter of the total impact of the terms of trade on macroeconomic outcomes.

JEL Classification Numbers: C32, E32, F41, Q33

Keywords: terms of trade, small open economy, real business cycle, stochastic volatility

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# Stochastic Terms of Trade Volatility in Small Open Economies

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## 1. Introduction

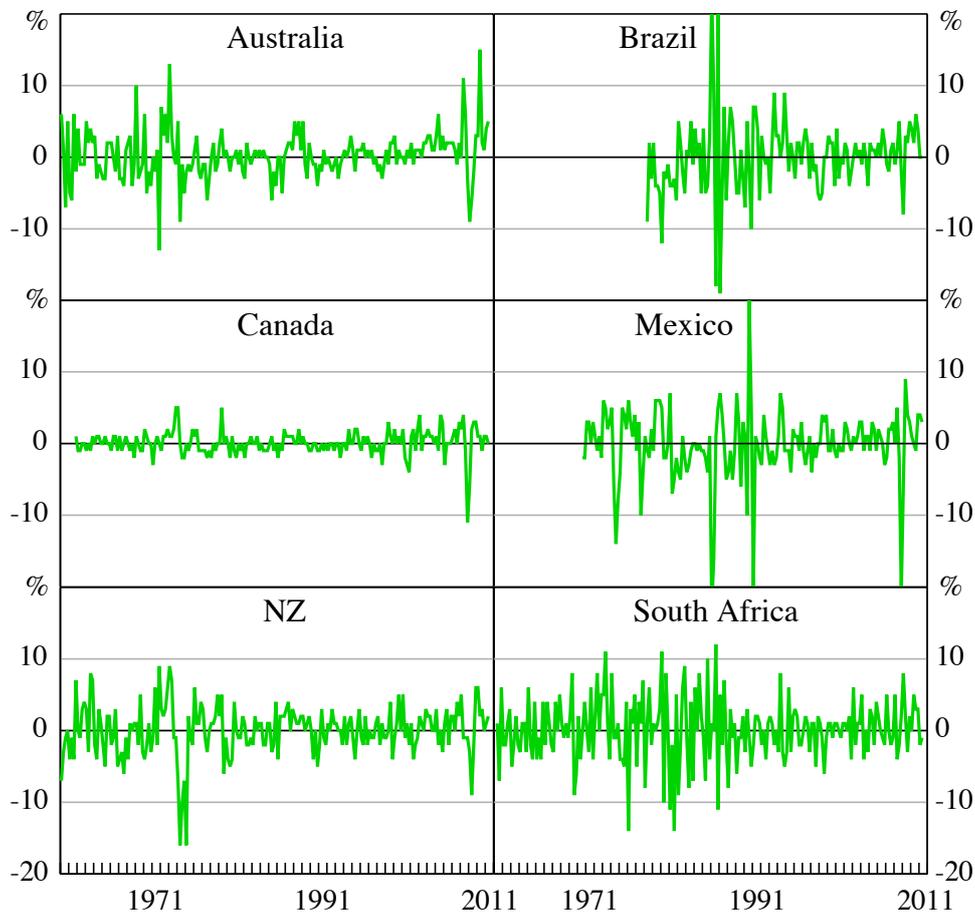
The terms of trade of many commodity-producing small open economies are subject to large shocks that can be an important source of macroeconomic fluctuations. Alongside times of high volatility, however, these economies also experience periods in which their terms of trade are comparatively stable. The effect of shocks to the level of the terms of trade has been widely studied. But little is known about the impact of changes in the volatility of terms of trade shocks. This is the focus of our paper.

To illustrate what we mean by changes in the volatility of terms of trade shocks, consider Figure 1. This shows the quarterly growth rate of the terms of trade for a selection of commodity-producing small open economies. At various times, each economy has experienced an increase or decrease in its terms of trade of more than 10 per cent, while fluctuations of 5 per cent or more are common. On average, these economies' terms of trade are highly volatile.

Figure 1 also reveals that terms of trade volatility varies over time. The economies in the figure have all endured episodes of extremely high terms of trade volatility, including during the 1970s for Australia and New Zealand, 1980s for Brazil, Mexico and South Africa, and 2000s for Australia and Canada. But these economies have also experienced lengthy periods in which their terms of trade were comparatively stable. Examples include the late 1990s for Australia, New Zealand and Mexico. We refer to these shifts in the absolute magnitude of changes in the terms of trade as volatility shocks.

To examine the economic implications of changes in terms of trade volatility, we first estimate the empirical process of the terms of trade for the six economies featured in Figure 1. We use the estimated time series of terms of trade volatility produced in this exercise to identify the effect of volatility shocks on output, consumption, investment, the current account and prices in a vector autoregression (VAR). We then set up and augment a small open economy real business cycle model to incorporate stochastic terms of trade volatility. We demonstrate that this

**Figure 1: Terms of Trade**  
Quarterly percentage change



Sources: See Appendix A

model can broadly replicate the empirical responses produced by the VAR and use it to explore the theoretical causes and sectoral impacts of these responses. Finally, we quantify the importance of terms of trade volatility shocks as a source of macroeconomic fluctuations.

Our empirical results suggest that, by itself, an increase in terms of trade volatility depresses domestic demand and leads to an improvement in the current account, leaving the response of aggregate output ambiguous. Our model successfully replicates these patterns. It also suggests that increased terms of trade volatility causes a shift in the composition of output from non-tradeables to tradeables and a substitution in factor inputs from capital to labour.

The effects of terms of trade volatility shocks are generally small. But, interacted with shocks to the level of the terms of trade, variance decompositions suggest

that these shocks have an economically meaningful impact. For a typical small open economy we find that shocks to volatility account for around one quarter of the total impact of terms of trade shocks on the standard deviations of output, consumption and investment.

## 2. Literature Review

Our paper complements other work studying the macroeconomic effects of uncertainty and time-varying volatility. Examples here include Bloom (2009), who explores the short-run fluctuations of output, employment and productivity growth after shocks to macroeconomic uncertainty, Justiniano and Primiceri (2008), who shed light on the sources of changes in US macroeconomic volatility, and Christiano, Motto and Rostagno (2010), who study the effects of idiosyncratic risk shocks on entrepreneurs' productivity. Also closely related to our paper are Fernández-Villaverde *et al* (2011), who examine shocks to the volatility of sovereign debt interest rates, and Fernández-Villaverde *et al* (2012), who study how changes in uncertainty about future fiscal policy affects aggregate economic activity. Our main contribution to this literature is empirical. We document time-varying volatility in a variable, the terms of trade, that has not previously been studied and explore the effects of changes in this volatility.

We also build on the literature examining the macroeconomic consequences of terms of trade shocks. Many papers in this literature have examined terms of trade shocks using calibrated business cycle models. These typically conclude that terms of trade shocks are an important driver of small open economy business cycles. For example, Mendoza (1995) concludes that terms of trade shocks account for around half of the fluctuations in GDP in developing countries and slightly less in advanced economies. In a model calibrated to match features of a standard developing economy, Kose and Riezman (2001) find that terms of trade shocks account for 45 per cent of output volatility and 86 per cent of investment volatility. And, in a model calibrated for Canada, Macklem (1993) finds that a 10 per cent temporary deterioration in the terms of trade – a large but not unprecedented shock for the economies in Figure 1 – reduces real GDP by almost 10 per cent and investment by almost 20 per cent.

Other papers in this literature have adopted a more reduced form approach and have examined the effects of terms of trade shocks in VAR models. These papers

typically conclude that terms of trade shocks have smaller effects than is implied by structural business cycle models. For example, using a panel VAR covering 75 developing countries, Broda (2004) concludes that a 10 per cent permanent deterioration in the terms of trade reduces the level of GDP by around 1 per cent, and that terms of trade shocks explain between 10 to 30 per cent of the volatility of GDP growth. Similarly, Collier and Goderis (2012) find that a 10 per cent rise in commodity prices increases the level of GDP by around 1 percentage point after two years for a typical developing country. Our contribution to this literature is to highlight an additional channel – volatility – through which the terms of trade can have macroeconomic effects.

Alongside the literature examining the dynamic effect of shocks to the level of the terms of trade, another empirical literature documents a link between terms of trade volatility and long-run economic growth. Using a panel of 35 advanced and developing economies over the period 1870 to 1939, Blattman, Hwang and Williamson (2007) conclude that, for commodity producers, a one standard deviation increase in terms of trade volatility (in their sample, from 8 per cent to 13 per cent per year) causes a 0.4 percentage point reduction in annual per capita GDP growth. In related work, Williamson (2008) attributes much of the gap in economic performance in the early 19th century between economies in Western Europe and those in Eastern Europe, the Middle East and east Asia to the fact that the latter regions experienced more terms of trade volatility. Focusing on more contemporary patterns, Bleaney and Greenaway (2001) estimate a cross-country panel regression using data from 14 sub-Saharan African countries over the period 1980 to 1995 and also conclude that terms of trade volatility, measured as the residuals from a GARCH model of the terms of trade, reduces GDP growth.

Papers in this literature have also explored links between terms of trade volatility and the volatility of other macroeconomic variables. For example, using a panel of countries, Easterly, Islam and Stiglitz (2000) show that times of high terms of trade volatility tend to be correlated with times of more volatile GDP growth, while Andrews and Rees (2009) also establish a link with consumption and inflation volatility. Our paper complements this literature by illustrating the links between terms of trade volatility and macroeconomic outcomes in a fully specified macroeconomic model, and by tracing out the dynamic effects of changes in terms of trade volatility on output, external accounts and prices.

### 3. Estimating the Law of Motion for the Terms of Trade

In this section, we estimate the empirical process for the terms of trade for six small open economies: Australia, Brazil, Canada, Mexico, New Zealand and South Africa. We select these countries based on two criteria. First, we focus on commodity-producing small open economies whose terms of trade are both volatile and plausibly exogenous to domestic economic developments. Second, we require countries to have reasonably long time series data for the terms of trade and other macroeconomic variables.

To support our contention that the terms of trade are exogenous, Table 1 provides descriptive statistics about the size and export composition of each economy. The six economies each account for a small share of world GDP and merchandise trade. This suggests that economic developments within these countries are unlikely to have a substantial effect on world economic activity. Moreover, the exports of these countries are geared towards agriculture, fuels and mining – that is, commodities – with these goods accounting for more than 50 per cent of merchandise export values for each country, except Mexico.<sup>1</sup> Commodities tend to be less differentiated, and more substitutable, than manufactured goods and commodity producers generally have less pricing power on world markets.<sup>2</sup> Further evidence to support our contention comes from the numerous studies that have used statistical techniques to examine the exogeneity of the terms of trade for small open economies. For example, using Granger causality tests, Mendoza (1995) and Broda (2004) conclude that the terms of trade is exogenous for a large sample of small open economies, including Brazil, Canada and Mexico.

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1 Even for Mexico, petroleum is the largest single export good at the three digit SITC 3 level, accounting for almost 12 per cent of total exports in 2010. Moreover, commodities accounted for the bulk of Mexico's exports in the early part of our sample, before the expansion of manufacturing exports that accompanied Mexico's trade liberalisation in 1986 and entry into NAFTA in 1994 (Moreno-Brid, Santamaría and Rivas Valdivia 2005).

2 While this is not strictly true for all commodity producers, such as large oil producers for example, it seems reasonable for the countries in our sample.

**Table 1: Summary Statistics – Merchandise Exports**  
2010, per cent

	Share of world merchandise exports	Export composition		
		Food items and agricultural raw materials	Fuels, ores and metals	Manufactured goods
Australia	1.4	13.1	69.2	12.8
Brazil	1.3	34.8	29.5	35.8
Canada	2.5	13.5	35.6	47.8
Mexico	2.0	6.3	18.7	74.5
New Zealand	0.2	63.3	10.1	22.9
South Africa	0.6	10.5	47.4	39.2

Source: UNCTAD (2011)

### 3.1 Estimation

For each country, we specify that the terms of trade,  $q_t$ , follow an AR(1) process described by:

$$q_t = \rho_q q_{t-1} + e^{\sigma_{q,t}} u_{q,t} \quad (1)$$

where  $u_{q,t}$  are normally distributed shocks with mean zero and unit variance.<sup>3</sup> The log of the standard deviation of the terms of trade shocks,  $\sigma_{q,t}$ , varies over time, according to an AR(1) process:

$$\sigma_{q,t} = (1 - \rho_\sigma) \sigma_q + \rho_\sigma \sigma_{q,t-1} + \eta_q u_{\sigma,t} \quad (2)$$

where  $u_{\sigma,t}$  are normally distributed shocks with mean zero and unit variance. To emphasise, innovations to  $u_{q,t}$  alter the level of the terms of trade. In contrast, innovations to  $u_{\sigma,t}$  alter the magnitude of shocks to the terms of trade, with no direct effect on its level. We refer to these as volatility shocks. The parameter  $\sigma_q$  is the log of the mean standard deviation of terms of trade shocks, while  $\eta_q$  is the standard deviation of shocks to the volatility of the terms of trade. The parameter  $\rho_q$  controls the persistence of shocks to the level of the terms of trade, while  $\rho_\sigma$  controls the persistence of terms of trade volatility shocks. Throughout, we assume that  $u_{q,t}$  and  $u_{\sigma,t}$  are independent of each other.

<sup>3</sup> Our estimation procedure requires stationary data. We discuss the transformations that we make to ensure that our terms of trade series are stationary in the Data section below.

Equations (1) and (2) represent a standard stochastic volatility model. Inference in these models is challenging because of the presence of two innovations, one to the level of the terms of trade and one to its volatility, that enter the model in a non-linear manner. To overcome this issue, we follow Fernández-Villaverde *et al* (2011) and use a sequential Markov chain Monte Carlo filter, also known as a particle filter, that allows us to evaluate the likelihood of the model using simulation methods. We estimate the model using a Bayesian approach that combines prior information with information that can be extracted from the data. For presentational ease, we confine the technical details of the estimation procedure to Appendix C.

Other methods of modeling time-varying volatility processes, including Markov switching models and GARCH models, also exist. Although these methods have advantages in other contexts, we do not believe that they provide a satisfactory description of terms of trade volatility. For example, a GARCH model does not sharply distinguish between innovations to the level of the terms of trade and its volatility. High levels of volatility are triggered only by large innovations to the terms of trade. In contrast, our methodology allows changes in the volatility of the terms of trade to occur independently of innovations to the level of the terms of trade. Nonetheless, for comparison Appendix D shows the results when we estimate the terms of trade processes using a GARCH model. A Markov switching model would require us to restrict the number of potential realisations of terms of trade volatility in a way that seems inconsistent with the patterns in Figure 1.

## 3.2 Data

The terms of trade for each country are measured quarterly, and defined as the ratio of the export price deflator to the import price deflator and sourced from national statistical agencies.<sup>4</sup> As we wish to estimate changes over time in the variance of the terms of trade, we require our data to be stationary. It is not clear whether real commodity prices (which drive the terms of trade for the countries in our sample) are stationary. Previous studies by Powell (1991), Cashin, Liang and McDermott (2000) and Lee, List and Strazicich (2006), among others, have concluded that real commodity prices are stationary. Others, including

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<sup>4</sup> Appendix A includes a full list of data sources and descriptions.

Kim *et al* (2003), Newbold, Pfaffenzeller and Rayner (2005) and Maslyuk and Smyth (2008) have found that they are not.

In light of the disagreement in the literature, we adopt a compromise approach and detrend our data using a band-pass filter that excludes cycles of longer than 30 years. This preserves all but the lowest frequency movements in the terms of trade for each country while ensuring that the data are stationary.<sup>5</sup>

### 3.3 Priors

Table 2 reports our priors for the parameters of the terms of trade process. For the persistence parameters,  $\rho_q$  and  $\rho_\sigma$ , we impose a Beta prior with mean of 0.9 and standard deviation of 0.1. The shape of this prior restricts the value of these parameters to lie between 0 and 1, consistent with economic theory. For the log of the mean standard deviation of terms of trade shocks,  $\sigma_q$ , we impose a Normal prior. For each country, we set the mean of this prior equal to the OLS estimate of this parameter calculated assuming an AR(1) process for the terms of trade without stochastic volatility. For the standard deviation of terms of trade volatility shocks,  $\eta_q$ , we use a truncated Normal prior to ensure that this parameter is positive. We experimented with alternative priors and found that these had very little impact on our results.

**Table 2: Prior Distribution of Parameters**

Parameter	$\rho_q$	$\sigma_q$	$\rho_\sigma$	$\eta_q$
Prior	$\beta(0.9, 0.1)$	$\mathcal{N}(\hat{\sigma}_{OLS}, 0.4)$	$\beta(0.9, 0.1)$	$\mathcal{N}^+(0.5, 0.3)$

Note:  $\beta$ ,  $\mathcal{N}$  and  $\mathcal{N}^+$  stand for Beta, Normal and truncated Normal distributions

### 3.4 Posterior Estimates

Table 3 reports the posterior medians of the parameter estimates and associated confidence bands. The first row shows the posterior estimates of  $\rho_q$ , the persistence of the terms of trade processes. This parameter lies above 0.9 for all countries

<sup>5</sup> We also estimated the models with HP-filtered data (see Appendix B for the results). The choice of detrending method has some effect on the estimated persistence of terms of trade shocks, but relatively little impact on the estimated magnitude of shocks to the terms of trade or the volatility processes. Appendix A provides figures (A1 and A2) of the terms of trade for each country in raw and filtered form to illustrate the effects of filtering methods on the underlying series.

except for South Africa, indicating that shocks to the terms of trade for these countries tend to be highly persistent.

**Table 3: Posterior Medians – Band-pass Filtered Data**

	Australia	Brazil	Canada	Mexico	New Zealand	South Africa
$\rho_q$	0.93 (0.88, 0.98)	0.96 (0.90, 0.99)	0.90 (0.85, 0.95)	0.92 (0.87, 0.97)	0.96 (0.89, 0.97)	0.81 (0.73, 0.86)
$\sigma_q$	-3.65 (-4.16, -3.03)	-3.22 (-3.74, -2.57)	-4.40 (-4.74, -4.04)	-3.43 (-3.78, -3.03)	-3.48 (-3.93, -2.90)	-3.30 (-3.81, -2.76)
$\rho_\sigma$	0.94 (0.83, 0.99)	0.92 (0.79, 1.00)	0.85 (0.57, 0.98)	0.85 (0.64, 0.94)	0.93 (0.84, 1.00)	0.97 (0.85, 1.00)
$\eta_q$	0.21 (0.12, 0.33)	0.21 (0.11, 0.36)	0.26 (0.13, 0.48)	0.38 (0.25, 0.54)	0.13 (0.07, 0.23)	0.10 (0.05, 0.23)

Note: 95 per cent credible sets in parantheses

The parameter estimates for  $\sigma_q$  reveal substantial differences in the average size of shocks to the terms of trade between countries. Converting the parameters into standard deviations, the results suggest that the magnitude of the average terms of trade shock varies from around 1.2 per cent for Canada to 4.0 per cent for Brazil.<sup>6</sup> The estimates for  $\rho_\sigma$  indicate that shocks to the volatility of the terms of trade are highly persistent for Australia, Brazil, New Zealand and South Africa, but somewhat less so for Canada and Mexico.

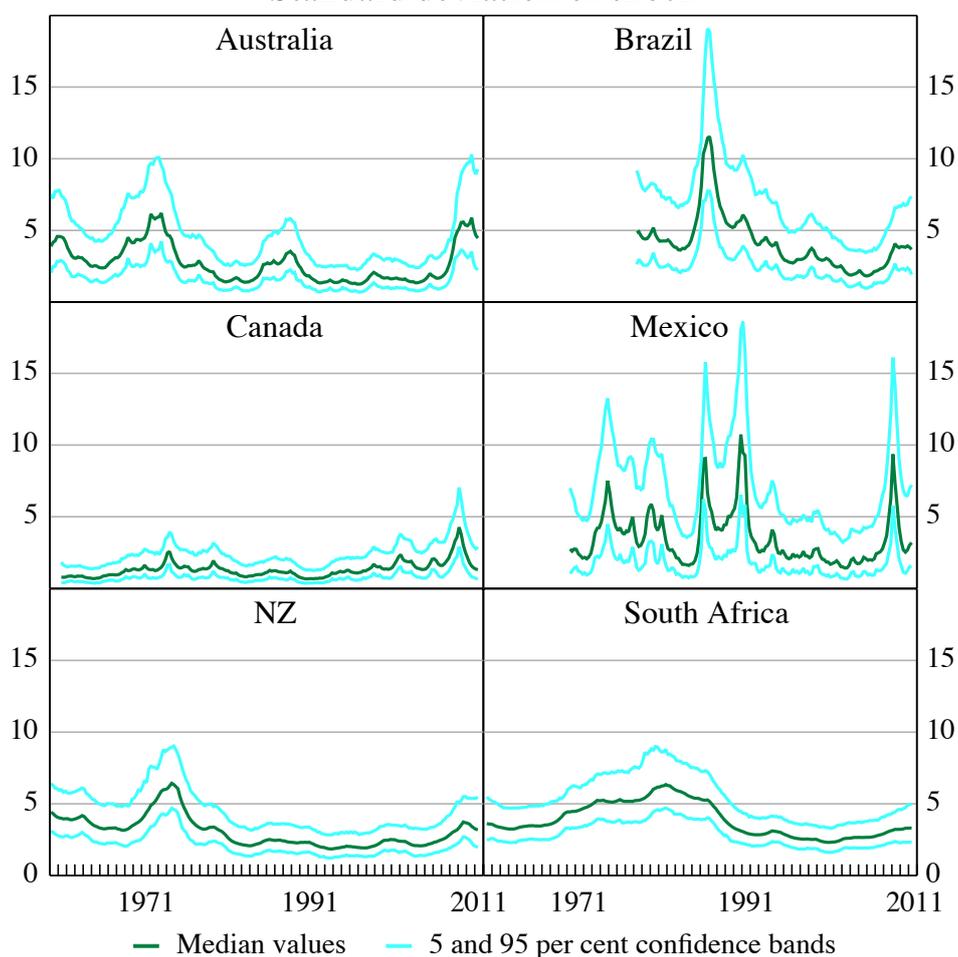
The final row of Table 3 confirms that the magnitude of shocks to the volatility of the terms of trade differs across countries. Of the countries in our sample, Mexico has tended to experience the largest volatility shocks, while New Zealand and South Africa have experienced the smallest. To put these numbers in context, a one standard deviation shock to  $u_{\sigma,t}$  increases the standard deviation of terms of trade shocks in Mexico from 3.2 per cent to 4.7 per cent and in South Africa from 3.7 per cent to 4.1 per cent.

To give a sharper insight into what our results imply for time-variation in terms of trade volatility, Figure 2 shows the model's estimates of the evolution of the standard deviations of terms of trade shocks for each country. The average level of volatility is higher for Brazil and Mexico than for the other countries in the sample, reflecting the fact that these countries have typically experienced larger terms of trade shocks. The changes in the level are also greatest for

6 Recall, that the standard deviation of shocks to the terms of trade is equal to  $\exp(\sigma_q)$ .

Mexico, as that country has experienced the largest shocks to the volatility of its terms of trade. In contrast, the standard deviation of shocks to Canada's terms of trade has typically been small and stable over time, at least compared to those experienced by other commodity exporters. The experiences of Australia, New Zealand and South Africa lie somewhere in between those of Canada and the Latin American countries. These economies have typically experienced large terms of trade shocks, with an average standard deviation of around 3 per cent. They have also experienced periods of heightened volatility, although not to the same extent as countries like Brazil and Mexico.

**Figure 2: Time-variation in Terms of Trade Shocks**  
Standard deviation of shock



Notes: Median values of  $\exp\left(\tilde{\sigma}_{q,t|T}\right)$  where  $\tilde{\sigma}_{q,t|T}$  are the estimated volatilities conditional on all information, calculated using the particle smoother

The patterns of volatility suggested by Figure 2 broadly conform to our understanding of macroeconomic developments over recent decades. For example,

the average magnitudes of terms of trade shocks increased in most countries during the mid 1970s, mid 1980s and late 2000s, while the 1990s was generally a period of low terms of trade volatility.

In sum, our results indicate that the volatility of shocks to the terms of trade for small open economy commodity producers varies over time. Historically, the variation has been largest for Latin American countries such as Brazil and Mexico, where the standard deviation of terms of trade shocks has at times increased from an average level of around 3 per cent to over 10 per cent. But countries like Australia, New Zealand and South Africa have also experienced shocks that have increased the standard deviation of their terms of trade shocks from 2 per cent to around 6 per cent.

## 4. The Impact of Volatility Shocks: Empirics

### 4.1 Vector Autoregression

To investigate the broader macroeconomic consequences of terms of trade volatility, this section models the responses of real GDP ( $y$ ), consumption ( $c$ ), investment ( $i$ ), the current account ( $ca$ ) and the GDP deflator ( $p$ ) to the terms of trade volatility shocks identified in the previous section. Because the economies in our sample have each experienced relatively few sizeable volatility shocks we pool the data for all six countries and estimate a VAR of the form:

$$Y_{it} = v + A(L)Y_{it} + B(L)X_{it} + u_{it} \quad (3)$$

where  $Y'_{it} = (y, c, i, ca, p)$  is a vector of stationary endogenous variables,  $v$  is a vector of constants,  $X'_{it} = (q_{it}, \sigma_{q, it})$  is a vector containing the level of the terms of trade as well as its volatility,  $u'_{it} = (u^y_{it}, u^c_{it}, u^i_{it}, u^{ca}_{it}, u^p_{it})$  is an error vector,  $A(L)$  and  $B(L)$  are matrix polynomials in the lag operator and  $var(u_{it}) = \Omega$ .<sup>7</sup> Note that although the variables in  $Y_{it}$  respond to the level of the terms of trade and its volatility, neither of the terms of trade variables appear as endogenous variables

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<sup>7</sup> In the results below, we include four lags of the endogenous variables and the contemporaneous value and one lag of the terms of trade variables. Experiments with alternative lag structures produced broadly similar results. We apply an HP filter to the macroeconomic variables to ensure stationarity.

in the VAR. This is consistent with our assumption in Section 3 that small open economies' terms of trade are exogenous with respect to domestic economic developments.

The empirical model described in Equation (3) can be thought of as a simplified reduced form version of a DSGE model with stochastic volatility, like the one described in Section 5 below. Of course, the empirical model cannot fully capture the non-linear relationships implied by a theoretical model. However, we argue that it nonetheless provides a meaningful indication of the relationships between terms of trade volatility shocks and macroeconomic outcomes that exist in the data, and serves as a useful benchmark against which to compare the results of our theoretical model. In Appendix E we provide additional evidence to support this contention.

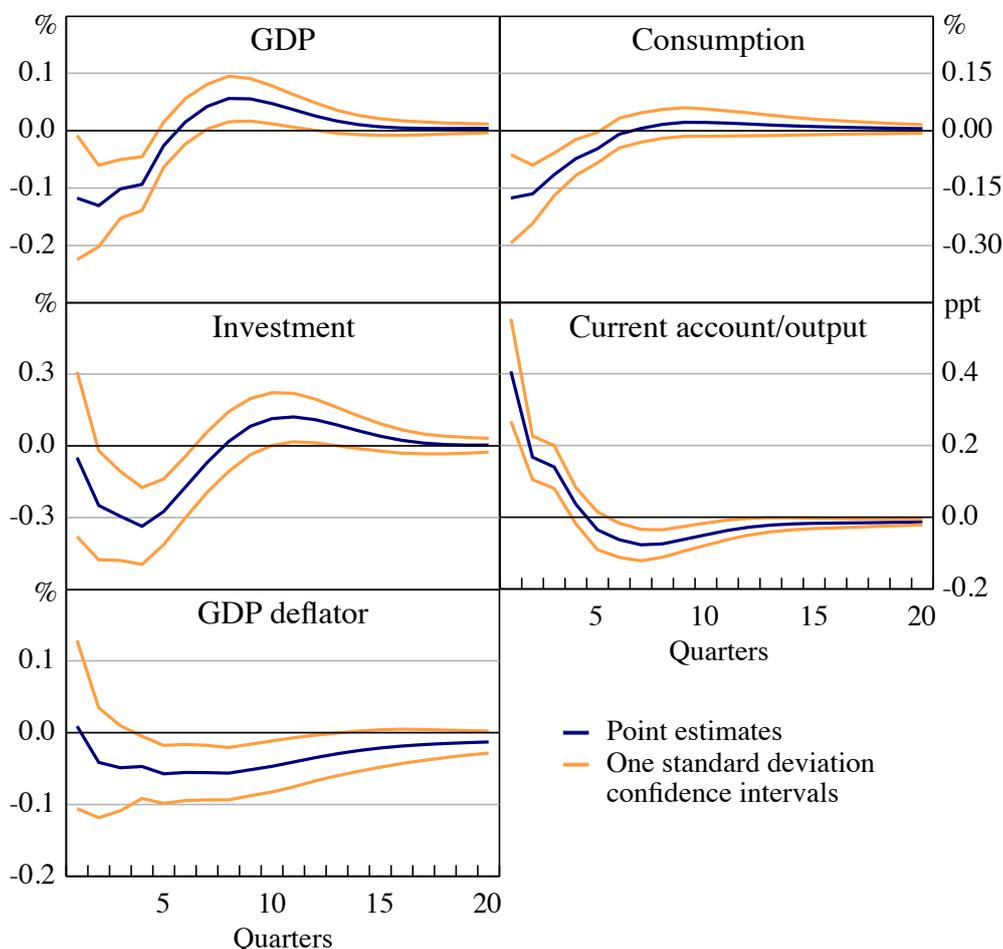
## 4.2 Results

To illustrate the consequences of a terms of trade volatility shock, we report the dynamic effects of an innovation to  $\sigma_q$  of 0.22, roughly equivalent to the average of  $\eta_q$  across the countries estimated in Section 3. After the initial shock, we allow  $\sigma_q$  to decay by 10 per cent per quarter, again broadly consistent with the estimates in Section 3.

Figure 3 shows the dynamic response of  $y_{it}$ ,  $c_{it}$ ,  $i_{it}$ ,  $ca_{it}$  and  $p_{it}$  to an increase in the volatility of terms of trade shocks, holding the level of the terms of trade fixed. Dark lines indicate the point estimates of the impulse response functions and the lighter lines represent one standard deviation (16th and 84th percentile) of the distribution of responses.

The volatility shock reduces both consumption and investment on impact. Although the investment response is not initially significant, it becomes so in later quarters, and is largest two quarters after the shock. Consumption and investment both return to their original levels eight quarters after the shock, and there is some evidence of a pick-up in domestic demand in later quarters. Aggregate output also decreases in the periods after the shock. The size of its response is smaller than the responses of consumption and investment, however, suggesting an offsetting response of net exports. This shows up in the current account balance (expressed as a ratio to GDP), which increases in the quarter in which the shock hits. It remains above its trend level for two subsequent quarters, before declining as domestic

**Figure 3: Impulse Responses to a Volatility Shock**  
Pooled sample



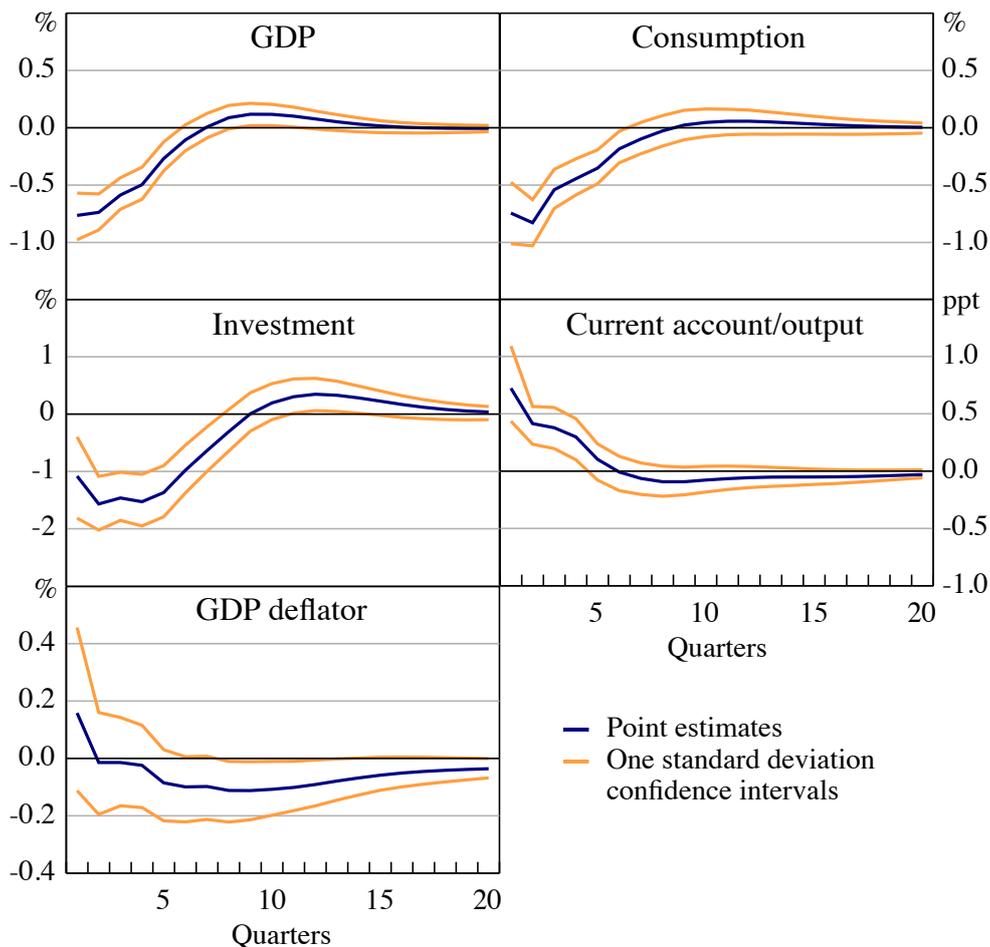
demand recovers. There is also a persistent decrease in the GDP deflator. As we have held the terms of trade constant in this exercise, absent nominal exchange rate movements this implies a fall in the price of non-tradeable goods relative to tradeables.

In reality, of course, both the level of the terms of trade and its volatility may change at any point in time. Empirical studies usually conclude that an increase in the terms of trade has an expansionary effect while a contraction in the terms of trade contracts activity (Broda 2004). Our results suggest that a terms of trade boom that is accompanied by an increase in volatility will have a less expansionary effect than would be the case if terms of trade volatility were to remain constant. In contrast, a fall in the terms of trade will have a larger contractionary effect on activity if it is accompanied by an increase in volatility. In sum, increased terms

of trade volatility dampens the effect of terms of trade booms but exacerbates the effect of terms of trade declines.

A possible criticism of our empirical approach is that pooling data conceals cross-country heterogeneity in the impact of volatility shocks. In particular, one might expect that economies in which households and firms are less able to hedge the risks associated with terms of trade volatility will be more responsive to these types of shocks. As a first step to answering this question, Figures 4 and 5 show responses to volatility shocks when we separate our sample into emerging economies – Brazil, Mexico and South Africa – and advanced economies – Australia, Canada and New Zealand.

**Figure 4: Impulse Responses to a Volatility Shock**  
Emerging economy sample

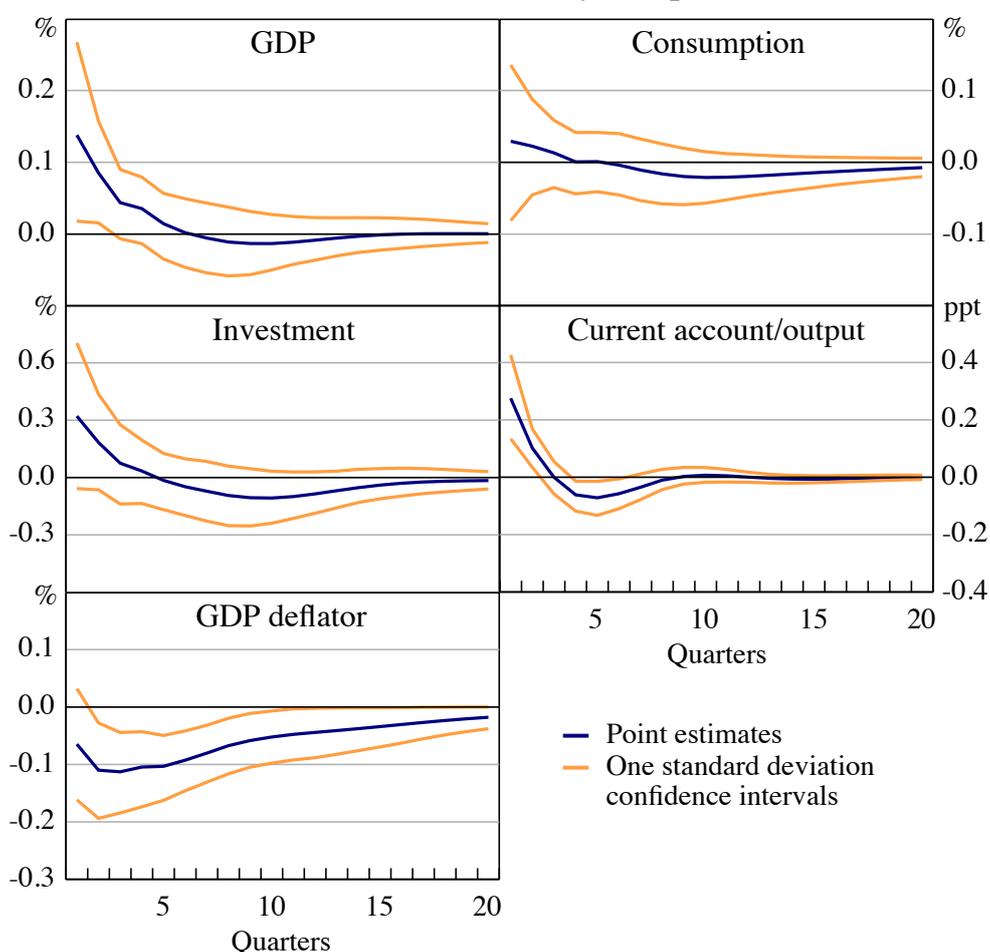


As Figure 4 shows, the effect of volatility shocks on output and its components appears to be larger when we estimate the model on a sample including only the emerging economies. The responses of these variables is roughly four times as large and the changes in investment and GDP are now significant from the quarter of impact. It also takes an additional quarter or two for these variables to return to trend after the shock. The current account-to-GDP ratio continues to increase following the shock, consistent with the contraction in domestic demand exceeding the decrease in GDP. The point estimate of the response of the GDP deflator is qualitatively similar to the pooled response, although it is only marginally statistically significant.

Figure 5 reveals a somewhat different response to the shock among the advanced economies. The responses of investment and consumption for these economies are not significantly different from zero. The initial response of aggregate output is also positive, albeit only significant in the period in which the shock hits. In contrast, the response of the GDP deflator remains negative and significant, and is larger relative to the response of the other variables than in Figure 4. These economies also experience a substantial increase in their current account following the volatility shock.

In sum, the empirical results suggest that, holding steady the level of the terms of trade, an increase in terms of trade volatility triggers a decrease in domestic demand that is partly offset by an increase in net exports, leading to a relatively small impact on aggregate output. These shocks also cause a decrease in the domestic price level which, given that we have held the level of the terms of trade constant, suggests a relative decrease in the price of domestic non-tradeable goods. There is some evidence that the response of output and its components is larger in emerging economies, while the price response is larger in advanced economies. However, given the relatively small number of countries in our sample, we are reluctant to place too much weight on this conclusion. In the following section, we show that a standard international real business cycle model, augmented with stochastic terms of trade volatility, is broadly able to replicate these responses. We then use the model to shed light on the sectoral implications and theoretical causes of these responses.

**Figure 5: Impulse Responses to a Volatility Shock**  
Advanced economy sample



## 5. The Impact of Volatility Shocks: Theory

In this section, we embed stochastic terms of trade volatility in an otherwise standard small open economy real business cycle model. In the model, households choose consumption, saving and labour supply to maximise expected lifetime utility. Households consume three goods – non-tradeable, home- and foreign-produced tradeable goods – and can invest in three assets – a one-period risk-free bond traded in international capital markets and physical capital in the two domestic sectors. On the production side, firms seek to maximise profits by producing goods using capital, which is sector-specific, and labour, which is mobile across sectors. As well as terms of trade shocks, we also include productivity shocks in the model. These shocks help the model to match key features of the data, but play little role in the analysis.

## 5.1 Households

The economy features a representative household that maximises its expected lifetime utility given by:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\zeta}}{1+\zeta} \right) \quad (4)$$

where  $C$  is an aggregate consumption bundle comprising tradeable and non-tradeable goods and  $L$  represents the household's supply of labour. The consumption bundle is given by:

$$C_t \equiv \left[ \omega_T^{\frac{1}{\vartheta}} \left( C_t^T \right)^{\frac{\vartheta-1}{\vartheta}} + (1 - \omega_T)^{\frac{1}{\vartheta}} \left( C_t^{NT} \right)^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}} \quad (5)$$

where the elasticity of substitution between tradeables and non-tradeables is  $\vartheta$ , the weight of tradeables in the consumption basket is  $\omega_T$  and  $C^{NT}$  is the household's consumption of non-tradeables.  $C^T$  is the household's consumption of tradeable goods, which is itself a composite of home- and foreign-produced tradeable goods:

$$C_t^T \equiv \left[ \omega_H^{\frac{1}{\eta}} \left( C_t^H \right)^{\frac{\eta-1}{\eta}} + (1 - \omega_H)^{\frac{1}{\eta}} \left( C_t^F \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (6)$$

where the elasticity of substitution between the two tradeable goods is  $\eta$ , the weight of home-produced goods is  $\omega_H$ ,  $C^H$  is the household's consumption of home-produced tradeable goods and  $C^F$  is the household's consumption of foreign-produced tradeable goods.

To smooth consumption across time, households have access to three assets: a one-period risk-free bond, denominated in units of the foreign-tradeable good, and physical capital in the non-tradeable and home-tradeable sectors. Reflecting the fact that the domestic economy is small relative to the rest of the world, we assume that the interest rate that agents face when they borrow or lend abroad,  $r$ , is exogenous.

Household capital holdings,  $K^{NT}$  and  $K^H$ , are sector specific. We assume that the prices of all capital goods are denominated in units of the foreign-produced tradeable good.

We take the price of the foreign good as numeraire and set it equal to one. With this normalisation, the household's budget constraint is given by:

$$\begin{aligned} & C_t^F + e^{q_t} C_t^H + P^{NT} C_t^{NT} + I_t^{NT} + I_t^H + d_t(1+r) \\ & \leq W_t L_t + R_t^{NT} K_t^{NT} + R_t^H K_t^H + d_{t+1} - \frac{\psi}{2} (d_{t+1} - d)^2 \end{aligned} \quad (7)$$

where  $e^{q_t}$  is the price of home-produced tradeable goods in terms of foreign-produced tradeable goods – the terms of trade – and  $P^{NT}$  is the relative price of non-tradeable goods. The terms of trade is exogenous in the model, while  $P^{NT}$  is determined endogenously.  $I_t^{NT}$  and  $I_t^H$  are investment in the non-tradeable and home-tradeable sectors.  $W$ ,  $R^{NT}$  and  $R^H$  are the wage rate and return on capital in the non-tradeable and home-tradeable sectors. Note that as labour is mobile between the two sectors, firms in each sector pay the same wage. The final term on the right-hand-side of the equation represents portfolio adjustment costs that households must pay when holding foreign debt at a different level than its steady-state level,  $d$ . These ensure that the economy's foreign debt level is stationary and prevent precautionary savings diverging to infinity.<sup>8</sup>

The capital stock of each sector evolves according to the law of motion:

$$K_{t+1}^j = (1 - \delta)K_t^j + \left( 1 - \frac{\phi}{2} \left( \frac{I_t^j}{I_{t-1}^j} - 1 \right)^2 \right) I_t^j \quad (8)$$

for  $j = \{NT, H\}$ . The parameter  $\delta$  represents the depreciation rate of capital, while the parameter  $\phi$  controls the size of investment adjustment costs. We include these to ensure that the model economy does not deliver excessive investment volatility in response to shocks.

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<sup>8</sup> Portfolio adjustment costs are one of the several ad hoc methods commonly used to close small open economy models. Others include a debt-elastic interest rate premium or a time preference rate that varies with aggregate consumption. Schmitt-Grohé and Uribe (2003) show that all of these methods deliver almost identical dynamics at business-cycle frequencies. Another way of attaining a stationary asset distribution is to assume that the rate of time preference is smaller than the interest rate as in Aiyagari (1994).

Household optimisation implies that the demand for home- and foreign-produced tradeable goods is given by:

$$C_t^H = \omega_H \left( \frac{e^{q_t}}{P_t^T} \right)^{-\eta} C_t^T; C_t^F = (1 - \omega_H) \left( \frac{1}{P_t^T} \right)^{-\eta} C_t^T \quad (9)$$

where  $P_t^T \equiv \left[ \omega_H (e^{q_t})^{1-\eta} + (1 - \omega_H) \right]^{\frac{1}{1-\eta}}$  is the traded goods price index. The demand for tradeable and non-tradeable goods is:

$$C_t^T = \omega_T \left( \frac{P_t^T}{P_t} \right)^{-\vartheta} C_t; C_t^{NT} = (1 - \omega_T) \left( \frac{P_t^{NT}}{P_t} \right)^{-\vartheta} C_t \quad (10)$$

where  $P_t \equiv \left[ \omega_T (P_t^T)^{1-\vartheta} + (1 - \omega_T) (P_t^{NT})^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}$  is the consumer price index (CPI).

Using the household's decisions over different good types, we can re-write the household's budget constraint as:

$$\begin{aligned} & P_t C_t + I_t^{NT} + I_t^H + d_t(1+r) \\ & \leq W_t L_t + R_t^{NT} K_t^{NT} + R_t^H K_t^H + d_{t+1} - \frac{\Psi}{2} (d_{t+1} - d)^2 \end{aligned} \quad (11)$$

The household's optimal choice over consumption, labour supply and asset holdings implies the following intra and intertemporal conditions:

$$C_t^\sigma L_t^\zeta = \frac{W_t}{P_t} \quad (12)$$

$$1 = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t (1+r + \Psi(d_{t+1} - d))}{P_{t+1}} \right] \quad (13)$$

and

$$\varphi_t^j = \beta E_t \left[ (1 - \delta) \varphi_{t+1}^j + R_t^j \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right] \quad (14)$$

$$\begin{aligned} \frac{C_t^{-\sigma}}{P_t} &= \varphi_t^j \left[ 1 - \frac{\phi}{2} \left( \frac{I_t^j - I_{t-1}^j}{I_{t-1}^j} \right)^2 - \phi \frac{I_t^j}{I_{t-1}^j} \left( \frac{I_t^j - I_{t-1}^j}{I_{t-1}^j} \right) \right] \\ &+ \beta E_t \left[ \varphi_{t+1}^j \phi \left( \frac{I_{t+1}^j}{I_t^j} \right)^2 \left( \frac{I_{t+1}^j - I_t^j}{I_t^H} \right) \right] \end{aligned} \quad (15)$$

for  $j \in \{H, NT\}$  where  $\varphi_t^j$  is the Lagrangian associated with capital in sector  $j$ .

## 5.2 Firms

The home-tradeable and non-tradeable sectors both feature perfectly competitive firms that maximise profits, which are given by:

$$\pi_t^H = e^{q_t} Y_t^H - W_t L_t^H - R_t^H K_t^H \quad (16)$$

$$\pi_t^{NT} = P_t^{NT} Y_t^{NT} - W_t L_t^{NT} - R_t^{NT} K_t^{NT} \quad (17)$$

Firms in each sector produce output using a Cobb-Douglas production function:

$$Y_t^H = e^{a_t} (K_t^H)^\alpha (L_t^H)^{1-\alpha} \quad (18)$$

$$Y_t^{NT} = e^{a_t} (K_t^{NT})^\alpha (L_t^{NT})^{1-\alpha} \quad (19)$$

where  $e^{a_t}$  is a productivity shifter that is common to both sectors.

Profit maximisation by firms implies that factor prices are equated to the value of marginal products:

$$W_t = (1 - \alpha) e^{q_t} \frac{Y_t^H}{L_t^H} \quad (20)$$

$$W_t = (1 - \alpha) P_t^{NT} \frac{Y_t^{NT}}{L_t^{NT}} \quad (21)$$

$$R_t^H = \alpha e^{q_t} \frac{Y_t^H}{K_t^H} \quad (22)$$

$$R_t^{NT} = \alpha P_t^{NT} \frac{Y_t^{NT}}{K_t^{NT}} \quad (23)$$

### 5.3 Shock Processes

The model features three exogenous processes. First, productivity evolves according to an AR(1) process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad (24)$$

Second, the terms of trade and its volatility evolve according to the processes described in the empirical section and repeated here for convenience:

$$q_t = \rho_q q_{t-1} + e^{\sigma_{q,t}} u_{q,t} \quad (25)$$

$$\sigma_{q,t} = (1 - \rho_\sigma) \sigma_q + \rho_\sigma \sigma_{q,t-1} + \eta_q u_{\sigma,t} \quad (26)$$

The interpretation of the parameters is also as described in the empirical section.

### 5.4 Equilibrium Definition

A competitive equilibrium is given by an allocation  $\{C_t, L_t, L_t^H, L_t^{NT}, K_t^H, K_t^{NT}, I_t^H, I_t^{NT}, d_t, \varphi^H, \varphi^{NT}\}_{t=0}^\infty$  and goods and factor prices  $\{W_t, R_t^H, R_t^{NT}, P_t^{NT}, P_t\}_{t=0}^\infty$  where (i) consumers' satisfy their optimality conditions (Equations (12) to (15)) and capital evolves as per Equation (8); (ii) firms' zero-profit conditions given in Equations (20) to (23) hold; (iii) productivity and the terms of trade,  $a_t$ ,  $q_t$  and  $\sigma_{q,t}$ , follow the exogenous processes in Equations (24) to (26); and (iv) factor and goods markets clear.

Regarding factor market clearing, labour is fully mobile across sectors. Hence, its market clearing condition is given by:

$$L_t = L_t^H + L_t^{NT} \quad (27)$$

Goods market clearing implies that all production in the non-tradeable and tradeable sectors is consumed:

$$Y_t^{NT} = C_t^{NT} \quad (28)$$

$$Y_t^H = C_t^H + C_t^{H*} \quad (29)$$

where  $C_t^{H*}$  is consumption of the home-produced tradeable good by foreigners. The latter can be expressed in terms of home variables only. To do so, we use the equation for the evolution of foreign debt  $d_{t+1} - d_t = rd_t - NX_t$  where  $NX_t$  denotes net exports, defined as nominal exports minus nominal imports:

$$e^{q_t} C_t^{H*} = NX_t + C_t^F + I_t^{NT} + I_t^H \quad (30)$$

Substituting this equation into the tradeable goods market clearing condition and replacing net exports with the debt evolution equation we obtain the condition for home-produced goods market clearing in terms of home variables only:

$$e^{q_t} Y_t^H = (1+r)d_t - d_{t+1} + P_t^T C_t^T + I_t^{NT} + I_t^H + \frac{\Psi}{2} (d_{t+1} - d)^2 \quad (31)$$

## 5.5 Model Solution and Calibration

We solve the model using perturbation methods, taking a third-order approximation of the policy functions of the agents and the law of motion of the exogenous variables around the model's steady state. As Fernández-Villaverde *et al* (2011) discuss, in models with stochastic volatility it is necessary to take a third-order approximation of the model to capture the effects of volatility shocks independent of the other innovations in the model.<sup>9</sup>

We fix the value of a number of parameters using values generally found in the literature (Table 4). For households, we set the discount rate,  $\beta$ , equal to 0.99, the inverse of the elasticity of substitution,  $\sigma$ , and the inverse of the Frisch elasticity,  $\zeta$ , both equal to 2, consistent with values commonly used in the literature. We base the values of  $\vartheta$  and  $\eta$  on available estimates for the elasticity of substitution between traded and non-traded goods. For the elasticity of substitution between tradeables and non-tradeables,  $\vartheta$ , we use the estimate by Mendoza (1995), calculated for a sample of industrialised countries, and set that elasticity equal to 0.74. For the elasticity of substitution between home- and foreign-tradeables,

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<sup>9</sup> Specifically, a first-order approximation eliminates all of the effects of volatility shocks as certainty equivalence holds. A second-order approximation captures the effects of volatility shocks only through their interaction with shocks to the level of the terms of trade. It is only in a third-order (or higher) approximation that stochastic volatility shocks enter as independent arguments in the policy functions.

$\eta$ , we use the estimate of Corsetti, Dedola and Leduc (2008) and select a value of 0.85. We set the share of traded goods in the households' consumption basket,  $\omega_T$ , equal to 0.5, consistent with the estimates of Stockman and Tesar (1995). We also set the share of home goods in the tradeable goods basket equal to 0.5.

**Table 4: Parameter Values**

Parameter	Description	Value	Comments
$\beta$	Discount factor	0.99	Standard value
$\sigma$	Inverse elasticity of substitution	2	Standard value
$\zeta$	Inverse of Frisch elasticity	2	Standard value
$\omega_T$	Share of tradeables in consumption basket	0.5	As in Stockman and Tesar (1995)
$\omega_H$	Share of home goods in tradeable consumption basket	0.5	Imports share of consumption equals 25 per cent
$\vartheta$	Elasticity of substitution between tradeable and non-tradeable goods	0.74	As in Mendoza (1995)
$\eta$	Elasticity of substitution between home- and foreign-tradeable goods	0.85	As in Corsetti <i>et al</i> (2008)
$\alpha$	Capital share of income	$\frac{1}{3}$	Standard value
$\rho_a$	Persistence of shock to productivity	0.95	As in Fernández-Villaverde <i>et al</i> (2011)
$\psi$	Portfolio adjustment cost	$10^{-3}$	Small value to limit impact on dynamics

On the firm side, we set the capital share of income,  $\alpha$ , equal to  $1/3$  for both sectors. We follow Fernández-Villaverde *et al* (2011) in setting the persistence of productivity shocks,  $\rho_a$ , equal to 0.95. This choice has little effect on our results as we merely use this shock to calibrate the model. Finally, we set  $\psi$ , the portfolio adjustment cost of foreign debt, equal to  $10^{-3}$  for all the countries. This small value ensures that the foreign debt level is stationary, without significantly affecting the dynamic properties of the model (Schmitt-Grohé and Uribe 2003; Fernández-Villaverde *et al* 2011).

Conditional on these choices, we pick the remaining three parameters to match moments of the ergodic distribution generated by simulating the model to moments of the data. The three parameters are: (i)  $\sigma_a$ , the standard deviation of productivity shocks; (ii)  $\phi$ , the adjustment cost of investment; and (iii)  $d$ , the parameter that controls the average stock of foreign debt. The moments of the data that we match are: (i) output volatility; (ii) the volatility of investment relative to

output; and (iii) the ratio of net exports to output.<sup>10</sup> Table 5 provides the resulting parameter values.

Parameter	Australia	Brazil	Canada	Mexico	New Zealand	South Africa
$d$	-2.11	1.98	3.94	-3.27	2.63	7.54
$\phi$	7.89	17.20	1.87	10.88	4.04	1.66
$\sigma_a$	$1.00 \times 10^{-2}$	$1.14 \times 10^{-2}$	$1.08 \times 10^{-2}$	$1.85 \times 10^{-2}$	$1.01 \times 10^{-2}$	$1.07 \times 10^{-2}$

In general, the calibration assigns higher values of  $\sigma_a$  to economies whose output is more volatile. The parameter  $d$  helps the model to match the average ratio of net exports to output. Because in the model the real interest rate is greater than the average growth rate, economies who have, on average over the sample, run trade deficits are assigned a value of  $d$  that is less than 0, while economies that have run trade surpluses are assigned a value of  $d$  that is greater than 0.<sup>11</sup> The parameter  $\phi$  then varies to match the volatility of investment relative to output, conditional on the variances of productivity and the terms of trade.

<sup>10</sup> Because the moments are affected by a non-linear combination of parameters, we choose the parameters to minimise the sum of the quadratic distance between the model moments and the moments from the data. Specifically, for each economy, we simulate a sample of 200 observations and calculate moments based on these observations. We then repeat this procedure 200 times and calculate the mean of each moment across the 200 draws.

<sup>11</sup> In reality, some countries in our sample, such as Australia, have typically run net export deficits while being net debtors to the rest of the world, implying that  $d$  is greater than 0. In our model such a persistent pattern could exist if, for example, an economy's expected long-run growth rate was greater than that of the rest of the world (Engel and Rogers 2006).

## 6. Results

In this section, we analyse the quantitative implications of our model. First, we demonstrate that the model can successfully match some broad features of the macroeconomic data. Second, we show how an increase in terms of trade volatility affects the other variables in the model. Third, we quantify the contribution of terms of trade volatility shocks to the variance of the key macroeconomic variables in the model.

### 6.1 Moments

Table 6 compares the moments of the model to those of the data. The model matches the three calibrated moments – the variance of output, the relative variance of investment and the level of net exports relative to output – successfully for all countries. The model also comes reasonably close to matching the correlation of output with consumption. However, it is less successful at replicating some of the other moments of the data. In particular, the volatility of consumption relative to output is generally lower in the model than it is in the data. This is a common finding in small open economy real business cycle models and is generally resolved by assuming the absence of wealth effects on labour supply or adding trend growth shocks to the model (Correia, Neves and Rebelo 1995; Aguiar and Gopinath 2007).<sup>12</sup> The model also produces too much correlation between net exports and income and too little correlation between investment and income. The latter result might be due, in part, to our assumption that all investment goods are imported. We examine this issue in the robustness checks section below.

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<sup>12</sup> In our model, one can induce greater consumption volatility by increasing the magnitude of the portfolio adjustment cost,  $\psi$ . This makes it more costly for households to borrow and lend, which reduces consumption smoothing. However, we find that an extremely high value of  $\psi$  – generally in the order of 0.1 – is required for the volatility of consumption in the model to match that found in the data. And, with  $\psi$  at such a high level, the effect of portfolio adjustment costs on the model's dynamics cease to be negligible.

**Table 6: Empirical Second Moments**

	Australia		Brazil		Canada	
	Data	Model	Data	Model	Data	Model
$\sigma_y$	1.35	1.36	1.52	1.52	1.38	1.38
$\sigma_c/\sigma_y$	0.81	0.52	1.06	0.47	0.83	0.46
$\sigma_i/\sigma_y$	2.97	2.96	3.67	3.67	2.97	2.97
$\rho_{c,y}$	0.49	0.49	0.62	0.80	0.80	0.81
$\rho_{i,y}$	0.67	0.20	0.87	0.12	0.71	0.44
$\rho_{nx,y}$	-0.22	0.61	-0.26	0.55	0.18	0.57
$nx/y$	-0.94	-0.94	0.30	0.30	1.41	1.41
	Mexico		New Zealand		South Africa	
	Data	Model	Data	Model	Data	Model
$\sigma_y$	2.43	2.43	1.39	1.39	1.60	1.60
$\sigma_c/\sigma_y$	1.16	0.50	1.04	0.54	1.29	0.69
$\sigma_i/\sigma_y$	1.82	1.82	4.46	4.46	3.70	3.70
$\rho_{c,y}$	0.97	0.66	0.76	0.43	0.76	0.00
$\rho_{i,y}$	0.83	0.20	0.83	0.26	0.64	0.37
$\rho_{nx,y}$	-0.07	0.63	-0.18	0.58	-0.52	0.64
$nx/y$	-1.53	-1.53	0.80	0.80	2.53	2.53

Notes:  $\sigma_j$  indicates the standard deviation of variable  $j$ ;  $\rho_{i,j}$  indicates the correlation between variable  $i$  and variable  $j$

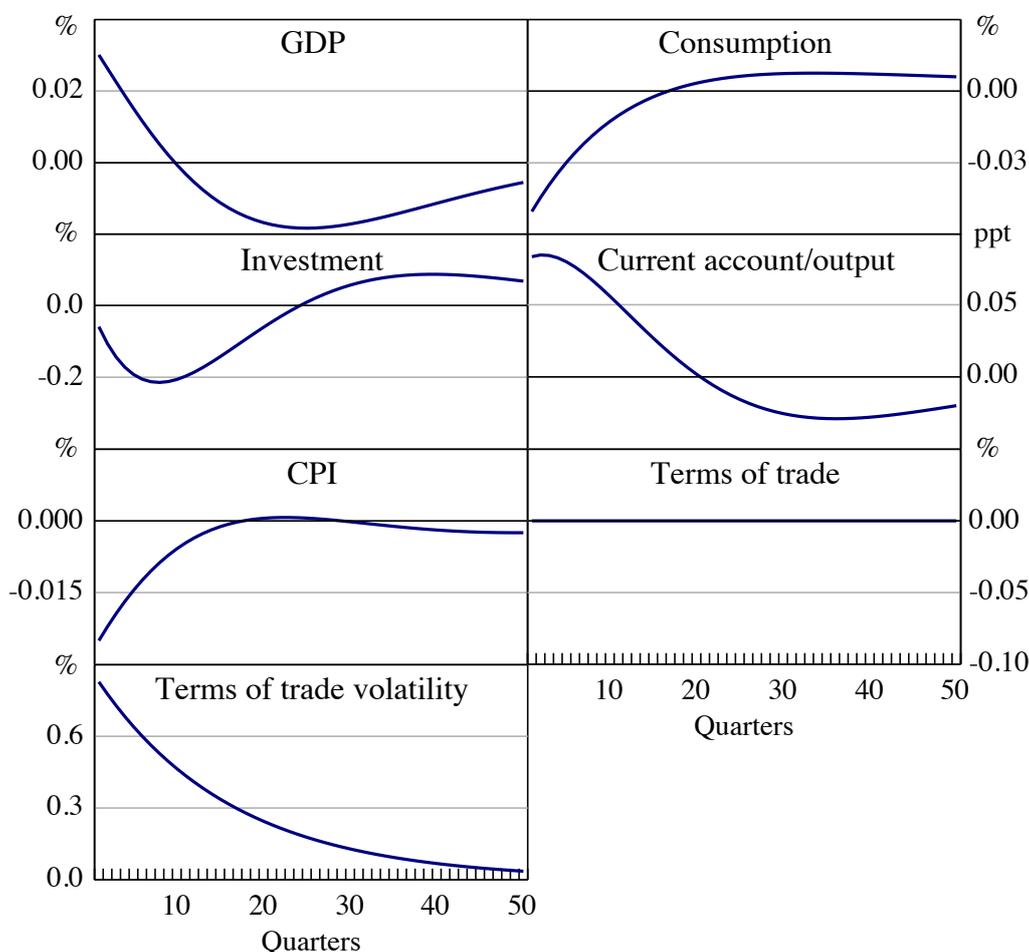
## 6.2 Impulse Response Functions

We now turn to the dynamic response of the economies to a shock to terms of trade volatility. As the pattern of the responses is broadly similar across economies, we focus and describe the Australian case in detail.

Figure 6 shows the estimated response of the Australian economy to a shock that increases the standard deviation of terms of trade volatility from 2.6 per cent to 6.0 per cent. To put this in context, the shock is of broadly the same magnitude as the estimated increase in Australian terms of trade volatility in the mid 2000s reported in Figure 2.

By construction, the volatility shock has no direct effect on the level of the terms of trade,  $q_t$ . Despite this, the shock induces a contraction in investment of around 0.2 per cent and in consumption of just under 0.1 per cent. As in the VAR responses, investment displays a ‘hump’ shaped response to the shock. The current account-to-output ratio also increases by around 0.1 per cent following the

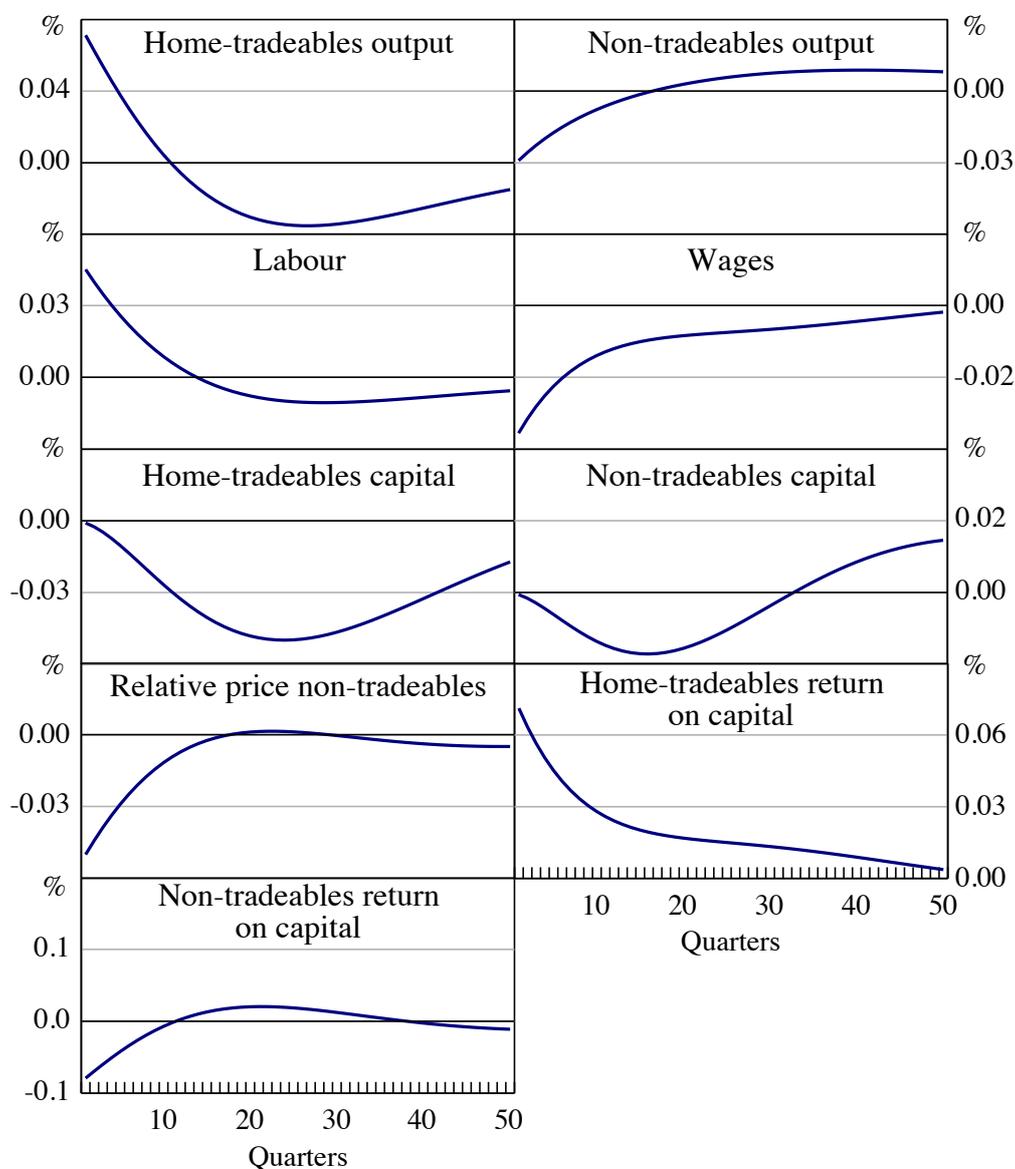
**Figure 6: Impulse Responses to a Volatility Shock**  
Australia



shock, while the price level decreases. Because the terms of trade does not change following the shock, this implies a decrease in the GDP deflator. The movements in domestic demand and net exports partly offset each other. Consequently, the absolute change in GDP is smaller than the changes in its individual expenditure components. In sum, the model qualitatively matches the empirical responses to a terms of trade volatility shock identified in the VAR, although the the responses are somewhat smaller.

The theoretical model also allows us to trace out the implications of the shock for factor utilisation and the sectoral composition of production (Figure 7). The results suggest that a positive volatility shock leads to an increase in hours worked and a decrease in investment in both the domestic sectors. This change in factor utilisation is associated with a decrease in real wages, while returns to capital in

**Figure 7: Impulse Responses to a Volatility Shock**  
Australia



the tradeable goods sector increases. The return to capital in the non-traded goods sector falls, reflecting the contraction in demand for the output of that sector.

Turning to the production side of the economy, the volatility shock brings about a change in the sectoral composition of output away from non-tradeables towards tradeables. The contraction in non-tradeable production follows directly from the decrease in consumption, although this is somewhat mitigated by a fall in the relative price of non-tradeable goods. Domestic consumption of tradeable goods also contracts. However, as wages have decreased relative to the price of tradeable goods, home-tradeable firms find it profitable to expand production. They export

this increased production, which contributes to the improvement in the trade and current account balances.

The intuition for the model responses comes from the household's Euler equation:

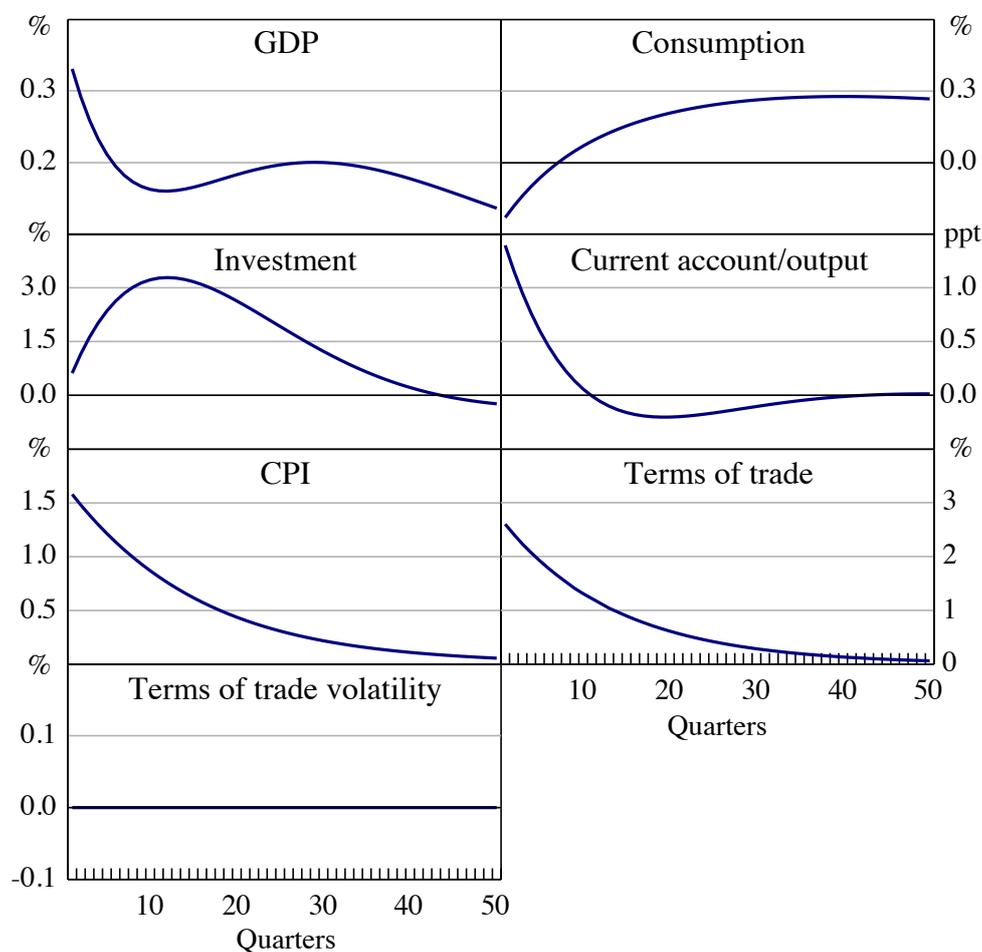
$$1 = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1+r+\psi(d_{t+1}-d)) P_t}{P_{t+1}} \right]$$

A shock to terms of trade volatility does not affect the expected level of consumption directly. But it does make agents more uncertain about their future income. All other things constant, this increases the expected marginal utility of future consumption,  $E_t \{ C_{t+1}^{-\sigma} \}$ . In response to this, agents reduce current consumption. This increases the marginal utility of consumption today and frees up more resources for future consumption, thereby also reducing the expected marginal utility of consumption in the future. In addition, the reduction in consumer demand lowers prices today relative to future prices, which decreases  $E_t \{ P_t/P_{t+1} \}$ . Because the terms of trade is exogenous, the adjustment in prices must occur through changes in the relative price of non-tradeable goods. Finally, an increase in volatility also increases the attractiveness of holding foreign assets, which provide a hedge against a large adverse change in the relative price of domestic tradeables in the future. In addition to the decrease in consumption, this explains why the volatility shock is associated with an increase in the economy's current account balance.

It is instructive to compare the response of the economy to a volatility shock to its response to a shock to the level of the terms of trade, shown in Figure (8). That shock brings about (i) a prolonged increase in consumption; (ii) an investment boom; (iii) an improvement in the current account; (iv) an increase in home-produced goods output and a temporary decrease in non-tradeable goods production; and (v) an increase in hours worked and real wages. These results are consistent with the findings in Mendoza (1995).

Although both shocks lead to an increase in domestic tradeable output and a reduction in foreign debt, the terms of trade level shock is far more favourable to domestic agents. This shock encourages firms to invest in order to increase production to take advantage of temporarily high relative goods prices. The resulting increase in the capital-to-labour ratio drives the increase in real wages,

**Figure 8: Impulse Responses to a Terms of Trade Shock  
Australia**



which triggers the expansion in labour supply. However, our results demonstrate that the extent of the expansionary impact of a terms of trade boom will depend upon the degree of terms of trade volatility that an economy experiences. A given increase in the terms of trade will be less expansionary if it occurs during a time of high terms of trade volatility and more expansionary if it occurs when volatility is low.

### 6.3 Variance Decompositions

In this section we study the contribution to aggregate fluctuations of each of the three shocks in our model. Because we calculate a non-linear approximation to the policy function, it is not possible to assign the total variance to individual shocks as in a linear model. Therefore, in this exercise, we set the realisations of one or

two of the shocks to zero and measure the volatility of the economy when we simulate the economy with the remaining shocks.

We study four macro-aggregates: output, consumption, investment and net exports and explore four scenarios: (i) all shocks; (ii) terms of trade level shocks only; (iii) terms of trade level and volatility shocks jointly; and (iv) terms of trade volatility shocks only.<sup>13</sup>

Table 7 reports the variance decompositions for all six countries. For each, productivity shocks are the main contributor for output fluctuations, while shocks to the level and volatility of the terms of trade are key drivers of fluctuations in investment and net exports.<sup>14</sup>

By themselves, volatility shocks account for only a very small portion of the standard deviation of output and consumption for all of the countries in our sample. The impact of these shocks for investment and net exports is somewhat greater, although still modest given the high variance of these series.

However, interacted with shocks to the level of the terms of trade, volatility shocks are estimated to have a meaningful impact on macroeconomic outcomes. For example, with only shocks to the level of the terms of trade, the standard deviation of Australian investment is estimated to be 2.83 per cent. With shocks to the volatility as well as the level of the terms of trade, the standard deviation of investment is estimated to be 3.77 per cent, that is, 33 per cent greater.

---

13 Note that in each decomposition, agents in the model believe that the shocks are distributed according to the law of motion specified in the previous section. Consequently, they will respond to volatility shocks even when the realisation of shocks to the level of the terms of trade is always zero.

14 This is likely to overstate the importance of productivity shocks, which in our model account for all variation in macroeconomic aggregates not driven by terms of trade shocks.

**Table 7: Variance Decomposition**

<b>Australia</b>				
	All	TOT only	TOT + volatility	Volatility only
$\sigma_y$	1.35	0.39	0.52	0.01
$\sigma_c$	0.71	0.33	0.44	0.02
$\sigma_i$	4.03	2.83	3.77	0.06
$\sigma_{nx}$	2.56	1.84	2.43	0.03
<b>Brazil</b>				
	All	TOT only	TOT + volatility	Volatility only
$\sigma_y$	1.52	0.37	0.51	0.06
$\sigma_c$	0.71	0.17	0.26	0.09
$\sigma_i$	5.57	4.07	5.46	0.21
$\sigma_{nx}$	3.18	2.31	3.05	0.15
<b>Canada</b>				
	All	TOT only	TOT + volatility	Volatility only
$\sigma_y$	1.38	0.21	0.26	0.00
$\sigma_c$	0.63	0.21	0.27	0.00
$\sigma_i$	4.09	2.28	2.81	0.00
$\sigma_{nx}$	1.35	0.88	1.09	0.00
<b>Mexico</b>				
	All	TOT only	TOT + volatility	Volatility only
$\sigma_y$	2.43	0.51	0.72	0.01
$\sigma_c$	1.21	0.44	0.62	0.02
$\sigma_i$	4.41	2.74	3.80	0.04
$\sigma_{nx}$	3.62	2.37	3.32	0.03
<b>New Zealand</b>				
	All	TOT only	TOT + volatility	Volatility only
$\sigma_y$	1.39	0.46	0.56	0.01
$\sigma_c$	0.74	0.41	0.49	0.02
$\sigma_i$	6.19	4.85	5.85	0.09
$\sigma_{nx}$	2.67	2.14	2.55	0.04
<b>South Africa</b>				
	All	TOT only	TOT + volatility	Volatility only
$\sigma_y$	1.60	0.76	0.87	0.00
$\sigma_c$	1.10	0.82	0.94	0.00
$\sigma_i$	5.95	4.38	5.06	0.03
$\sigma_{nx}$	3.56	3.03	3.47	0.01

Notes: TOT indicates terms of trade;  $\sigma_j$  indicates the standard deviation of variable  $j$

Indeed, for countries like Australia, Brazil and Mexico, the volatility of the key macroeconomic variables is between 20 and 30 per cent higher when there are both volatility and level shocks than it is when terms of trade level shocks operate alone. That is, for these countries between a fifth and a third of the effect of the terms of trade on macroeconomic volatility comes through changes in the volatility of terms of trade shocks. For countries like Canada, New Zealand and South Africa, the contribution of volatility to the overall effects of terms of trade shocks is smaller. However, even for these countries, our results suggest that between 10 and 20 per cent of the impact of the terms of trade on the key macroeconomic variables is due in part to volatility in the terms of trade.

Because of the non-linear structure of our model, it is difficult to isolate the exact channels through which interactions between the level and volatility of the terms of trade affect the macroeconomy. However, much of the explanation may come from the fact that stochastic volatility increases the variance of the terms of trade and larger shocks to the terms of trade imply greater macroeconomic volatility. To see the impact of stochastic volatility on the variance of the terms of trade, first note that in the absence of stochastic volatility, that is if  $\eta_q = 0 \forall t$ , then the variance of the terms of trade,  $var(q_t)$  is:

$$var(q_t) = \frac{\exp(2\sigma_q)}{1 - \rho_q^2} \quad (32)$$

In contrast, when stochastic volatility is present, the variance of the terms of trade is:

$$var(q_t) = \frac{\exp(2\sigma_q + 2\chi^2)}{1 - \rho_q^2} \quad (33)$$

where  $\chi^2 = \frac{\eta_q^2}{1 - \rho_\sigma^2}$ . For Australia, the presence of stochastic volatility increases the standard deviation of movements in the terms of trade by almost 50 per cent, from 7 per cent to 10 per cent. Although other effects are likely to exist, this direct impact appears about large enough to explain the change in macroeconomic volatility between the scenario with terms of trade level shocks only and the scenario with both terms of trade level and volatility shocks.

## 7. Robustness Checks

In this section we examine the robustness of our results to alternative parameter assumptions and modeling choices.

### 7.1 Alternative Parameter Values

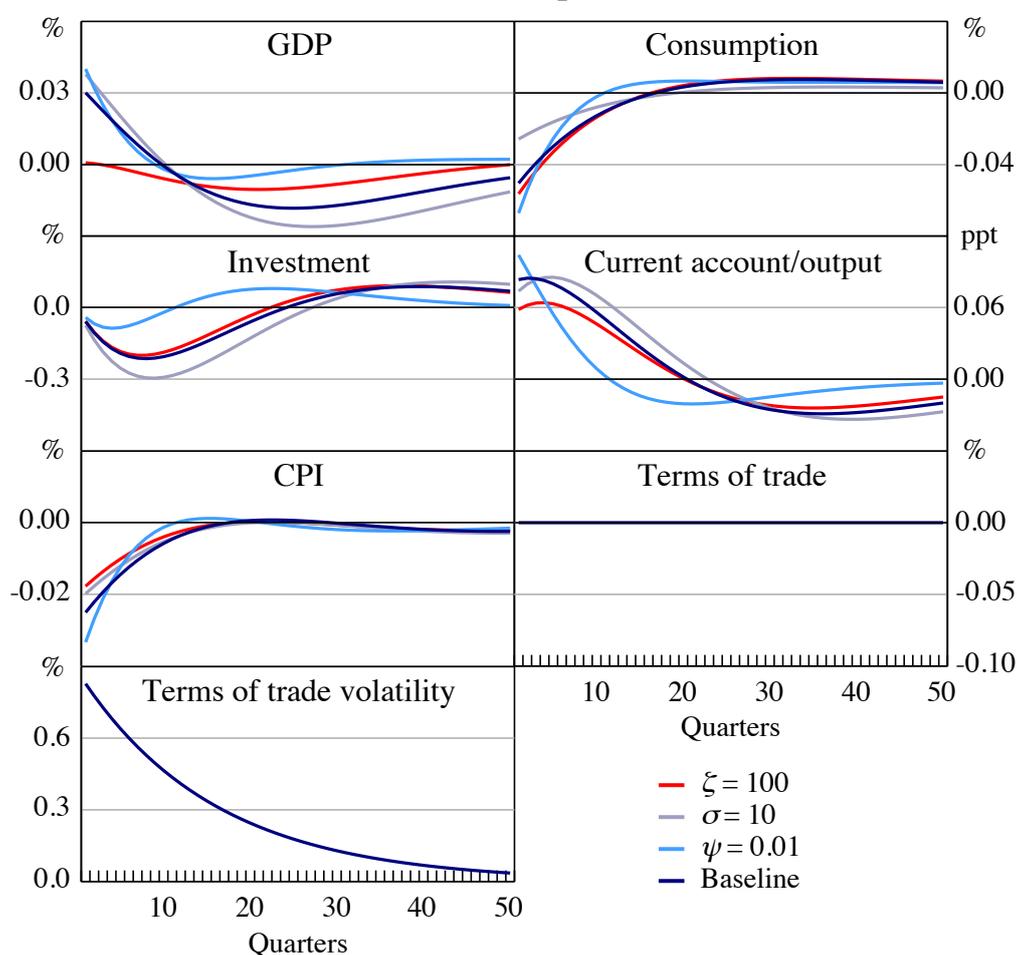
As a first exercise, we test the sensitivity of our model's dynamics to alternative parameter values. In particular we consider: (i) increasing the inverse of the labour supply elasticity,  $\zeta$ , from 2 to 100; (ii) increasing the inverse of the elasticity of substitution,  $\sigma$ , from 2 to 10; and (iii) increasing the parameter governing the sensitivity of the risk-free interest rate to the foreign debt level,  $\psi$ , from  $10^{-3}$  to  $10^{-2}$ . We examine each of these alternative parameter choices separately, leaving the other parameters at the same level as in the baseline model presented above. Figures 9 and 10 show impulse responses to a one standard deviation terms of trade volatility shock in Australia under the alternative parameter values.

An increase in  $\zeta$  decreases the willingness of households to adjust their labour supply in response to macroeconomic shocks. Relative to the baseline case, an increase in terms of trade volatility now triggers a smaller increase in tradeables output, larger contraction in non-tradeables output and consumption, and a larger improvement in the current account. That is, because households are less willing to respond to adverse income shocks by increasing their labour supply, they engage in more precautionary foreign asset accumulation. Aggregate GDP also decreases following the shock in this scenario, consistent with the empirical responses in Section 4.

In contrast, a decrease in the intertemporal elasticity of substitution (equivalent to an increase in  $\sigma$ ) reduces the consumption response to changes in terms of trade volatility. This largely occurs through a smaller decline in non-tradeables consumption, leaving the response of the other variables largely unaffected.

In our baseline results, we assumed an extremely low value of  $\psi$  in order to minimise the effect of this parameter on the dynamics of the model. Other papers that have estimated the value of this parameter for various countries have tended

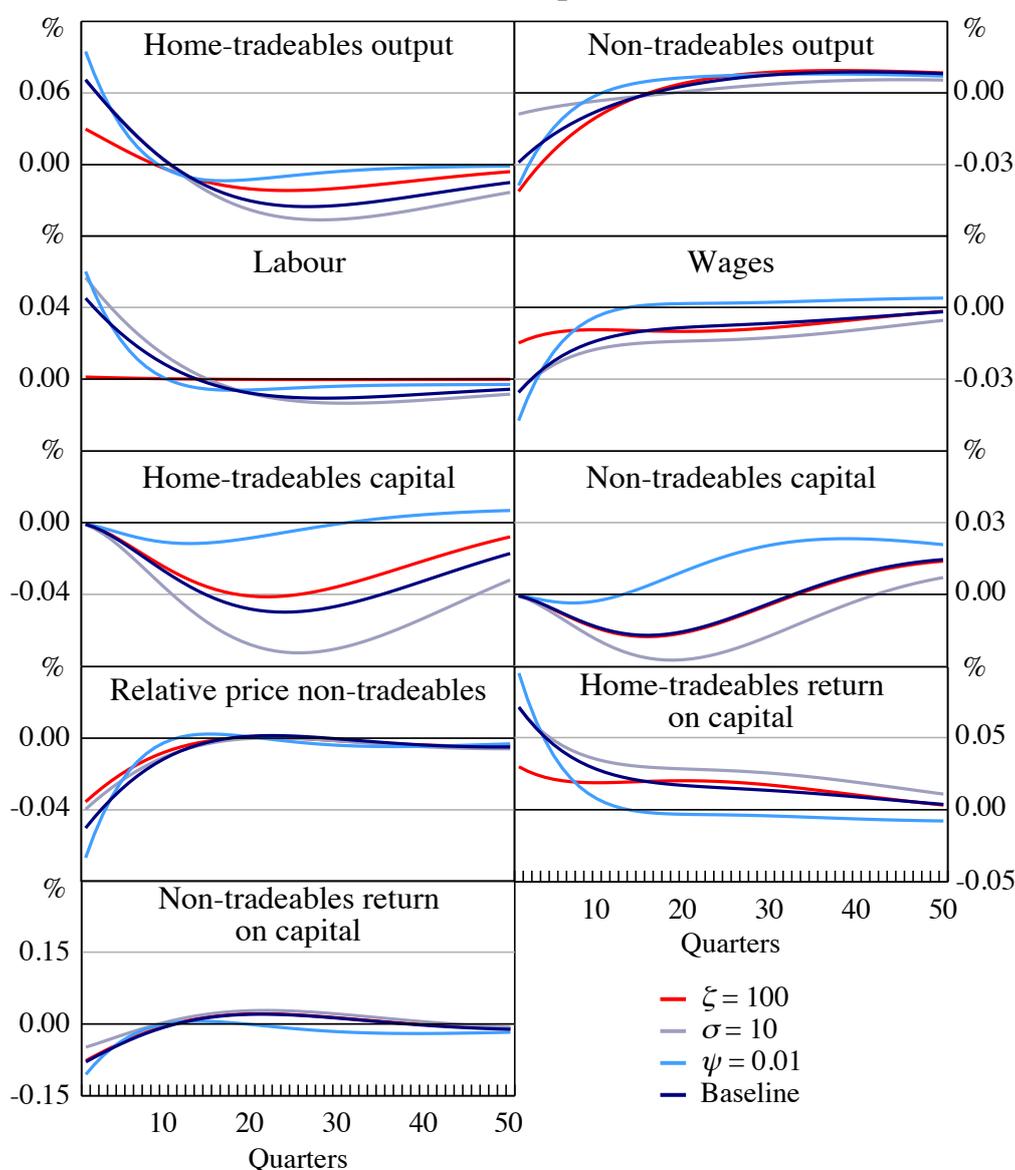
**Figure 9: Impulse Responses to a Volatility Shock**  
Australia – alternative parameter values



to find higher values.<sup>15</sup> A higher value of  $\psi$  penalises the economy for varying its stock of foreign assets. Setting this parameter to  $10^{-2}$  reduces the amount of time that it takes for the model to converge to its steady state. However, the initial responses of the variables to the terms of trade volatility shock are broadly similar to the baseline model.

<sup>15</sup> For example, Fernández-Villaverde *et al* (2011) for a selection of South American economies and Jääskela and Nimark (2011) for Australia, while Justiniano and Preston (2010) calibrated this parameter to  $10^{-2}$  in models of Australia, Canada and New Zealand

**Figure 10: Impulse Responses to a Volatility Shock**  
Australia – alternative parameter values



## 7.2 Home-produced Components of Investment

In our baseline model we assumed that the investment good was priced in units of the foreign tradeable good. This choice was motivated by the stylised fact that prices of investment goods differ less across countries than the prices of consumption goods (Hsieh and Klenow 2009). In this exercise, we instead assume that the investment good is priced in the same units as the economy's consumption good. That is, we allow the prices of home-produced goods to affect prices of investment goods. To do this, we assume that the investment good is a constant

elasticity of substitution aggregate of tradeable and non-tradeable goods and that the tradeable component is itself an aggregate of home and foreign tradeable goods.<sup>16</sup>

Because we are interested in how including home-produced investment goods affects the fit of the model, as well as the dynamics, we first recalibrate the new model to match the same moments of the data as we did with the baseline model. Table 8 shows the resulting model moments for Australia. Including a home-produced component of investment goods increases the correlation between investment and output marginally, which improves the fit of the model in that dimension. However, it also reduces the volatility of consumption relative to that of output.

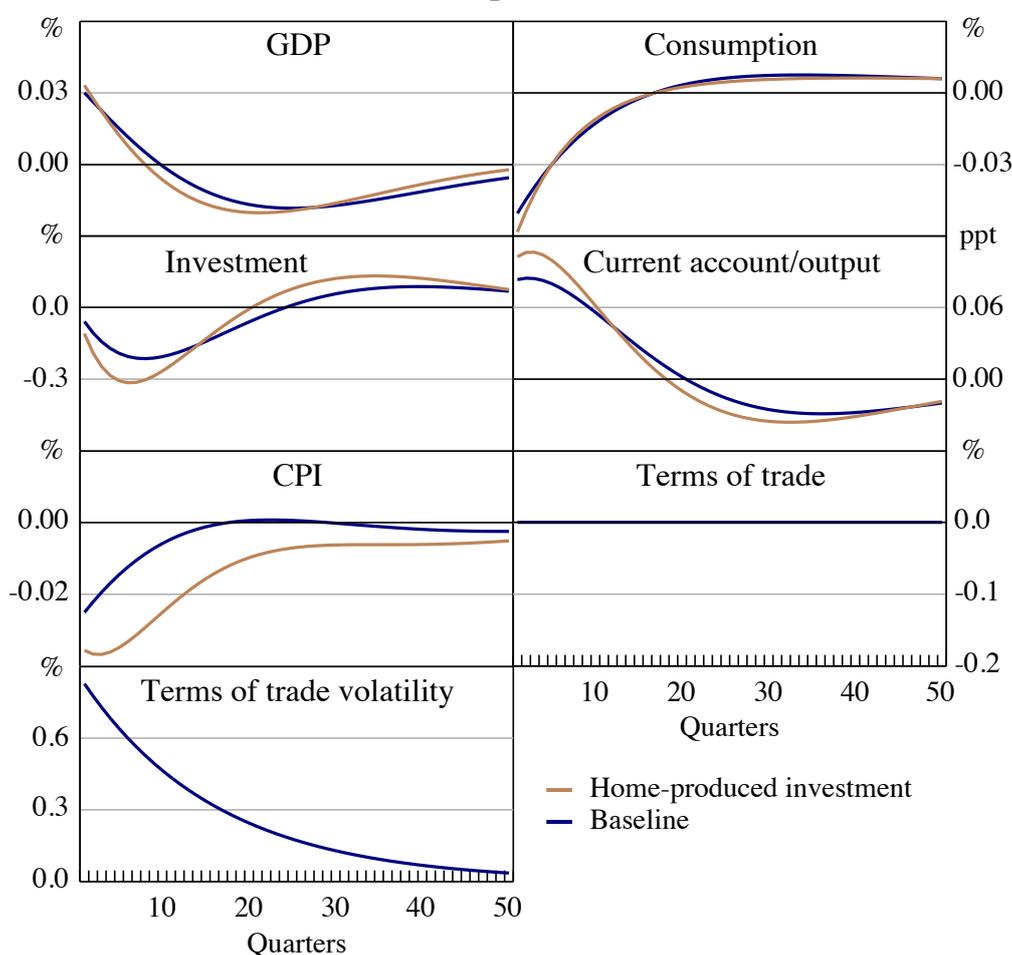
**Table 8: Moments – Home-produced Investment Model**

	$\sigma_y$	$\sigma_c/\sigma_y$	$\sigma_i/\sigma_y$	$\rho_{c,y}$	$\rho_{i,y}$	$\rho_{nx,y}$	$nx/y$
Data	1.35	0.81	2.97	0.49	0.67	-0.22	-0.94
Model	1.36	0.46	2.97	0.56	0.28	0.55	-0.94

Next, we examine the dynamic response of the economy to a terms of trade volatility shock, shown in Figure 11. Including a home-produced component of investment has very little effect on the response of the economy to a volatility shock. In sum, in terms of both model fit and dynamics there seems to be little to choose between the two model specifications.

16 To be precise:  $I_t \equiv \left[ \omega_T^{\frac{1}{\vartheta}} \left( I_t^T \right)^{\frac{\vartheta-1}{\vartheta}} + (1 - \omega_T)^{\frac{1}{\vartheta}} \left( I_t^{NT} \right)^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}}$  and  $I_t^T \equiv \left[ \omega_H^{\frac{1}{\eta}} \left( I_t^H \right)^{\frac{\eta-1}{\eta}} + (1 - \omega_H)^{\frac{1}{\eta}} \left( I_t^F \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$ , where  $\omega_T$  is the share of tradeable goods in the aggregate investment good, while  $\omega_H$  is the share of the home-produced goods in the tradeables goods aggregate.

**Figure 11: Impulse Responses to a Volatility Shock**  
Australia – home-produced investment



## 8. Conclusion

This paper has contributed to the literature examining time-varying volatility in macroeconomics by studying the effects of changes in the volatility of the terms of trade, a plausibly exogenous relative price for small open commodity-exporting economies.

Our empirical estimates for six of these economies demonstrate that the magnitude of terms of trade shocks varies considerably over time. Using a panel VAR we demonstrate that a volatility shock reduces both consumption and investment. Aggregate output also decreases following the shock and the current account balance increases when the shock hits, and remains above trend before decreasing as domestic demand recovers. There is also a persistent decrease in the price level.

Our small open economy real business cycle model can replicate the responses to the volatility shock generated by the VAR. We use the model to explore the mechanisms behind these responses and to examine their sectoral impacts. In the model, a shock to terms of trade volatility reduces consumption, causes a boom in the tradeable sector at the expense of the non-tradeable sector and triggers a shift in the factor intensity of production away from capital towards labour. The decrease in domestic absorption and the increase in tradeables production leads to an increase in the trade balance that allows the economy to reduce its foreign borrowing.

The model allows us to quantify the contribution of terms of trade volatility shocks to the fluctuations of macro-aggregates. Although the direct contribution of terms of trade volatility shocks to the variance of key variables is rather small, we find that these shocks have a meaningful economic effect in interaction with shocks to the level of the terms of trade. Our estimates suggest that terms of trade volatility shocks account for between one-fifth and one-third of the total effect of the terms of trade on the volatility of output, consumption, investment and net exports in the countries in our sample.

Our results point to a number of promising avenues for further research. The disaggregated VAR results hint that, for emerging economies, the response to volatility shocks occur mainly through quantities while, for advanced economies, the response occurs mainly through prices. More detailed empirical work using a larger sample of economies could shed light on the robustness of this result. And, if it does turn out to be robust, further work would be needed to understand why this occurs.

## **Appendix A: Data Sources, Definitions and Transformations**

### **Terms of trade data**

All terms of trade data were sourced from national statistical agencies, except for Canada, for which data were from the OECD. For Australia, Brazil, New Zealand and South Africa, published terms of trade indices were used. For Canada, we constructed a terms of trade index by dividing the exports of goods and services deflator by the imports of goods and services deflator. For Mexico, we constructed a terms of trade index by dividing the exports price index by the imports price index. The raw data for Australia, Canada, New Zealand and South Africa were quarterly. For Brazil and Mexico, we constructed a quarterly series using quarterly averages of monthly data. Samples and sources for the individual countries are:

Australia: sample – 1959:Q3–2011:Q2; source – ‘Australian National Accounts: National Income, Expenditure and Product’, Australian Bureau of Statistics Cat No 5206.0 ([www.abs.gov.au](http://www.abs.gov.au))

Brazil: sample – 1978:Q1–2011:Q2; source – Institute for Applied Economic Research ([www.ipeadata.gov.br](http://www.ipeadata.gov.br))

Canada: sample – 1961:Q1–2011:Q2; source – OECD ([www.oecd.org](http://www.oecd.org))

Mexico: sample – 1970:Q1–2011:Q2; source – Instituto Nacional de Estadística y Geografía ([www.inegi.org.mx](http://www.inegi.org.mx))

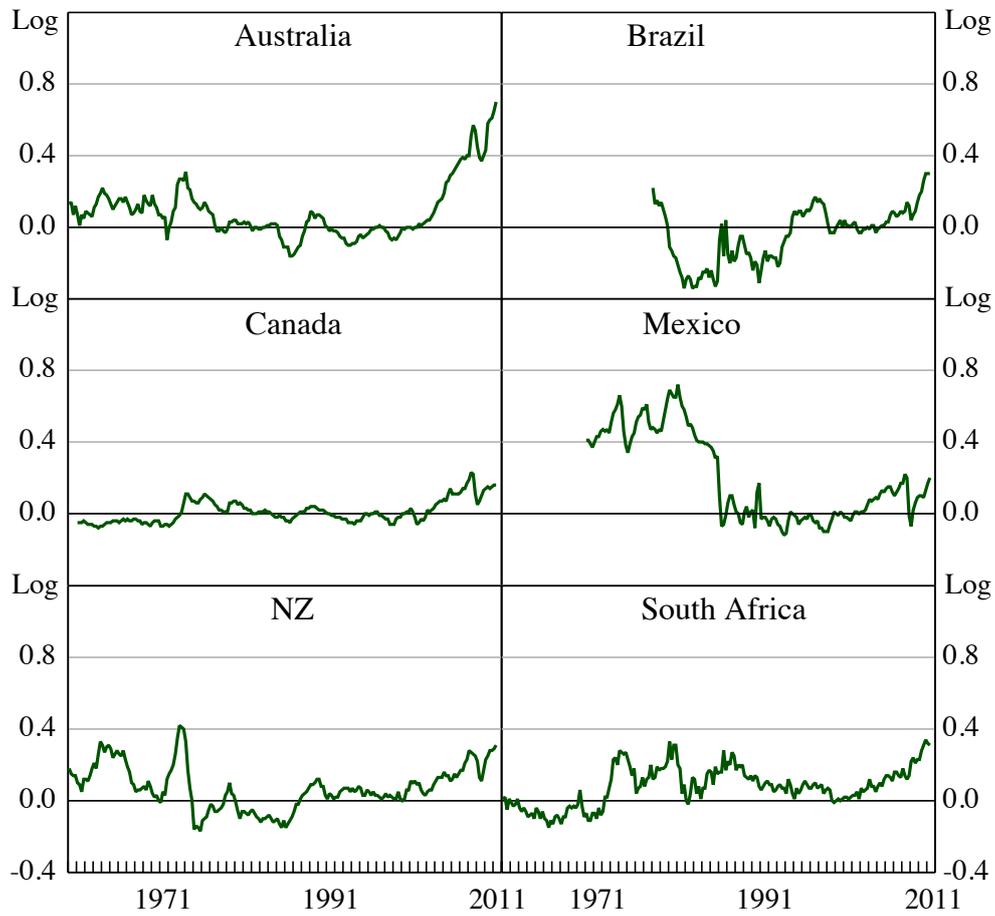
New Zealand: sample – 1957:Q1–2011:Q2; source – Statistics New Zealand ([www.stats.govt.nz](http://www.stats.govt.nz))

South Africa: sample – 1960:Q1–2011:Q2; source – South African Reserve Bank ([www.resbank.co.za](http://www.resbank.co.za))

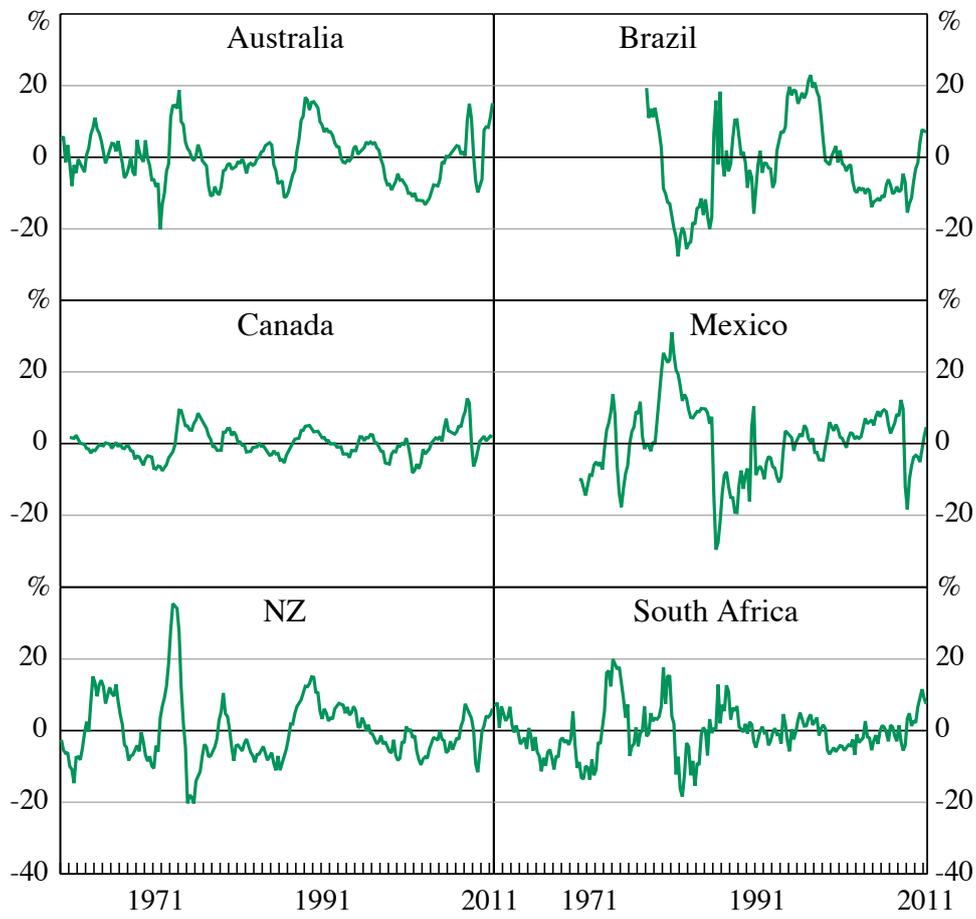
### **National accounts data**

For all countries, data for gross domestic product and its components were sourced from the OECD ‘Economic Outlook Database’ ([www.oecd.org](http://www.oecd.org)). All national accounts data are HP-filtered using a smoothing parameter of 1 600.

**Figure A1: Terms of Trade**  
2000:Q1 = 100



**Figure A2: Terms of Trade**  
Band-pass filter



## Appendix B: Terms of Trade Processes – HP-filtered Data

Table B1 shows the results for the empirical estimation when the terms of trade processes are HP-filtered using a smoothing parameter of 1 600.

<b>Table B1: Posterior Medians – HP-filtered Data</b>						
	Australia	Brazil	Canada	Mexico	New Zealand	South Africa
$\rho_q$	0.84 (0.81, 0.89)	0.77 (0.65, 0.89)	0.83 (0.77, 0.91)	0.78 (0.71, 0.89)	0.83 (0.77, 0.92)	0.69 (0.60, 0.80)
$\sigma_q$	-3.69 (-4.19, -2.95)	-3.36 (-3.87, -2.74)	-4.45 (-4.82, -4.00)	-3.46 (-3.88, -3.02)	-3.53 (-3.96, -2.91)	-3.38 (-3.95, -2.81)
$\rho_\sigma$	0.94 (0.78, 0.99)	0.93 (0.78, 1.00)	0.89 (0.62, 0.99)	0.87 (0.66, 0.97)	0.95 (0.86, 1.00)	0.97 (0.88, 1.00)
$\eta_q$	0.20 (0.12, 0.35)	0.21 (0.11, 0.38)	0.23 (0.11, 0.45)	0.31 (0.20, 0.46)	0.14 (0.08, 0.24)	0.10 (0.05, 0.21)

Note: 95 per cent credible sets in parantheses

## Appendix C: Estimating Stochastic Volatility

This appendix describes our procedure for estimating the stochastic volatility of the terms of trade. For a more detailed description of the use of the particle filter to estimate macroeconomic models, see Fernández-Villaverde and Rubio-Ramírez (2007).

Denote the vector of parameters to be estimated as  $\Psi = \{\rho_q, \rho_\sigma, \sigma_q, \eta_q\}$  and the log of the prior probability of observing a given vector of parameters  $\mathcal{L}(\Psi)$ . The function  $\mathcal{L}(\Psi)$  summarises what is known about the parameters prior to estimation. The log-likelihood of observing the dataset  $q^T \equiv \{q_1, \dots, q_T\}$  for a given parameter vector is denoted  $\mathcal{L}(q^T | \Psi)$ .

The likelihood of the data given the parameters factorises to:

$$\exp\left(\mathcal{L}(q^T | \Psi)\right) = p(q^T | \Psi) = \prod_{t=1}^T p(q_t | q^{t-1}; \Psi)$$

The final term in this expression expands as follows:

$$\prod_{t=1}^T p(q_t | q^{t-1}; \Psi) = \prod_{t=1}^T \int p(q_t | q_{t-1}, \sigma_{q,t}; \Psi) p(\sigma_{q,t} | q^{t-1}; \Psi) d\sigma_{q,t} \quad (\text{C1})$$

Computing this expression is difficult because the sequence of conditional densities  $\left\{p(\sigma_{q,t} | q^{t-1}; \Psi)\right\}_{t=1}^T$  has no analytical characterisation. A standard procedure, which we follow, is to substitute the density  $p(\sigma_{q,t} | q^{t-1}; \Psi)$  with an empirical draw from it. To obtain these draws, we follow Algorithm 1, which we borrow from Fernández-Villaverde *et al* (2011).

### Algorithm 1

*Step 0: initialisation*

Sample  $N$  particles,  $\left\{\sigma_{q,0}^i\right\}_{i=1}^N$  from the initial distribution  $p(\sigma_{q,0} | \Psi)$ .

*Step 1: prediction*

Sample  $N$  one-step-ahead forecasted particles  $\left\{ \sigma_{q,t|t-1}^i \right\}_{i=1}^N$  using  $\left\{ \sigma_{q,t-1|t-1}^i \right\}_{i=1}^N$ , the law of motion for the states (Equation (2)) and the distribution of shocks  $u_{\sigma_{q,t}}$ .

*Step 2: filtering*

Assign each draw  $\left( \sigma_{q,t|t-1}^i \right)$  the weight  $\omega_t^i$ , where:

$$\omega_t^i = \frac{p\left(q_t | q_{t-1}, \sigma_{q,t|t-1}^i; \Psi\right)}{\sum_{i=1}^N p\left(q_t | q_{t-1}, \sigma_{q,t|t-1}^i; \Psi\right)} \quad (\text{C2})$$

*Step 3: resampling*

Generate a new set of particles by sampling  $N$  times with replacement from  $\left\{ \sigma_{q,t|t-1}^i \right\}_{i=1}^N$  using the probabilities  $\left\{ \omega_t^i \right\}_{i=1}^N$ . Call the draw  $\left\{ \sigma_{q,t|t}^i \right\}_{i=1}^N$ . In effect, this step builds the draws  $\left\{ \sigma_{q,t|t}^i \right\}_{i=1}^N$  recursively from  $\left\{ \sigma_{q,t|t-1}^i \right\}_{i=1}^N$  using the information on  $q_t$ .

If  $t < T$ , set  $t = t + 1$  and return to Step 1. Otherwise stop.

Using the law of motion for the terms of trade in Equation (1), we can evaluate  $p\left(q_t | q_{t-1}, \sigma_{q,t|t-1}^i; \Psi\right)$  for any  $\sigma_{q,t|t-1}^i$ . Moreover, from the law of large numbers we know that:

$$\int p\left(q_t | q_{t-1}, \sigma_{q,t}; \Psi\right) p\left(\sigma_{q,t} | q^{t-1}; \Psi\right) d\sigma_{q,t} \approx \frac{1}{N} \sum_{i=1}^N p\left(q_t | q_{t-1}, \sigma_{q,t|t-1}^i; \Psi\right)$$

Algorithm 1 provides a sequence of  $\left\{ \sigma_{q,t|t-1}^i \right\}_{i=1}^N$  for all  $t$ . Consequently, the algorithm gives us the information needed to evaluate Equation (C1).

To calculate the posterior distribution of the parameters, we repeat this procedure 25 000 times. At each iteration, we update our parameter draw using a random walk Metropolis-Hastings procedure, scaling the proposal density to induce an

acceptance ratio of around 25 per cent. We discard the initial 5 000 draws and conduct our posterior inference on the remaining draws. For each evaluation of the likelihood we use 2 000 particles.

## Appendix D: Terms of Trade Processes – GARCH Estimation

To examine the robustness of our assumption about the functional form of the empirical terms of trade process, we also estimated exponential generalised autoregressive conditional heteroskedasticity (EGARCH) models of the form:

$$q_t = \rho_{q,t-1} + e^{\sigma_{q,t}} u_{q,t} \quad (\text{D1})$$

$$\begin{aligned} \sigma_{q,t} = & (1 - \rho_\sigma) \sigma_q + \rho_\sigma \sigma_{q,t-1} \\ & + \alpha (|u_{t-1}/\sigma_{q,t-1}| - E|u_{t-1}/\sigma_{q,t-1}|) + \beta (u_{q,t-1}/\sigma_{q,t-1}) \end{aligned} \quad (\text{D2})$$

The EGARCH model differs from our baseline stochastic volatility model in two respects. First, in the EGARCH model the equation for the volatility of the terms of trade shocks does not include an error term. Instead, increases in volatility can only occur because of large shocks to the terms of trade. In particular, if the parameter  $\alpha > 0$ , the model implies that a deviation of  $u_{t-1}$  from its expected value causes the variance of shocks to the volatility of the terms of trade to be larger than otherwise. The second difference is that we allow positive and negative shocks to have an asymmetric effect on volatility. If  $\beta = 0$  then a positive shock to the terms of trade has the same effect on volatility as a negative shock. In contrast, if  $\beta > 0$  then a positive surprise increases volatility by more than a negative surprise.

The model was estimated assuming a Student- $t$  distribution for the errors of the terms of trade.<sup>17</sup> We estimated this model using maximum likelihood methods without imposing any priors on the parameter values. Thus, this exercise also provides a check on the restrictiveness of the priors in the Bayesian estimation of the baseline stochastic volatility model. Table D1 shows the result of this estimation.

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<sup>17</sup> Estimates assuming a Gaussian distribution produced very similar results.

**Table D1: EGARCH Estimation**

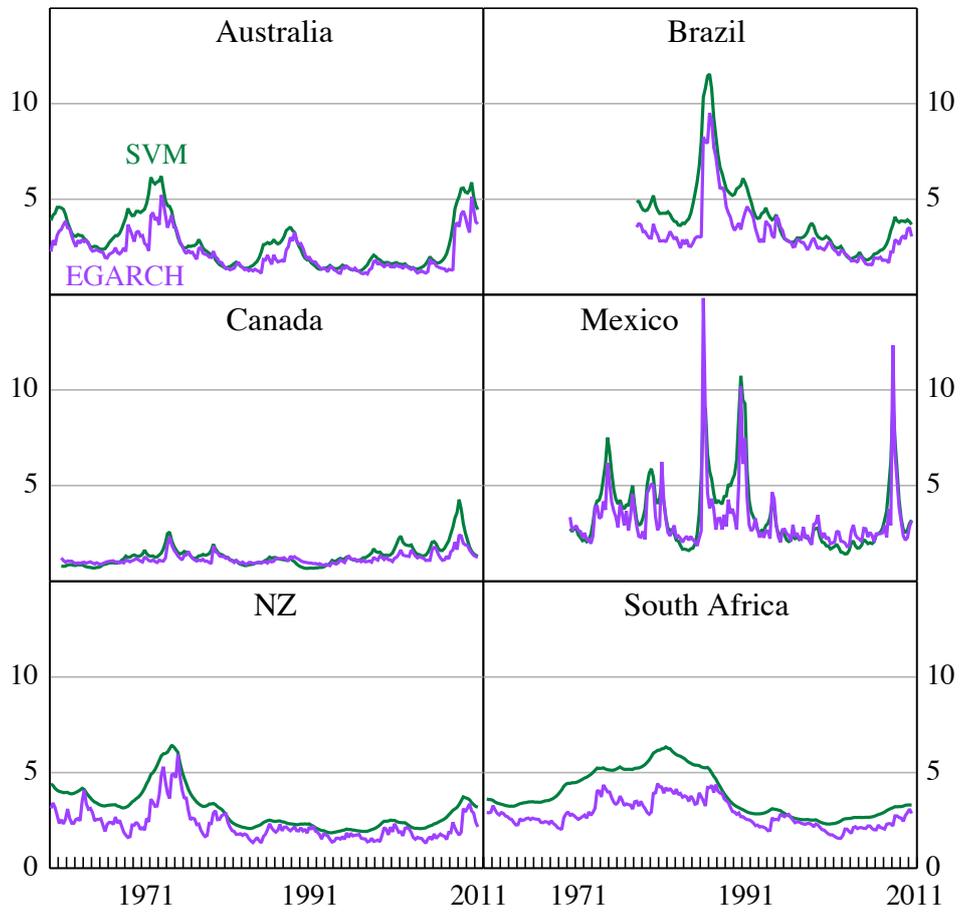
	Australia	Brazil	Canada	Mexico	New Zealand	South Africa
$\rho_q$	0.94*** (0.02)	0.96*** (0.02)	0.91*** (0.03)	0.91*** (0.02)	0.92*** (0.02)	0.81*** (0.05)
$\sigma_q$	-3.20*** (0.51)	-3.08*** (0.28)	-4.00*** (0.32)	-3.05*** (0.28)	-3.50*** (0.17)	-3.18*** (0.20)
$\rho_\sigma$	0.95*** (0.04)	0.92*** (0.06)	0.81*** (0.17)	0.65*** (0.17)	0.93*** (0.05)	0.96*** (0.04)
$\alpha$	0.39*** (0.14)	0.35** (0.14)	0.33** (0.16)	0.77** (0.27)	0.41*** (0.14)	0.19** (0.10)
$\beta$	0.03 (0.07)	0.08 (0.09)	0.08 (0.09)	0.09 (0.16)	0.03 (0.06)	0.06 (0.07)

Notes: \*, \*\* and \*\*\* denote significance from zero at the 10, 5 and 1 per cent levels, respectively; standard errors are in parentheses

Pleasingly, the persistence of the terms of trade and volatility processes, as well as the estimated mean volatility from the EGARCH estimation, are similar to those produced in our baseline stochastic volatility model. The results also indicate that positive and negative shocks to the terms of trade appear to have a symmetric effect on future terms of trade volatility. We interpret these results as supporting our choice to model the terms of trade using a stochastic volatility model.

As a final check on the plausibility of our baseline results, Figure D1 compares the implied standard deviation of terms of trade shocks derived from our EGARCH estimation to the median estimates implied by the stochastic volatility model (SVM). For each of the countries, the two methods imply a similar time series of terms of trade volatility.

**Figure D1: Time-variation in Terms of Trade Shocks**  
Standard deviation of shock

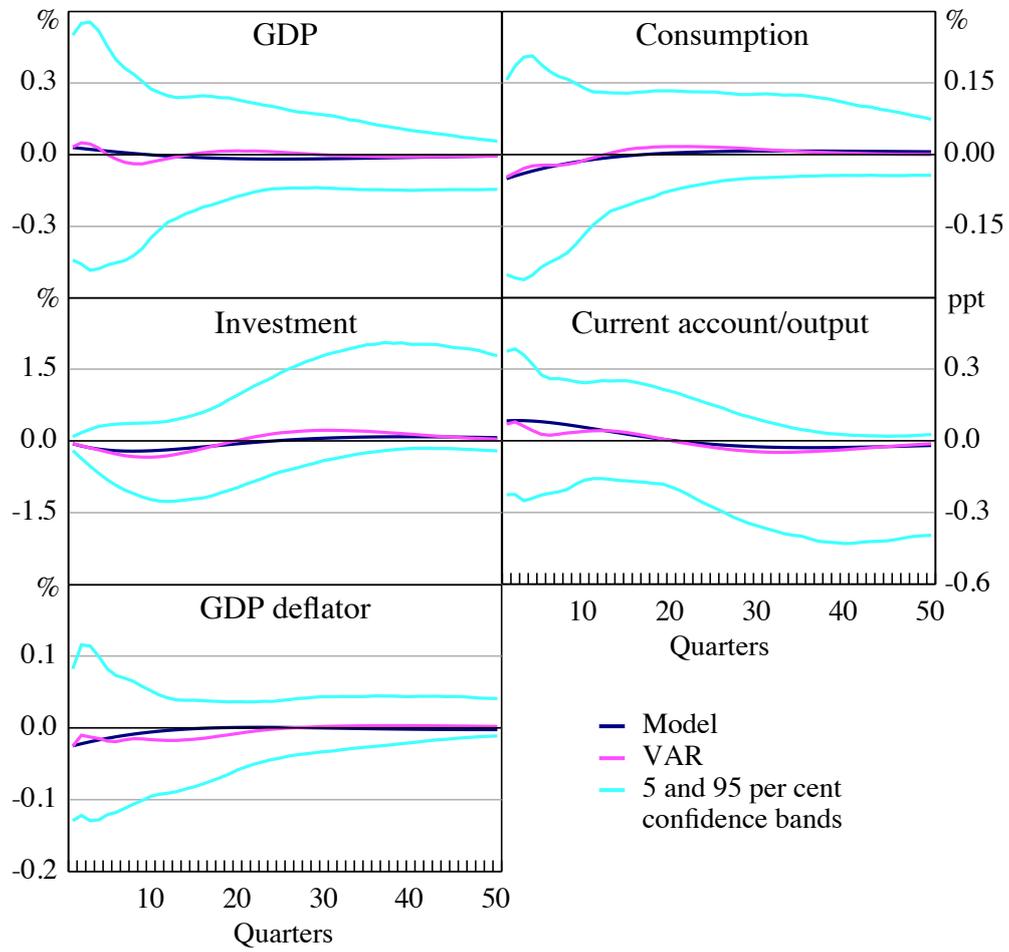


## **Appendix E: What Does the Empirical VAR Capture?**

In this appendix, we demonstrate the ability of our empirical VAR exercise to capture the macroeconomic impacts of exogenous shocks to terms of trade volatility. To do this, we compare impulse responses from our empirical VAR estimated using simulated data to the impulse responses to exogenous terms of trade volatility shocks generated by our model. Specifically, we simulate our model for 200 periods, setting all parameters at their baseline values for Brazil. We then estimate our empirical VAR using this data and calculate impulse responses to an innovation to the terms of trade volatility variable as in Section 4. We repeat this process 50 000 times to characterise the distribution of VAR responses.

Figure E1 shows the median, 5 and 95 per cent responses of the simulated VAR for each variable as well as the theoretical responses to a terms of trade volatility shock from the model. Despite its linear structure, the VAR comes extremely close to matching the theoretical model responses. This gives us some confidence that our empirical model reflects a response to an exogenous terms of trade volatility shock.

**Figure E1: Impulse Responses to a Volatility Shock**  
Model and VAR



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