IS THE PHILLIPS CURVE A CURVE?
SOME EVIDENCE AND IMPLICATIONS FOR AUSTRALIA

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Abstract

The Phillips curve has generally been estimated in a linear framework. This paper investigates the possibility that the Phillips curve is indeed a curve, and shows that a convex short-run Phillips curve may be a more accurate representation of reality than the traditionally used linear specification.

The paper also discusses the policy implications of convexity in the Phillips curve. These include the need for policy to be forward-looking and to act pre-emptively. Convexity provides a strong rationale for stabilisation policy, and it reinforces the need for policy-makers to proceed cautiously. It also implies that deep recessions may have only a marginally greater disinflationary impact than shallower ones, unless they induce large credibility bonuses.

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‘The relation between unemployment and the rate of change of wage rates is therefore likely to be highly non-linear.’


1. Introduction

Phillips’ original article indicated that he thought it likely that the Phillips curve was indeed a curve, not a straight line. However, subsequent estimates of the Phillips curve have generally been conducted in a linear framework. Using such a framework, Robert Gordon provided regular estimates of the Phillips curve in the 1970s in the United States (Gordon 1970, 1975, 1977). The Phillips curve fell into a period of neglect in academic circles during the 1980s, while remaining an important tool for policy-makers.¹ More recently, the Phillips curve has again been the subject of intensive debate (for example, the symposium in the *Journal of Economic Perspectives*²). This debate has focused on the usefulness of the Phillips curve as an analytical tool for monetary policy, given the uncertainties associated with its estimation.

Like the historical estimation of Phillips curves, the recent debate has been generally conducted in a linear framework. However, a separate stream of analysis has sought to restore the ‘curve’ in the short-run Phillips curve, and has investigated the empirical evidence for, and implications of, a non-linear model of the Phillips curve. Laxton, Meredith and Rose (1994); Turner (1995); Clark, Laxton and Rose (1996); and Debelle and Laxton (1997) all investigate the possibility that the Phillips curve is convex in various G7 countries. In contrast, Eisner (1996) finds evidence that the Phillips curve is concave. This has somewhat perverse implications for monetary policy which will be discussed below in Section 5. Akerlof, Dickens and Perry (1996) provide evidence that the long-run Phillips curve may be negatively sloped at

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¹ Leeson (1997) provides a comprehensive summary of the Phillips curve debate.
low levels of inflation, although Groshen and Schweitzer (1997) present some counterevidence.

This paper builds on the existing literature on non-linear Phillips curves. It highlights the features of a non-linear model of the Phillips curve and examines the policy implications of non-linearities. A simple ‘horse race’ is conducted between parsimonious linear and non-linear models of the Phillips curve using Australian data. It is shown that the non-linear model appears to outperform the linear model, when plausible priors are placed on the two models.

Our purpose in this paper is not to estimate a definitive model (either linear or non-linear) of the Phillips curve. Rather our aim is simply to investigate the possibility that the Phillips curve is non-linear, and use our derived estimates of a non-linear Phillips curve as an expository device to demonstrate the implications of an asymmetrical Phillips curve.

In the next section, the basic linear and non-linear models of the short-run Phillips curve are presented and previous work estimating the Phillips curve in both Australia and the United States is summarised. Section 3 describes the technique we use to estimate the Phillips curve and the data used. The results of our estimation are presented in Section 4, and the non-linear and linear models are compared. In Section 5, we discuss the policy implications of a non-linear Phillips curve. Section 6 concludes.
2. Models of the Short-run Phillips Curve

2.1 Linear Model

The standard linear models of the short-run Phillips curve that underlie most of the existing theoretical and empirical literature have generally been of the following form,

\[ \pi_t = \pi_t^e + \gamma (u^* - u_t) + \epsilon_t, \tag{1} \]

where \( \pi \) is the inflation rate, \( u \) is the unemployment rate and \( u^* \) is the non-accelerating inflation rate of unemployment (the NAIRU): when the unemployment rate equals \( u^* \), inflation equals inflation expectations. Inflation expectations \( \pi^e \) are generally assumed to be a linear combination of a backward- and forward-looking component (Buiter and Miller 1985). The backward-looking component may reflect inertia in the inflation process, or may be motivated by an overlapping-contract model such as Fischer (1977). Thus,

\[ \pi_t^e = \lambda A^{-1}(L)\pi_{t-i} + (1 - \lambda)B(L)\pi_{t-i}, \tag{2} \]

where \( A(L) \) and \( B(L) \) are polynomial lag operators.

Robert Gordon has estimated numerous versions of Equation (1) for the United States. To measure inflation expectations, Gordon generally has used lagged values of the inflation rate for up to two years. A number of dummy variables are also included on the right-hand side to control for various supply shocks (such as the OPEC petroleum shocks) and other events such as the Nixon price controls in the early 1970s. In nearly all the specifications that Gordon has estimated, the specification has been entirely linear. Furthermore, the NAIRU \( u^* \) has been assumed to remain constant over the estimation period. More recently, Tootell (1994) and Fuhrer (1996) have estimated Phillips curves with similar specifications, including the assumption of a constant natural rate of unemployment.

Staiger, Stock and Watson (1997) employ a wide variety of techniques to estimate the natural rate of unemployment in the United States using the Phillips curve framework. They allow for a number of differing specifications for the natural rate, including that it remains constant over the whole period, is a constant with
occasional shifts, a random walk, and a function of various labour-market variables. They conclude that the natural rate can only be estimated with a large degree of uncertainty, so that the smallest 95 per cent confidence interval for the natural rate in 1994 was 4.8 to 6.6 per cent (around a central estimate of 5.7 per cent).

Consequently, Staiger, Stock and Watson conclude that this uncertainty means that the Phillips curve may not be a useful guide for monetary policy in and of itself, although the large degree of uncertainty does not imply that the concept of the NAIRU is irrelevant. From a policy perspective, they conclude that a downward-sloping short-run Phillips curve *does* exist so that loose monetary policy will result in a rise in inflation, and tighter monetary policy will reduce inflation.

Early estimates of Phillips curves in Australia were calculated by Parkin (1973) and Jonson, Mahar and Thompson (1974). In these and other later estimates, a wage-inflation Phillips curve was estimated. Our analysis will focus on the price-inflation version of the Phillips curve. In estimating wage Phillips curves in Australia, there has been debate over the appropriate way to incorporate the centralised wage-determination system in Australia that existed for most of the sample period (Gregory 1986). Consequently, a number of specifications have included award wage growth explicitly on the right-hand side of the equation (Mitchell 1987).

As estimates of the Phillips curve appeared to break down in the late 1970s and early 1980s, various attempts were made to rescue them. Simes and Richardson (1987) and Gregory and Smith (1985) include measures of overtime to better capture labour-market pressure from ‘insiders’. Simes and Richardson also allow for ‘speed-limit’ effects by including the rate of change of unemployment in addition to the level of unemployment, as does Mitchell (1987). Cockerell and Russell (1995) find that only the change in, not the level of, ‘insider’ unemployment matters in a model of price inflation that also controls for unit labour cost growth. We investigate the role of speed-limit effects in the context of our non-linear model in Section 4.4 below. Including the rate of change of unemployment implies that the NAIRU is not constant in the short run, only in the long run.

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3 Providing another example of Goodhart’s Law, Kirby (1981) using data up to 1978 provides evidence of a stable Phillips curve including an estimate of the NAIRU of 2.2 per cent.
A number of attempts have been made to allow for shifts in the NAIRU over the sample period. Gregory and Simes and Richardson allow for a time trend in their specification, although it is not apparent whether this is necessarily capturing the shifts in the NAIRU. The Treasury model (TRYM) allows for a once-off level shift in the NAIRU in 1974 (Commonwealth Treasury 1996). As will be shown later, our results suggest that this is a reasonable approximation.

More recently, Crosby and Olekalns (1996) reach similar conclusions to Staiger, Stock and Watson using a similar methodology to derive estimates of the natural rate of unemployment in Australia. Crosby and Olekalns estimate a version of Equation (1) which allows lagged values of the unemployment gap to affect inflation, and measures inflation expectations using three lags of the inflation rate. Their model is estimated over the period 1959–1995 and over various sample periods to allow for possible variation in the natural rate of unemployment over time.

They find that over the whole sample period, a 95 per cent confidence interval for the natural rate is 20 percentage points wide (around a central estimate of 7.7 per cent). When the sample period is split into three sections, a smaller confidence interval for the natural rate is obtained, and the central estimate of the natural rate is found to rise through time, from 2.3 in the period from 1959–1973 to 9.6 in the period from 1984–1995. However, the confidence interval still remains relatively large. For example, over the 1984–1995 period, a 95 per cent confidence interval for the natural rate ranged from 3.7 per cent to 15.5 per cent.

However, all of the models discussed in this section adopt two important assumptions in their estimation which we relax in this paper. Firstly, they assume that the natural rate of unemployment is constant over the sample period, or alternatively that it changes only occasionally by discrete amounts. A comparison of the actual rates of unemployment in Australia and the United States (Figure 1) suggests that the assumption of a constant NAIRU over the sample period may be appropriate in the United States, but it is clearly heroic in Australia. In this paper, we allow the natural rate to vary through time, that is, \( u^* \) in Equation (1) is allowed to be time-varying.\(^4\)

\(^4\) Gordon (1997) estimates a version of his traditional linear model with a time-varying NAIRU, using a similar technique to that here.
Secondly, the models assume that the Phillips curve specification is linear. Next, we present a specification of a Phillips curve that allows for the possibility that the unemployment gap affects the inflation rate in a non-linear manner.

### 2.2 Non-linear Model

The non-linear specification that we assume for the Phillips curve is of the following simple form,

$$
\pi_t = \pi_t^e + \gamma \left( \frac{u_t^* - u_t}{u_t} \right) + \epsilon_t.
$$

(3)

A similar specification is included in the TRYM model. In the TRYM model, the NAIRU is determined directly by labour-market factors such as the degree of search effectiveness and institutional features of the wage-determination process. We adopt an alternative approach in our estimation by allowing the information implicit in movements in the inflation rate to help identify shifts in the NAIRU.
The convexity implies that the rise in (unexpected) inflation associated with a positive output gap is greater than the fall in inflation associated with an equally sized negative output gap. Thus, the non-linear specification of Equation (3) captures the implications of a traditional upward-sloping aggregate supply curve. Bottlenecks develop in the economy as the unemployment rate falls below the NAIRU. These bottlenecks result in further increases in aggregate demand (decreases in unemployment) causing ever-increasing rises in inflation. In the limit, as unemployment approaches zero, inflation increases without limit. Note that we have adopted the conservative assumption that this asymptote is reached at zero, when it is likely in reality to be some small positive number. There will be greater convexity in the curve if we use a positive asymptote.


An important implication of a non-linear specification is that there is a distinction between the natural rate of unemployment and the non-accelerating inflation rate of unemployment (NAIRU). This is most easily demonstrated if we assume a slightly different functional form for the non-linear Phillips curve:

$$\pi_t = \pi_t^e + \exp(\gamma (u^* - u_t)) - 1 + \varepsilon_t, \quad (4)$$

where the error term $\varepsilon_t$ is distributed normally with mean zero.

As before, the NAIRU is given by $u^*$. That is, inflation is equal to inflation expectations when the rate of unemployment is equal to $u^*$. However, taking expectations in Equation (4) shows that the average rate of unemployment is given by $u^* + \text{var}(u_t)/2$. We define the natural rate of unemployment to be this stochastic steady state average, which accords with the definition of the natural rate of unemployment given by Milton Friedman in his 1967 American Economic Association Presidential address:

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5 Dupasquier and Ricketts (1997) also discuss some theoretical foundations for non-linearity.
‘The “natural rate of unemployment” … is the level that would be ground out by the
Walrasian system of general equilibrium equations, provided there is imbedded in
them the actual structural characteristics of the labor and commodity markets,
including market imperfections, stochastic variability in demands and supplies, the
cost of gathering information about job vacancies and labor availabilities, the costs
of mobility, and so on.’ Friedman (1968, p. 8).

Both the NAIRU and the natural rate of unemployment are functions of the
institutional structure of the economy. However, the natural rate of unemployment is
also dependent on the shocks which affect the variability of unemployment, some of
which may be policy-induced.

Figure 2 illustrates the general implications of convexity. $PP'$ is the short-run
Phillips curve and $u^*$ is the NAIRU. Comparing the points $L$ and $L'$ shows that a
one percentage point increase in inflation is associated with a smaller gap between
unemployment and the NAIRU $(u_1 - u^*)$, than the unemployment gap $u_2 - u^*$
needed to reduce inflation by one percentage point. In this simple world, where the
only shocks are +1 percentage point shocks to inflation, the Friedman natural rate of
unemployment is the point where $LL'$ crosses the $x$-axis. Larger shocks to inflation
will shift the line $LL'$ out, thus increasing $\alpha$, the difference between the NAIRU and
the natural rate.

In the linear framework, the NAIRU and the natural rate are one and the same.
Taking expectations in Equation (1), it is clear that the stochastic steady state value
of unemployment is the same as $u^*$, the rate of unemployment where inflation is
equal to inflation expectations and is constant. However, in a convex world, the
natural rate of unemployment will always exceed the NAIRU. The extent to which it
is greater will depend on the variability in the actual rate of unemployment. We
discuss the implications of this non-linearity for monetary policy-making in
Section 5 below.
3. Estimating Linear and Non-linear Phillips Curves

In this section, we describe the approach we use to estimate a non-linear Phillips curve for Australia and compare its empirical properties to a similarly estimated linear Phillips curve. The methodology we use is that adopted by Debelle and Laxton (1997) to estimate Phillips curves for Canada, the United Kingdom and the United States. A key feature of the approach is the use of model-consistent estimates of the NAIRU in comparing the linear and non-linear models. Previous attempts to detect the presence of non-linearities in Phillips curves have generally used measures of the NAIRU (or equivalently the output gap) which have been derived in a linear framework, thereby introducing bias into the tests for non-linearities. Laxton, Meredith and Rose (1994) and Clark, Laxton and Rose (1996) document the size of this bias.

3.1 Data

We estimate the linear and non-linear models of the Phillips curve presented in Section 2 (Equations (1) and (3)). Quarterly data from 1959:Q3 until 1997:Q1 is used to estimate the models. We use the four-quarter-ended growth in the (underlying) consumer price index to measure inflation. The unemployment rate is
the quarterly average of the monthly seasonally adjusted unemployment rate in the ABS Labour Force Survey\(^6\) from 1966 onwards, and from the NIF-10 database prior to that.

Inflation expectations are measured in a number of different forms. The backward-looking component is captured by the inclusion of a lagged four-quarter-ended inflation term. For the forward-looking component we use two different measures: an estimate derived from bond-market yields, and the Melbourne Institute measure of consumer inflation expectations. We also estimate a model with only a backward-looking component, where we include four lags of the inflation rate.

The measure of inflation expectations derived from bond-market yields is obtained by subtracting a measure of the equilibrium world real interest rate from the 10-year bond yield. The series for the world real interest rate is based on empirical work that relates the equilibrium world real interest rate to movements in the stock of world government debt (Ford and Laxton 1995; Tanzi and Fanizza 1995). Debelle and Laxton (1997) show that the results are not sensitive to the precise calculation of the world real rate.

### 3.2 Kalman Filter

As discussed in Section 2, we allow the estimate of the NAIRU to be time-varying. Consequently, we estimate the two models using the Kalman filter. The following provides a short overview of this approach.\(^7\) Consider the following system,

\[
y_t = X_t' \beta_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2 I) \tag{5}
\]

\[
\beta_t = T * \beta_{t-1} + \mu_t \quad \mu_t \sim N(0, \sigma^2 I) \tag{6}
\]

\(^6\) *Labour Force, Australia*, ABS Cat. No. 6203.0.

\(^7\) For more information on the Kalman filter, see Chapter 13 of Hamilton (1994). More details on our use of the filter are in Appendix A. The model was estimated using the KALMAN command in RATS version 4.2. The results were replicated using the KFILTER and KSMOOTH procedures available on the ESTIMA homepage (http://www.estima.com/).
The parameter (state) vector $\beta_t$ is time-varying in a manner determined by the transition matrix $T$. In our estimation we assume that $T$ is such that all parameters are constant except for the NAIRU which follows a random walk.\(^8\) The Kalman filter produces estimates of this system by minimising the sum of the squared one-step prediction errors of $y_t$.

In terms of the non-linear model we estimate, $y_t = (\pi_t - \pi_{t-1})$. $X_t = [\pi_t^e - \pi_{t-1}, 1/\mu_t, 1]$ and $\beta_t = [\delta, \gamma u^*_t, -\gamma]'$, while in the linear model $X_t = [\pi_t^e - \pi_{t-1}, u_t, 1]$. Both $\delta$ and $\gamma$ are assumed to be unchanging over time. Estimates of the NAIRU at each point in time can be calculated by taking the negative of the ratio between the second and third elements of $\beta_t$.

We need to place some restrictions on the ratio of the two variances in Equations (5) and (6), the signal-to-noise ratio. Multiplying the matrices $H$, $Q$ and $\Sigma$ (where $\Sigma$ is the initial covariance matrix of $\beta$) by a scalar leaves the system unchanged. We thus treat the variance of $\varepsilon_t$ as the numeraire, setting it equal to 1, and alter the signal-to-noise ratio by changing the magnitude of the non-zero element in $Q$. In our empirical work, this value is usually found through an optimisation procedure, but is generally around 0.4. The elements of $\Sigma$ are set to large values,\(^9\) reflecting lack of knowledge about the ‘true’ value of the NAIRU.

The Kalman filter can be used in two ways, which can broadly be referred to as prediction (one-sided estimation) and smoothing (two-sided estimation).\(^10\) In prediction mode, the filter computes estimates of the model parameters at time $t$ based on information up to time $t$. The filter recursively estimates through the entire sample in this manner. In this way, it simulates the process of estimating the NAIRU in real time. In smoothing mode, estimates of the NAIRU at each point are based on information over the entire sample. This two-sided estimate allows the benefit of hindsight in obtaining an estimate of the NAIRU in past time periods. For the last period in the sample, the two approaches will give the same estimate of the NAIRU.

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\(^8\) This does not mean that we necessarily believe that the NAIRU is indeed a random walk. Rather it is an empirically convenient way to model it.

\(^9\) The diagonal elements are set to a magnitude of 4.

\(^10\) See Kuttner (1992) for a more comprehensive description.
because information over the entire sample is available when obtaining the one-sided NAIRU estimates.

4. Empirical Results

This section presents empirical results from the estimation of linear and non-linear Phillips curves for Australia using model-consistent estimates of the NAIRU. As described above, the NAIRU is modelled as a time-varying parameter which follows a random walk through time.

4.1 Basic Results

Results for the two models of the Phillips curve are shown in Table 1. For the results reported in this section, we measure the forward-looking component of inflation expectations by the difference between the Australian 10-year bond yield and an estimated global real interest rate (as discussed in Section 3).

The table shows estimated parameters for both the linear and non-linear models and the value of the likelihood function associated with each model. Standard errors are reported for $\gamma$ and $\delta$. Given the presence of autocorrelation, in part induced by the use of a four-quarter-ended inflation rate (rather than a quarterly inflation rate), we correct the raw standard errors using the Newey-West (1987) approach. Therefore, the first line in parentheses in the table reports the unadjusted t-statistics while the second reports the Newey-West t-statistics.

The parameter $\alpha$ measures the difference between our estimate of the NAIRU ($\mu^*$) and the natural rate of unemployment which we proxy by the average value of unemployment over the sample.\textsuperscript{11} We also show the maximum size of the unemployment gap over the estimation period, and the largest change in the NAIRU in any one quarter.

\textsuperscript{11} This assumes that the model is in stochastic steady state over the whole sample period. It is likely that the economy has moved from one stochastic steady state to another over the sample, thus $\alpha$ may itself be time-varying.
Table 1: Basic Model Estimates

Phillips curve:

\[ \pi_t = \pi_t^e + \gamma \frac{(u_t^* - u_t)}{u_t} + \epsilon_t \]

where \( \pi_t^e = \delta \pi_t^e (LTE) + (1 - \delta) \pi_{t-1} \)

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>LLF</th>
<th>( \sigma^2 )</th>
<th>( \alpha )</th>
<th>( \text{max}[u-u^*] )</th>
<th>( \text{max}[\Delta u^*] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>0.09</td>
<td>16.27</td>
<td>0.49</td>
<td>1.17</td>
<td>6.01</td>
<td>1.46</td>
</tr>
<tr>
<td>(3.08)</td>
<td>(2.54)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.47)</td>
<td>(1.60)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Phillips line:

\[ \pi_t = \pi_t^e + \gamma (u_t^* - u_t) + \epsilon_t \]

where \( \pi_t^e = \delta \pi_t^e (LTE) + (1 - \delta) \pi_{t-1} \)

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>LLF</th>
<th>( \sigma^2 )</th>
<th>( \alpha )</th>
<th>( \text{max}[u-u^*] )</th>
<th>( \text{max}[\Delta u^*] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>0.17</td>
<td>26.92</td>
<td>0.18</td>
<td>0</td>
<td>9.31</td>
<td>4.47</td>
</tr>
<tr>
<td>(1.34)</td>
<td>(2.19)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.26)</td>
<td>(2.18)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

t-statistics in parentheses: unadjusted, and Newey-West corrected

\( \gamma \) labour-market sensitivity parameter

\( \delta \) weight given to forward-looking component of inflation expectations

LLF value of the log-likelihood function

\( \sigma^2 \) standard error of the equation

\( \alpha \) average value of \( u-u^* \)

\( \pi_t^e (LTE) \) forward-looking inflation expectations based on bond yields, equal to the 10-year bond rate less the world real interest rate

\( \delta \), the weight that wage and price setters place on the forward-looking component of inflation expectations, is 0.09 for the non-linear model, and 0.17 for the linear model. In each case, \( \delta \) is statistically significant at the 5 per cent level, using the unadjusted standard errors, but is insignificant in the non-linear model when the standard errors are adjusted to take account of the autocorrelation. The low value of \( \delta \) implies that for the most part inflation expectations are formed adaptively. The value of \( \delta \) presented here is consistent with estimates of 0.15, 0.20 and 0.08.
reported in Debelle and Laxton (1997) for Canada, the United Kingdom and the United States respectively.

In both models, $\gamma$ has the expected sign. The value of $\gamma$ in the non-linear model measures the degree of convexity in the curve. The functional form that we estimate implies that the response of inflation to a one percentage point change in the unemployment rate varies with the level of both the NAIRU and the actual unemployment rate. For example, assuming that the NAIRU is 6 per cent, a one percentage point fall in the unemployment rate from an initial value of 6 per cent leads to a rise in inflation (above expectations) of 0.22 percentage points. However, if unemployment falls a further one percentage point, inflation increases by a further 0.31 percentage points. The convexity implies that further falls in unemployment lead to even larger rises in inflation. In the linear model, a one percentage point fall in the unemployment rate always leads to a 0.27 percentage point rise in inflation, regardless of the level of unemployment or the NAIRU.

Thus, in the region of the NAIRU, the linear and non-linear models are approximately equivalent. The important distinction is when the economy is further away from equilibrium.

Convexity in the Phillips curve implies that the NAIRU will always be less than the expected value of the unemployment rate in the stochastic steady state. $\alpha$ depends on the amplitude of the fluctuations of unemployment around its expected value. These results suggest that for Australia over the sample period, the unemployment rate was on average 1.17 percentage points above the NAIRU. This compares to estimates of 0.86 for Canada, 0.57 for the United Kingdom and 0.33 for the United States, presented in Debelle and Laxton (1997). One explanation for the higher value for Australia might be that the size of the shocks affecting the Phillips curve equation were larger here than in other countries.

As discussed above, given the estimate of $\gamma$, we can derive time paths for the NAIRU. These are shown for the linear and non-linear model in Figures 3 and 4. For both models, both the one-sided and two-sided estimates of the NAIRU are shown (Section 3.2).
Figure 3: Unemployment and the NAIRU
Non-linear model

Unemployment rate

Two-sided NAIRU
One-sided NAIRU


Figure 4: Unemployment and the NAIRU
Linear model

Unemployment rate

Two-sided NAIRU
One-sided NAIRU

In the non-linear model, the one-sided estimate of the NAIRU remains at approximately 2 per cent through the 1960s, before rising to a level around 6 per cent around the time of the sharp increase in real wages in 1974. After falling slightly during the rest of the 1970s, the estimate again increases at the turn of the decade. Thereafter, the estimate of the NAIRU fluctuates in the range between 5 and 8 per cent. The most recent estimate of the NAIRU (1997:Q1) produced by the non-linear model is 6.9 per cent. This figure is slightly lower than the point estimates of most researchers, although inside the confidence bounds reported in Crosby and Olekalns (1996).

The two-sided estimate is clearly smoother than the one-sided estimate. When there is a difference between the two estimates, the estimated value of the NAIRU based on information available at the time has been revised in the light of later data. In periods where the two-sided NAIRU is above the one-sided NAIRU, the central bank would have run policy which in retrospect might appear too loose. For example, during the early 1970s, a policy-maker using this framework would have underestimated the level of the NAIRU in real time (using the one-sided estimates) and thus would have erred on the side of overly loose policy. Conversely, in the period 1986–1989, the real time estimate of the NAIRU lay below the smoothed estimate implying that if one used this framework, policy would have been relatively tight. However, the one-sided estimate always lies within the confidence interval of the two-sided estimate, highlighting the large degree of uncertainty associated with estimating the NAIRU.

The non-linear estimates of the NAIRU suggest that the actual unemployment rate has been above the NAIRU throughout most of the 1990s. This is because inflation expectations have exceeded inflation over this period. Figure 5 plots the difference between actual inflation and the level of inflation expectations in the non-linear model.\(^{12}\) In this framework, if inflation expectations were lower, then a smaller unemployment gap would be consistent with maintaining inflation at the targeted level. This demonstrates that inflation expectations have been slow to adjust downwards to the new low inflation environment. The slow adjustment of expectations increases the adjustment cost in terms of output/unemployment.

\(^{12}\) Other measures of inflation expectations yield a similar picture, if not an even larger gap between actual and expected inflation.
4.2 The Linear versus the Non-linear Model

The time path of the NAIRU exhibits considerably larger fluctuations in the linear model than in the non-linear case. In the linear model, the NAIRU increases rapidly from 2 per cent in 1970 to over 10 per cent in 1974, before falling back to around 5 per cent in the mid 1980s. The final point estimate of the NAIRU using this model is 7.1 per cent.

The largest quarterly movement in the NAIRU in the linear model is 4.5 per cent, compared to only 1.4 per cent for the non-linear model. The larger movements in the NAIRU, such as those in the latter part of the 1980s shown in Figure 4, are necessary for the linear model to obtain a good fit of the data. As mentioned above, the convexity in the non-linear model allows it to capture periods of a large difference between actual and expected inflation with a relatively small gap between the actual unemployment rate and the NAIRU. The linear model fits such periods by moving the NAIRU more.

Movements in the NAIRU ought to reflect the changing structural features of the labour market. Our priors would thus suggest that the series should therefore evolve...
slowly over time, rather than exhibiting the large quarterly fluctuations in the linear model specification. On this basis, the non-linear model represents a path for the NAIRU which is more in accordance with the way the labour market is generally thought to operate. Nevertheless, the linear model has a higher value of the log-likelihood function than the non-linear specification.

Since estimating the linear model results in unrealistically large quarterly variations in the NAIRU in order to obtain a better fit to the data, we estimate a restricted model where the signal-to-noise-ratio is adjusted to restrict the variation in the NAIRU. This restricted model considerably reduces the value of the likelihood statistic, that is the degree of fit, to 9.92 – less than the non-linear model (although the difference between the two models is not statistically significant). Thus, imposing a reasonable amount of time variation in the NAIRU on the linear model can only be achieved with a substantial loss of fit.

4.3 Alternative Estimates of Inflation Expectations

In this section we examine how the above results are affected by alternative estimates of inflation expectations. We consider three alternative processes. In the first, a richer dynamic structure is used for the formation of the backward-looking component of inflation expectations. In the second, expectations are formed entirely adaptively, and are modelled only using lags of the rate of change of the consumer price index. Thirdly, expectations are modelled using the Melbourne Institute inflation expectations series. This series, which is available from 1973 onwards, is based on survey data.

A comparison of actual underlying inflation, as well as inflation expectations based on bond yields and the Melbourne Institute survey, is presented in Figure 6.13 For reference, we also include an expectations series derived from indexed bonds, although this time series is not long enough to include in our estimation. Two features of this figure are notable:

---

13 The median (rather than the average) inflation expectations response from the Melbourne Institute survey is used in Figure 6.
Both the bond yields and survey measures produce estimates of inflation expectations which overstate actual inflation through most of the 1980s.

The Melbourne Institute series tracks the bond-yield and indexed-bond measures of inflation expectations quite closely, especially since the mid 1980s.

**Figure 6: Measures of Inflation Expectations**

In the first alternative specification, the model is extended to include additional lags of the inflation rate, so that the process generating inflation expectations is:

\[
\pi_t^e = \delta \pi_t^e (LTE) + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \theta_3 \pi_{t-3} + \theta_4 \pi_{t-4}.
\]  

(7)

If inflation is best characterised as a unit root process, then it may be appropriate to restrict the sum of the right-hand side variables to equal one. In the estimation conducted here, this restriction is not imposed on the model.\(^{14}\) Note however, that in a successful inflation-targeting regime, the inflation rate may more likely be stationary.

\(^{14}\) The unrestricted sum of coefficients in the model is 1.12.
Summary results for including this specification for inflation expectations in the non-linear model are presented in Table 2.

### Table 2: Alternative Specification for Inflation Expectations

\[
\pi_t = \pi_t^e + \gamma \frac{(u_t^* - u_t)}{u_t} + \varepsilon_t
\]

where \( \pi_t^e = \delta \pi_t^e (LTE) + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \theta_3 \pi_{t-3} + \theta_4 \pi_{t-4} \)

<table>
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<tr>
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<th>( \delta )</th>
<th>LLF</th>
<th>( \sigma^2 )</th>
<th>( \alpha )</th>
<th>max[( u-u^* )]</th>
<th>max[( \Delta u^* )]</th>
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</tbody>
</table>

t-statistics in parentheses: unadjusted, and Newey-West corrected

- \( \gamma \): labour-market sensitivity parameter
- \( \delta \): weight given to forward-looking component of inflation expectations
- LLF: value of the log-likelihood function
- \( \sigma^2 \): standard error of the equation
- \( \alpha \): average value of \( u-u^* \)
- \( \pi_t^e (LTE) \): forward-looking inflation expectations based on bond yields, equal to the 10-year bond rate less the world real interest rate

Only the first of the four lagged inflation terms was significant at the 5 per cent level of significance. The final point estimate of the NAIRU based on this model specification is 6.5 per cent. The time path of the NAIRU is also somewhat different than in the previous case, with the main rise in the NAIRU occurring in the second half, rather than the first half, of the 1970s; followed by a further rise in the early part of the 1990s. More generally, the time path of the NAIRU, as well as its final point estimate were quite sensitive to the lag length chosen for the adaptive component of the inflation expectations variable.

The results for the other two expectations processes are presented in Tables B.1 and B.2 in Appendix B. In both cases, the NAIRU became more volatile than in the case where bond yields were used to model forward-looking expectations. This was
especially so when the Melbourne Institute series for inflation expectations was used in the model.\textsuperscript{15}

4.4 Speed Limits in the Phillips Curve Specification

To capture the possibility of ‘speed-limit’ effects, an unemployment change variable is included in the non-linear and linear Phillips curve specifications in Table 1. The specification of the non-linear model then becomes,

\[ \pi_t = \pi_t^c + \gamma \left( u_t^* - u_t \right) + \chi \frac{\Delta u_t}{u_t} + \varepsilon_t. \]  

(8)

Such a specification is compatible with the view that rapid reductions in the unemployment rate (such as that which occurred in Australia during 1994–1995) are associated with increased inflationary pressure. Thus, \( \chi \) would be expected to be negatively signed. However, \( \chi \) is not significantly different from zero at the 5 per cent level of significance in either model, and actually has a positive sign (the results are in Table B.3 in Appendix B). When the models were re-estimated using data from 1980 onwards, \( \chi \) had the expected sign in the non-linear model, but was still not statistically significant, while in the linear model it remained slightly positive.

4.5 Sensitivity of the NAIRU Estimates

In addition to changing the specification of inflation expectations, we also included a number of dummy variables to account for various exogenous shocks to inflation over the estimation period, that ought not necessarily be reflected in movements in the NAIRU. We include a dummy variable for the period 1973:Q2 to 1975:Q2 to take account of the first OPEC oil price rise and the large wage increases in 1974. We also include dummy variables for the 1982 wage freeze, and the wage-tax trade-offs of the late 1980s. Only the OPEC dummy variable is significant. Including the dummies results in a substantially smaller rise in the NAIRU in the mid 1970s in the non-linear model. In the linear model, the spike in the NAIRU in 1974 was reduced from around 22 per cent to around 12 per cent. The estimation results are in Table B.4 in Appendix B.

\textsuperscript{15} Gregory (1986) finds similar problems in using the Melbourne Institute series.
Figure 7 plots the different estimates of the NAIRU for the linear and non-linear models using the basic specifications in Section 4.1 and the models including the dummy variable in 1973–1975. The smaller spike in the NAIRU in the mid 1970s is evident in both models when the dummy variable is included. The NAIRU in the non-linear model lies below that for the basic specification through the 1980s and 1990s because there is less convexity in the model with dummies ($\gamma$ is smaller), implying that a larger unemployment gap is needed to explain the disinflation over the period. This is achieved by generating a lower NAIRU. Including the dummy in the linear model does not have a major impact on the volatility of the NAIRU.

To more directly control for the impact of exogenous oil price movements, we included a measure of oil prices\textsuperscript{16} as an explanatory variable. We found that (lagged) oil price inflation was significant in both the linear and non-linear models. Again, the path of the NAIRU in the mid 1970s was slightly different from that in Figures 3 and 4.

In general, we find that our estimation is sensitive to changes in the model specification, such as employing different specifications for forward-looking inflation expectations, or changing the lag structure of the backward-looking component. It is also slightly sensitive to the choice of starting values for the Kalman filter.

\textsuperscript{16} We use the rate of change in West Texas intermediate oil prices.
Figure 8 shows that the two standard error confidence bounds on the NAIRU for 1997:Q1 are 5.2 and 8.6. This shows the degree of uncertainty for the one-sided estimate of the NAIRU in our preferred specification for the non-linear model. A wider band would be obtained if we also incorporated model uncertainty. The confidence-band width of 3.4 percentage points compares to Staiger, Stock and Watson’s estimate of 1.8 percentage points for the US.

In light of these findings, we concur with the conclusion of Crosby and Olekalns (1996) that considerable uncertainty exists regarding the ‘correct’ level of the NAIRU in Australia, although our estimate of the confidence interval is considerably less than their estimate. This uncertainty over the size of the NAIRU complicates its effectiveness as a tool for determining the appropriate stance of monetary policy. However, one result does appear to be robust in the results presented above, namely that in each of the model specifications investigated, the
NAIRU has risen appreciably over time, with the bulk of the increase occurring during the 1970s.

5. Policy Implications of Non-linearities

The basic policy prescription of the linear natural-rate framework is to tighten (loosen) policy whenever unemployment exceeds (is less than) the natural rate/NAIRU (recall that these are equal in the linear model).\textsuperscript{17} In the simple linear model, while symmetric policy mistakes will affect the variability of unemployment, they do not affect its average level (De Long and Summers 1988). If the model is extended to allow for asymmetric adjustment of inflation expectations there may be a linkage between the mean of unemployment and its variability. Nevertheless, in general, the concerns raised by Staiger, Stock and Watson (1997) about the uncertainty in the linear natural-rate framework are of second-order importance,

\textsuperscript{17} This prescription has recently been challenged by Chang (1997) and Espinosa and Russell (1997) who argue that the uncertainty about the value of the NAIRU renders the framework inoperative from a policy perspective.
unless there is some additional cost of output/unemployment variability, for example, if output variability is directly in the policy-maker’s objective function.

In contrast, in the non-linear natural-rate framework, the variability of the unemployment rate has a direct effect on the average level of unemployment.\(^{18}\) This means that policy mistakes that increase the variability of unemployment will increase the average level of unemployment (which we refer to as the natural rate of unemployment). For example, if monetary policy is slow to respond to a rise in inflationary pressures caused by unemployment falling below the NAIRU, the convexity of the curve requires that to return inflation to its targeted level, unemployment will have to be above the NAIRU for a longer period (or alternatively will need to be higher) than it was below it. Thus, the average unemployment rate – the Friedman natural rate – will be higher. Good stabilisation policy, on the other hand, may reduce the gap between the natural rate and the NAIRU by reducing the variability of unemployment.

Consequently, convexity of the short-run Phillips curve also points to the need for pre-emptive monetary policy. Monetary policy needs to be forward-looking so that inflationary pressures are quickly doused (to prevent inflation expectations from rising), in order to avoid costly recessions down the track. Clark, Laxton and Rose (1995) present estimates of the gains from a forward-looking monetary policy approach in a non-linear world, in terms of the ability to avoid larger recessions. These results also provide support for a conservative approach to policy-making, as the inflationary consequences of a slight overheating of the economy are greater than the consequences of a slight underheating.

While a non-linear Phillips curve provides support for pre-emptive monetary policy, it also implies that deep recessions should be avoided. The convexity means that the extra disinflationary impact from a slightly deeper recession is likely to be marginal, so that deep recessions are unnecessarily costly. Consequently, non-linearity implies that a gradualist approach to disinflation is preferable. In contrast, Ball (1994) exposit a model where a ‘cold turkey’ approach is preferable. However, his results depend on the effects of monetary policy actions on the public’s expectations through credibility channels which we do not model here.

\(^{18}\) This basic implication of convexity was noted by Mankiw (1988) in commenting on De Long and Summers (1988).
In the recent *Journal of Economic Perspectives* volume, a number of authors argue that the Federal Reserve Board could test the limits of the economy without any adverse long-run consequences (Gordon 1997 and Stiglitz 1997). Such a conclusion may be justified in a linear world. However, in a non-linear world such a prescription could be dangerous. If the central bank, in testing out the limits of the economy, induces overheating, again, the resultant period of depressed activity necessary to bring down inflation would more than offset the gains from the period of higher activity. Given the large uncertainty about the true level of the NAIRU, there may be some justification in attempting to test the limits, but the convexity implies that such an approach must proceed cautiously.

Moreover, an assumption of concavity in the short-run Phillips curve (as suggested by Eisner (1996)) would be dangerous if, in fact, there is convexity. In the limit, concavity implies that increasing the variance of unemployment (that is, larger amplitudes of business cycles) will reduce the average level of unemployment. This implication does not square well with the developments in the Australian economy over the past twenty years.

6. Conclusion

This paper has demonstrated that a non-linear specification for the short-run Phillips curve may be a more accurate representation of reality than the traditionally used linear specification. That is, the Phillips curve in Australia may indeed be a curve rather than a line. The estimates of the NAIRU that are implicit in our estimation are highly sensitive to the specifications that we use and have very wide confidence intervals around them. Thus, our estimation should only be regarded as indicative of the presence of non-linearities.

If the Phillips curve is in reality a curve, there are important implications for monetary policy. Firstly, it provides a stronger justification for stabilisation policy than is present in a linear framework. Secondly, it reinforces the need for policy to be forward-looking and to act pre-emptively to offset inflationary pressures. Thirdly, it suggests that deep recessions may have only a marginally greater disinflationary impact than shallower ones, unless they induce large credibility bonuses. Finally, it reinforces the need for policy-makers to proceed cautiously, particularly if the economy is close to its potential.
Appendix A: The Kalman Filter

In order to execute the Kalman filter, the following information needs to be provided to the model:

1. $\beta_0$, the initial values of the state vector;

2. the initial covariance matrix of $\beta$;

3. the variance of the measurement equation (variance of $\varepsilon_t$); and

4. the variance of the transition vector (variance of $\mu_t$).

Note that if $Q$, $H$ and $\Sigma_t$ are all multiplied by a constant, the constant does not affect estimates of the state-space vector $\beta$. It is only the ratio between the variances that affects the estimates of $\beta$. A usual procedure is to normalise $H = 1$. This procedure is followed below.

The elements in $\Sigma_0$ are set to relatively large values, reflecting lack of knowledge about the covariance between elements of the state space equation. $Q$ is a 3x3 matrix (because there are three elements in the state vector), however it has only one non-zero element, because only the second element of $\beta_t$ is time-varying. This value, as well as the three initial values for the coefficients in $\beta_t$, are estimated using a non-linear maximum-likelihood optimisation procedure. We estimated starting values for the model parameters using a numerical optimisation procedure in RATS, which chooses these values by maximising a concentrated log-likelihood function.\(^\text{19}\) This exercise was tractable because of the parsimonious model specification used.

\(^\text{19}\) The form of the likelihood function used is $L = -0.5^* (\log \sigma_t^2 + T(\log(y_t - X_t \beta_{t-1})^2 / \sigma_t^2))$. See Doan (1992) for more details.
### Appendix B: Further Estimation Results

#### Table B.1: Adaptive Expectations

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<tr>
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<th>Phillips curve:</th>
<th>Phillips line:</th>
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Where $\pi_t^e = A(L)\pi_{t-1}$

- $\pi_t = \pi_t^e + \gamma \frac{(u_t^* - u_t)}{u_t} + \varepsilon_t$

- $\gamma$ labour-market sensitivity parameter
- LLF value of the log-likelihood function
- $\sigma^2$ standard error of the equation
- $\alpha$ average value of $u-u^*$

**t-statistics in parentheses: unadjusted, and Newey-West corrected**
Table B.2: Melbourne Institute Expectations

Phillips curve:

\[
\pi_t = \pi_t^{e} + \gamma \left( \frac{u_t^{*} - u_t}{u_t} \right) + \epsilon_t
\]

\[
\pi_t^{e} = \delta \pi_t^{e} (LTE) + (1 - \delta) \pi_{t-1}
\]

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<td>max[(u-u^{*})]</td>
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<td>max[(\Delta u^{*})]</td>
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Phillips line:

\[
\pi_t = \pi_t^{e} + \gamma (u_t^{*} - u_t) + \epsilon_t
\]

\[
\pi_t^{e} = \delta \pi_t^{e} (LTE) + (1 - \delta) \pi_{t-1}
\]

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<th>Parameter</th>
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T-statistics in parentheses: unadjusted, and Newey-West corrected

- \(\gamma\): labour-market sensitivity parameter
- \(\delta\): weight given to forward-looking component of inflation expectations
- LLF: value of the log-likelihood function
- \(\sigma^2\): standard error of the equation
- \(\alpha\): average value of \(u-u^{*}\)
- \(\pi_t^{e} (LTE)\): forward-looking inflation expectations based on the Melbourne Institute survey of inflation expectations

Note: There is an extremely large amount of variation in the NAIRU under both the linear and non-linear model in the early part of the sample. Figures quoted here for \(\alpha\), max[\(u-u^{*}\)] and max[\(\Delta u^{*}\)] exclude observations from the first three years of the sample for which the Melbourne Institute survey was available (1973 to 1975).
Table B.3: Model with Speed Limits

Phillips curve:

\[
\pi_t = \pi_t^e + \gamma \left(\frac{u_t^* - u_t}{u_t}\right) + \chi \frac{\Delta u_t}{u_t} \varepsilon_t
\]

where \( \pi_t^e = \delta \pi_t^e (LTE) + (1 - \delta) \pi_t-1 \)

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<th>( \delta )</th>
<th>( \chi )</th>
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<th>( \alpha )</th>
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Phillips line:

\[
\pi_t = \pi_t^e + \gamma (u_t^* - u_t) + \chi \Delta u_t \varepsilon_t
\]

where \( \pi_t^e = \delta \pi_t^e (LTE) + (1 - \delta) \pi_t-1 \)

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t-statistics in parentheses: unadjusted, and Newey-West corrected

- \( \gamma \): labour-market sensitivity parameter
- \( \delta \): weight given to forward-looking component of inflation expectations
- \( \chi \): parameter on speed limit term
- LLF: value of the log-likelihood function
- \( \sigma^2 \): standard error of the equation
- \( \alpha \): average value of \( u-u^* \)
- \( \pi_t^e (LTE) \): forward-looking inflation expectations based on bond yields, equal to the 10-year bond rate less the world real interest rate
Table B.4: Model with Dummy Variables

Phillips curve:
\[ \pi_t = \pi_t^e + \gamma \left( u_t^* - u_t \right) + D_1 + \varepsilon_t \]

where \( \pi_t^e = \delta \pi_t^e (LTE) + (1 - \delta) \pi_{t-1} \)

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Phillips line:
\[ \pi_t = \pi_t^e + \gamma \left( u_t^* - u_t \right) + D_1 + \varepsilon_t \]

where \( \pi_t^e = \delta \pi_t^e (LTE) + (1 - \delta) \pi_{t-1} \)

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</table>

t-statistics in parentheses: unadjusted, and Newey-West corrected

- \( \gamma \): labour-market sensitivity parameter
- \( \delta \): weight given to forward-looking component of inflation expectations
- \( D_1 \): dummy variable for the period 1973:Q3–1975:Q2
- LLF: value of the log-likelihood function
- \( \sigma^2 \): standard error of the equation
- \( \alpha \): average value of \( u-u^* \)
- \( \pi_t^e (LTE) \): forward-looking inflation expectations based on bond yields, equal to the 10-year bond rate less the world real interest rate
References


