

**EXPLAINING FORWARD DISCOUNT BIAS:  
IS IT ANCHORING ?**

David W. R. Gruen and Marianne C. Gizycki

Research Discussion Paper  
9307

June 1993

Economic Research Department

Reserve Bank of Australia

We are grateful to seminar participants at the Reserve Bank of Australia, Princeton University, 1992 NBER Summer Institute, Division of International Finance, Board of Governors of the Federal Reserve, and IMF Research Department for helpful comments. We especially thank William Branson, Avinash Dixit, Jon Faust, Ken Froot, Atish Ghosh, Daniel Kahneman, Philip Lowe, Gordon Menzies, Paolo Pesenti and Ken Rogoff. The views expressed herein are those of the authors and do not necessarily reflect the views of the Reserve Bank of Australia. Any remaining errors are ours.

## **ABSTRACT**

Anchoring is a well-documented behaviour pattern. It occurs when agents form their expectations of an objective variable by only partially adjusting from some given starting value. We present a model of the foreign exchange market in which there are two types of traders: those who are fully rational and those whose expectations are anchored to the forward exchange rate. Under plausible conditions, a significant proportion of the anchored traders survive in the market in the long-run. The model explains both forward discount bias in the direction consistently observed in foreign exchange markets and the results of surveys of market participants' exchange rate expectations.

## TABLE OF CONTENTS

1. INTRODUCTION	1
2. IRRATIONAL AGENTS, ANCHORING AND FOREIGN EXCHANGE PUZZLES	4
2.1 Anchoring	5
2.2 Foreign Exchange Market Puzzles	6
3. ANCHORING AS AN EXPLANATION	9
4. A MODEL WITH ANCHORED AND RATIONAL TRADERS	12
4.1 The Stochastic Environment in the Foreign Exchange Market	13
4.2 The Anchored Traders' Expectations	15
4.3 The Traders' Asset - Demand Functions	15
4.4 The Supply of Domestic and Foreign Interest-Bearing Assets	16
4.5 Determining the Spot Exchange Rate	16
4.6 Market Reaction to Real and Nominal Shocks	17
4.7 Relative Asset Supplies Not Equal to the Traders' Minimum-Variance Portfolio	18
4.8 Discussion	18
5. EMPIRICAL RESULTS	19
5.1 Calibrating the Model	19
5.2 Endogenous Determination of the Proportion of Anchored Traders	22
5.3 Results and Discussion	23
5.4 Comparison with Other Work	25
6. CONCLUSION	27
APPENDIX A: REDUCED FORMS FOR THE STOCHASTIC ENVIRONMENT	29

APPENDIX B: THE TRADERS' ASSET - DEMAND FUNCTIONS	30
APPENDIX C: ASSET SUPPLIES NOT EQUAL TO THE TRADERS' MINIMUM-VARIANCE PORTFOLIO	32
APPENDIX D: AUGMENTED DICKEY-FULLER TESTS ON 3-MONTH NOMINAL INTEREST DIFFERENTIALS	33
APPENDIX E: COMPARING THE TRADERS' PERFORMANCE	34
APPENDIX F: REAL EXCHANGE RATE SHOCKS ARE AR(1)	35
DATA APPENDIX	36
TABLES AND FIGURES	37
REFERENCES	42

# EXPLAINING FORWARD DISCOUNT BIAS: IS IT ANCHORING ?

David W. R. Gruen and Marianne Gizycki

## 1. INTRODUCTION

The short-run behaviour of floating exchange rates has baffled economists for at least a decade. Along with the apparent inability of structural models of the exchange rate to outperform a random-walk (Meese and Rogoff (1983)) probably the best documented and most enduring puzzle is the bias of the forward discount as an estimate of the future exchange rate change. This paper provides a possible explanation for this bias.

If foreign exchange market participants are rational and risk-neutral and transaction costs in the market are small enough to be ignored, then the forward discount should be an unbiased estimate of the future exchange rate change. The overwhelming empirical evidence of forward discount bias is therefore a rejection of this joint hypothesis.<sup>1</sup>

There are four possible interpretations of the failure of this joint hypothesis. The first is that there is a time-varying risk premium required to hold assets denominated in different currencies. However, a model which incorporates risk premia and explains the bias of the forward discount has proven elusive (see, for example, Hodrick (1987), Cumby (1988), Baillie and Bollerslev (1990) and Froot (1990)). Furthermore, theory-based estimates of risk premia turn out to be very small indeed (see Frankel (1985), Frankel (1988) and Engel (1992)).

The second interpretation is that the sample used to test the joint hypothesis has been too small - either because of peso problems (Rogoff (1979), Krasker (1980)) or because rational agents need time to learn the true model of their economic environment (Lewis (1989)). But this interpretation seems increasingly strained the longer forward discount bias manifests itself in the data, and recent extensive evidence (Frankel and Chinn (1991)) shows no sign of its disappearance.

---

<sup>1</sup> See, for example, Hodrick (1987), Goodhart (1988) and Frankel and Chinn (1991).

A third possibility (Baldwin (1990)) is that small transaction costs combined with uncertainty can lead to an interest rate differential matched neither by a risk premium nor by an expected exchange rate change. Interest rate differentials within a small band do not set in motion the capital flows that would close the gap because transaction costs render the moving of capital sub-optimal. However, this analysis (which assumes the full rationality of market participants) seems inconsistent with results from surveys of market participants' expectations.

The final possible interpretation is that the foreign exchange market is not efficient. This is the interpretation we explore in this paper.<sup>2</sup> In particular, we examine the consequences for the exchange rate of the presence of market participants whose exchange rate expectations are *directly influenced* by the value of the forward exchange rate.

We justify this focus on the forward exchange rate by drawing on the work of cognitive psychologists. In Section 2 of the paper, we report some of this work and argue that the forward exchange rate fits very closely psychologists' definition of an "anchor" for exchange rate expectations. For brevity, we describe foreign exchange market participants whose expectations are influenced by the value of the forward rate as "anchored traders".

The paper presents a stochastic version of Dornbusch's 1976 sticky-price model in which two types of risk-averse traders, anchored and rational, jointly determine the exchange rate. The anchored traders use the forward rate as the anchor for their expectations, but they also have a view of the appropriate level of the exchange rate (and if the exchange rate is not at that level, their expectations adjust from the anchor).<sup>3</sup> By contrast, the rational traders understand the true nature of the foreign exchange market, how the anchored traders behave, as well as the proportion of market wealth managed by them. As a consequence, the rational traders have unbiased expectations of the future path of the exchange rate.

---

<sup>2</sup> Froot and Thaler (1990) are also drawn to this interpretation. They suggest that forward discount bias may be a consequence of some market participants responding sluggishly to changes in the interest differential. But again, it is hard to see how this analysis is consistent with the results of surveys of market participants' expectations (see later).

<sup>3</sup> The importance of assuming that the anchored traders respond to changes in the current exchange rate will become apparent when we describe the model in Section 4.

The traders manage the funds of "investors" who are assumed to base their investment decisions on the traders' relative performance over an exogenously specified finite horizon. The equilibrium proportion of anchored traders in the market is determined endogenously assuming a decision rule for investors based on the probability that the anchored traders' chosen portfolio out-performs the rational traders' portfolio over the investors' horizon.

The vast majority of future short-run movements in the exchange rate are unpredictable and hence, in particular, are unrelated to the value of the current forward discount.<sup>4</sup> This fact plays a key role in the model. It implies that the advantage gained from correctly understanding the relation between the forward discount and the spot exchange rate is not great. As a consequence, even when examined over quite long horizons, the anchored traders often out-perform the rational traders simply by chance. Provided investors have (even quite long) finite horizons over which they compare the relative performance of the traders, a significant proportion of anchored traders survive in the market in the long-run.

The model explains two puzzling features of the foreign exchange market. Firstly, it predicts forward discount bias in the direction consistently observed in foreign exchange markets. And secondly, it predicts that market participants' average exchange rate expectations are strongly correlated with the forward discount, as has been documented by Froot and Frankel (1989) and Frankel and Chinn (1991).

The paper proceeds as follows. Section 2 reviews some of the psychological evidence on anchoring and provides more detail on the puzzles in the foreign exchange market on which we focus. Section 3 explains why the presence of anchored traders in the market can help explain these puzzles. Section 4 presents a stochastic version of the Dornbusch (1976) sticky-price model in which risk-averse rational and anchored traders jointly determine the exchange rate. Data on interest rate differentials and exchange rate volatility are used in Section 5 to calibrate the model. This section also proposes a closure for the model which allows endogenous determination of the proportion of anchored traders in the market. Numerical results are presented which suggest that the model is empirically relevant and that it

---

<sup>4</sup> See, for example, Dornbusch (1980), Meese and Rogoff (1983), Cumby and Obstfeld (1984), Obstfeld (1985), Frankel and Meese (1987) and Campbell and Clarida (1987). Clarida and Taylor (1993) provide a dissenting view, though it is based on only a short run of out-of-sample testing.

generates several of the empirical regularities described in the literature. Finally, we examine a potentially serious simplification in the model. Section 6 concludes.

## **2. IRRATIONAL AGENTS, ANCHORING AND FOREIGN EXCHANGE PUZZLES**

The idea that irrational market participants exert an influence on market prices dates back a long time (for example, see Mackay (1841) on the seventeenth century tulipmania). But the stage was set for modern discussion of this issue by the debate between Nurske (1944) and Friedman (1953) about the importance of destabilizing speculation in the foreign exchange market. Recent interest in the relevance of irrational market participants has been spurred by evidence of 'excess volatility' in financial markets (LeRoy and Porter (1981), Shiller (1989)), by price movements which seem hard to reconcile with the efficient markets model (like the 1987 stockmarket crash or the aerobatics of the \$US in the 1980s) and by formal analysis of fads, fashions and 'noise traders' in financial markets (Kyle (1985), Summers (1986), Black (1986), Shiller (1989), De Long et. al. (1990a), (1990b), (1991) and Campbell and Kyle (1993)).

Formal analysis of irrational market agents is usually motivated by an appeal to anecdotal and/or psychological evidence. While this evidence is strongly suggestive, it often relates only loosely to the specification chosen for the irrational agents' behaviour. For example, De Long et. al.'s (1990a) noise trader is characterized by his "bullishness" about the return on a risky asset. But the stochastic properties of this bullishness are chosen for tractability and simplicity, rather than to conform closely to any body of evidence on how people actually behave. A similar comment applies to Frankel and Froot's (1990a) analysis of the swings in the \$US in the 1980s. As they point out, the expectations formed by their "chartists" and "fundamentalists" are chosen for simplicity rather than realism.

In this paper, we attempt to take a body of psychological evidence seriously and to base the behaviour of our irrational agents as closely as possible on this evidence. We now review the psychological evidence that "anchoring" is a common, robust and systematic behaviour pattern.

## 2.1 Anchoring

"In many [uncertain] situations, people make estimates by starting from an initial value that is adjusted to yield the final answer. The initial value, or starting point, may be suggested by the formulation of the problem, or it may be the result of a partial computation. In either case, adjustments are typically insufficient. That is, different starting points yield different estimates, *which are biased toward the initial values*. We call this phenomenon anchoring." (Tversky and Kahneman (1974), p. 1128, italics added).

Anchoring is a well-established behaviour pattern. The following experiment provides a stark example. A random number between 0 and 100 is generated by spinning a wheel of fortune in the subject's presence and s/he is asked to indicate if this number is higher or lower than the percentage of African countries in the UN. Then, s/he is asked to estimate this percentage. The median estimates of the percentage of African countries in the UN are 25 and 45 for groups that received 10 and 65, respectively, as (random) starting points. Payoffs for accuracy do not reduce the anchoring effect (Tversky and Kahneman (1974)).

While striking, this example prompts an obvious question: is anchoring also observed when subjects are knowledgeable about and familiar with the quantity they are estimating? The evidence of Wright and Anderson (1989) suggests that it is. In an experiment with undergraduate business school students, anchoring remained strong even when the students were asked to estimate quantities about which they had considerable knowledge and experience - like the grade point average of a randomly selected student in their program. Wright and Anderson's study also corroborates the earlier evidence that anchoring remains pronounced even when payoffs for accuracy are provided.

Perhaps most telling for our purposes is an example of anchoring in an information-rich, real-world setting (Northcraft and Neale (1987)). Subjects, who were either undergraduate business school students or professional real estate agents, were given a 10-page packet of information about a house currently for sale. The packet included *all* the information which local real estate agents claimed might be used to evaluate a piece of residential property. The information was all correct, with the exception of the seller's asking price for the house (which differed from one subject to the next and which, it was hypothesised, might act as an anchor for subjects'

estimates of the house's value). Subjects visited the house and its neighbourhood and then provided four estimates of its value (its appraised value; an appropriate advertised selling price; a reasonable price to pay for it and the lowest offer they would accept if they were the seller). For both amateurs and experts, the seller's asking price was a highly significant anchor for each of the estimates of the value of the house.<sup>5</sup>

As these examples imply, the forward exchange rate fits very closely psychologists' definition of an anchor for expectations of the future exchange rate. After all, the forward rate is the rate at which foreign exchange can be traded today for delivery at a particular future date. Thus there is a clear connection ("suggested by the formulation of the problem") between the forward rate and the value of the exchange rate at that future date.

Before explaining our model of anchored foreign exchange traders, we describe the foreign exchange market puzzles on which we focus.

## 2.2 Foreign Exchange Market Puzzles

The standard test for forward discount bias is the regression:

$$\Delta s_{t+k} = a + b \, fd_{t,k} + u_{t+k} \quad (1)$$

where  $\Delta s_{t+k}$  is the change in the log spot price of foreign exchange over the next  $k$  periods,  $fd_{t,k}$  is the current  $k$ -period forward discount (log of the current  $k$ -period forward exchange rate minus log of the spot rate) and  $u_{t+k}$  is a mean-zero error term. Ignoring transaction costs, the null hypothesis that market participants are risk-neutral and rational implies that  $b = 1$ .<sup>6</sup> However, when equation (1) is estimated for exchange rates between OECD economies with comparable inflation

---

<sup>5</sup> However, the experts seemed much less aware that they were being influenced by the anchor! When asked what factors had influenced their valuations, 56% of the amateurs mentioned the asking price, while only 24% of the experts did.

<sup>6</sup>  $a = 0$  is also sometimes included as part of the null, although it is not a general implication of risk-neutrality and rational expectations. Different assumed distributions of exchange rate changes imply different values for  $a$  because of Jensen's inequality (see, for example, Frankel (1983)).

rates and for  $k$  values up to about 12 months, the overwhelming empirical finding is that  $b < 1$  and often that  $b < 0$ .<sup>7</sup>

There is a stark contrast between these estimates of  $b$  and estimates of the coefficient on the forward discount,  $d$ , from the regression

$$\Delta s_{t+k}^e = c + d \, fd_{t,k} + v_t \quad (2)$$

where  $\Delta s_{t+k}^e$  is the average exchange rate change expected by market participants over the next  $k$  periods. When equation (2) is estimated under the conditions described above for equation (1), the overwhelming empirical finding is that  $d > 0$  and often that  $d$  is insignificantly different from one.<sup>8</sup> Taken together, equations (1) and (2) imply that

$$s_{t+k} - s_{t+k}^e = (a - c) + (b - d) \, fd_{t,k} + u_{t+k} - v_t$$

---

<sup>7</sup> To give two representative examples, Goodhart (1988) estimates equation (1) for nine datasets over time periods ranging from 1974-1980 to 1974-1986. He examines the £, DM, Swfr and ¥ all against the \$US, and studies both  $k =$  one month and  $k =$  three months. In six cases out of nine the point estimate of  $b$  is negative, and in five cases it is significantly (more than two standard errors) less than one. By contrast, in no case can he reject the null hypothesis that  $b = 0$ . Frankel and Chinn (1991) report pooled time series/cross section regressions of equation (1) for the exchange rates of 17 countries against the US using  $k =$  three months over the time period February 1988 - February 1991. (They also report qualitatively similar results for  $k = 12$  months.) The point estimate (GMM standard error) of  $b$  is  $b = -0.67$  (0.41) when the constant term,  $a$ , is constrained to be equal across countries and  $b = -2.88$  (0.65) when it is unconstrained.

<sup>8</sup>  $\Delta s_{t+k}^e$  is derived from surveys of market participants. To reduce the effect of outliers on the results, it is often the harmonic mean or the median of the survey responses, rather than the arithmetic mean. We assume that the surveys truly reflect average market expectations (at least up to a random measurement error). Froot and Frankel (1989) estimate equation (2) for a range of survey datasets. In seven cases out of nine, they accept the hypothesis that  $d = 1$  and conclude (p. 149) that: "Expectations seem to move very strongly with the forward rate." They also find that the point estimates of  $b$  in equation (1) for the time periods of their survey data are usually negative and statistically significantly less than 1. Frankel and Chinn (1991) report pooled time series/cross section regressions of equation (2) for survey data corresponding to the exchange rates and time period quoted in footnote 7. The point estimate (GMM standard error) of  $d$  for a 3-month horizon is  $d = 0.82$  (0.18) when  $c$  is constrained to be equal across countries and  $d = 0.42$  (0.20) when  $c$  is unconstrained.

with the empirical evidence strongly suggesting that  $b - d \neq 0$ . Thus, there is time  $t$  information,  $fd_{t,k}$  which helps to predict the time  $(t+k)$  prediction error,  $s_{t+k} - s_{t+k}^e$ . As has been repeatedly stressed, the average survey expectations are not rational expectations.

Figure 1 (p.39) shows exchange rate and expectations data which provide an example of these puzzles.<sup>9</sup> Two things stand out. First, it does appear from the figure that the average exchange rate change expected by market participants is correlated with the forward discount, an appearance supported by regression analysis (though the significance level is not high). Second, there is so much exchange rate volatility that it is not obvious from the figure that the forward discount or average market participants' expectations are biased estimates of the future exchange rate change, despite the formal statistical analysis which strongly suggests that they are. Compared to the range of possible outcomes for the actual exchange rate change, the average bias of market participants' expectations is very small.

Figure 1 does not show the substantial heterogeneity in market expectations (Frankel and Froot (1990b), Ito (1990), Takagi (1991)). To give an example from another market, the standard deviation of market participants' expectations of the one-month change in the ¥/\$US is 2.2% (Frankel and Froot (1990b)). That is a big number.<sup>10</sup> At any given time, different market participants have *widely* different exchange rate expectations. Only when these heterogeneous expectations are averaged over many participants and over a time span measured in years does it become apparent that, on average, market participants' expectations are biased. But as we have said, this average bias is very small.

Where does this small bias come from? For reasons explained above, we assume that some market participants are influenced by the value of the forward rate when forming their exchange rate expectations. In the rest of the paper, we maintain the

---

<sup>9</sup> For these data,  $\Delta s_{t+k} = -0.04 (0.02) - 6.2 (3.1) fd_{t,k}$ ;  $\Delta s_{t+k}^e = 0.0 (0.005) + 1.2 (0.7) fd_{t,k}$  with  $k = 4$  weeks and GMM standard errors in parentheses (Smith and Gruen (1989)).

<sup>10</sup> The average one-month change in the ¥/\$US is about 0.2% while its standard deviation is 3.2%. The former number is an average from 1974 to 1990; the latter is from Baille and Bollerslev (1989).

convenient fiction that there are only two possible types of market participants (traders), rational and anchored, and that all traders of a given type are identical.

### 3. ANCHORING AS AN EXPLANATION

To motivate the formal modelling to follow, here we describe how the presence of anchored traders can explain the foreign exchange puzzles described above. We consider a world like the one described by Dornbusch (1976) and make the following assumptions.

Model Assumptions:

There are two open economies, domestic and foreign, with a market-determined exchange rate between them. The foreign goods-price level is constant through time as is the foreign interest rate. The domestic economy is small in the world capital market and so takes the foreign interest rate as given. In the domestic economy, output is fixed and there is no trend money growth and no trend inflation but goods prices adjust slowly to money supply shocks. The domestic interest rate on short-term nominal assets clears the domestic money market and satisfies an international arbitrage condition. Finally, there is a forward market for foreign exchange with no default risk and no transaction costs so covered interest parity holds at all times,

$$f_t - s_t \equiv fd_t = (i - i^*)_t \quad (3)$$

where  $s_t$  and  $f_t$  are the time  $t$  log spot price of foreign exchange and 1-period forward rate respectively,  $fd_t$  is the 1-period forward discount and  $i_t$  and  $i^*$  are the one period interest rates on domestic and foreign nominal assets.<sup>11</sup>

We now compare the market reaction to a single unanticipated domestic monetary expansion when all foreign exchange traders are rational with the reaction when they are all anchored to the forward rate. To ease exposition, we assume all traders are risk-neutral, while in the formal model in the next section, we assume (for a reason that will become clear) that they are risk-averse. Before the monetary expansion, goods prices and the nominal exchange rate are in long-run equilibrium.

---

<sup>11</sup> Throughout, lowercase variables with the exception of interest rates are the logs of corresponding uppercase variables and foreign variables are denoted with a star.

That is,  $s_t = \bar{s}_0$  where  $\bar{s}_0$  is the (initial) long-run equilibrium level of the spot rate (see Figure 2). In this equilibrium, the risk-neutral foreign exchange agents ensure that domestic and foreign interest rates are equal ( $i_t = i^*$ ).<sup>12</sup>

It helps to imagine the events surrounding the monetary expansion as occurring in two sequential stages. At the first stage, at  $t = I^- \left( \lim_{t \uparrow I} = I^- \right)$ , the monetary expansion occurs, the domestic interest rate falls to clear the domestic money market and the forward rate moves to satisfy covered interest parity, but the spot rate remains unchanged i.e.,  $s_1 = \bar{s}_0$ . At the second stage, at  $t = I^+ \left( \lim_{t \downarrow I} = I^+ \right)$ , the foreign exchange market equilibrates. In general, the second stage involves movement of the spot rate, the forward rate and the domestic interest rate.<sup>13</sup> Note, however, that whatever movement of the spot rate is required to equilibrate the market, riskless arbitrage ensures that covered interest parity always holds.

While the first stage of this process is the same whether traders are rational or anchored, the second stage is not. As Dornbusch showed, with rational traders, the nominal exchange rate eventually fully reflects the monetary expansion but it initially overshoots (see Figure 2). To stress a key point: at  $t=1^+$  a spot rate  $s_{1^+} = \bar{s}_0$  is not consistent with market equilibrium because rational traders are aware of an excess return to be earned on the foreign asset at this spot rate. Rational traders' realisation of this excess return drives the spot rate onto its overshooting path.

Now assume that, rather than understanding their economic environment, all traders' expectation of the one-period-ahead spot rate is simply equal to the current 1-period forward rate. Now, at  $t=1^+$ , these anchored traders have no reason to drive the

---

<sup>12</sup> Although it is not clear at this stage, readers should take as given that the long-run equilibrium is the same whatever the nature of the foreign exchange traders.

<sup>13</sup> In a standard specification, movement of the spot rate affects the domestic price of imports, hence the domestic goods price level, hence the level of real balances and the domestic interest rate.

exchange rate onto the overshooting path because they are *unaware* of the excess return to be earned on the foreign asset.<sup>14</sup>

That much is clear. But, by itself, this argument does not determine either the level of the spot rate at  $t=1^+$  nor how it evolves to a new long-run. An auxiliary assumption is needed. The assumption we invoke is that, as well as being anchored to the forward rate, these traders have a target level for the real exchange rate. Then, in a market inhabited exclusively by such traders, there is no initial jump in the spot rate ( $s_{1^+} = \bar{s}_0$ ) and it gradually adjusts to long-run neutrality ( $\lim_{t \rightarrow \infty} s_t = \bar{s}_1$ ). This "anchored" adjustment path is shown in Figure 2.<sup>15</sup>

One point deserves emphasis. Our explanation of the anchored traders' behaviour *does not rely* on fleeting irrationality in the instant following a monetary expansion. There is nothing special about that instant. All along the "anchored" path, there is an excess return to holding the foreign asset but traders who use the forward rate as their expectation of the one-period-ahead spot rate are unaware of this excess return and hence never set off a Dornbusch-style overshooting adjustment.

What are the implications for the regressions (1) and (2)? With the foreign exchange market composed entirely of rational traders, both the expected exchange rate change and the actual exchange rate change are equal to the forward discount along the adjustment path. By contrast, with the foreign exchange market composed entirely of anchored traders, the expected exchange rate change is again equal to the

---

<sup>14</sup> These traders' expected nominal return on the foreign asset (measured in domestic currency) is  $i^* + (f_{1^+} - s_{1^+})$  while the nominal return on the domestic asset is  $i_{1^+}$ . Whatever the value of  $s_{1^+}$ , covered interest parity ensures that these expected returns are equal. Note that, rather than being anchored to the forward rate, one can interpret these traders as being "anchored" to a simple form of the efficient markets hypothesis so that they are again unaware of the excess return to be earned on the foreign asset. This interpretation was suggested by Avinash Dixit.

<sup>15</sup> All along this path, the real exchange rate is equal to the anchored traders' target and their expectation of the one-period-ahead spot rate is equal to the forward rate. A formal derivation of this adjustment path is presented in Section 4.6. One final clarifying point. As discussed earlier, the interest differential depends on the behaviour of the exchange rate. To avoid unnecessary clutter, Figure 2 ignores this dependence and shows a single path for the interest differential.

forward discount, but the actual exchange rate change is in *the opposite direction* to the prediction of the forward discount ( $b$  in equation (1) is negative).

Thus, if the foreign exchange market is composed *entirely* of anchored traders, this simple story predicts that  $b < 0$  and  $d = 1$ , as the empirical results in Section 2 suggest. This is a promising step, but it cannot be the whole story.

Firstly, if exchange rates adjusted smoothly and gradually to money shocks as in Figure 2, anchored traders would presumably learn that their expectations were systematically and repeatedly wrong. It strains credulity to argue that such expectations could survive in such circumstances. But, as discussed in the introduction and as highlighted in Figure 1, nominal exchange rates are subject to continual unpredictable shocks which are very much larger than (and uncorrelated with) the forward discount. In such an environment, it is not so implausible that some participants in the foreign exchange market are subject to a small psychologically-based bias in their exchange rate expectations.

Secondly, the foreign exchange market cannot be composed entirely of anchored traders. The bias of the forward discount has been well-established for at least a decade, and there are obviously participants in the market who can exploit it. Can the simple explanation presented above survive the presence of rational traders who are aware of the forward discount bias? The next section tackles this question.

#### 4. A MODEL WITH ANCHORED AND RATIONAL TRADERS

There are two features of the floating exchange rate environment we want to capture in a model. These features are the strong autocorrelation in short-term nominal interest differentials between countries and the very large unpredictable shocks to the nominal and real exchange rate. We model the former feature by assuming sticky goods-prices, so in response to nominal money shocks, the domestic-foreign nominal interest differential adjusts slowly back to long-run equilibrium. Following Campbell and Clarida (1987), the latter feature is modelled by assuming that the long-run real exchange rate follows a random-walk.<sup>16</sup> Beyond capturing these

---

<sup>16</sup> We examine the alternative of modelling these shocks as a stationary AR(1) process at the end of the paper in Appendix F. However, this alternative requires extra complication in the model.

features of the exchange rate environment, our model is as simple and stripped-down as possible.

The model is a two-country model in discrete time, with each period of four weeks duration. The government in each country issues local-currency-denominated interest-bearing nominal assets with a maturity of one period. The foreign exchange market contains two types of mean-variance optimising traders: rational and anchored. A fraction  $\alpha$  of market wealth is managed by the anchored traders with  $1 - \alpha$  managed by the rational traders. For the moment  $\alpha$  is specified exogenously, but in Section 5 it will be determined endogenously.

#### 4.1 The Stochastic Environment in the Foreign Exchange Market

The stochastic properties of goods-price adjustment and the domestic-foreign interest differential are derived as follows. We make the Model Assumptions (defined in Section 3) and also assume for the *domestic* economy that the money market has a standard LM curve, that the money supply is subject to independent shocks each period and that aggregate demand is a simplified version of the Dornbusch (1976) form. These assumptions lead to three reduced-form equations:

$$\bar{p}_{t+1} = \bar{p}_t + v_{t+1} \quad (4)$$

$$\mathbf{D} p_{t+1} = -\mathbf{1} (i - i^*)_t / \mathbf{q} \quad (5)$$

$$(i - i^*)_{t+1} = (i - i^*)_t [1 - \mathbf{1}/\mathbf{q}] - v_{t+1} / \mathbf{1} \quad (6)$$

where, in period  $t$ ,  $p_t$  and  $\bar{p}_t$  are logs of the domestic goods price level and its long-run equilibrium,  $(i - i^*)_t$  is the interest differential between domestic and foreign one-period nominal assets, and  $v_t$  is an i.i.d.  $N(0, \mathbf{S}_m^2)$  shock to the log domestic money supply.  $\lambda$  is the semi-elasticity of money demand with respect to the interest rate and  $1/\theta$  is the rate at which goods prices, and hence the domestic-foreign interest differential, adjust to money shocks.

Long-run neutrality implies equation (4) while (5) and (6) are derived in Appendix A. Here, we explain our use of a simplified form of the Dornbusch (1976) domestic demand function. Dornbusch assumes that demand depends on real income, the interest rate and the real exchange rate. We drop the dependence on the real exchange rate and as a consequence, must impose a constraint between exogenous variables in the model (see Appendix A). This greatly eases analysis of the stochastic environment in the foreign exchange market and allows the rational-expectation solution for the exchange rate to be derived straightforwardly. Crucially, the key features of the Dornbusch model are preserved. That is, in a fully-rational risk-neutral market, the model exhibits both exchange-rate overshooting and the absence of forward discount bias. Finally, we note that equations (5) and (6) are consistent with an arbitrage condition in the foreign exchange market. This arbitrage condition is introduced shortly (equation (10)).

Defining  $\bar{s}_t$  as the equilibrium value of  $s_t$  (the value of  $s_t$  if there were no goods-price stickiness) and setting the foreign goods price equal to one, the equilibrium real exchange rate at time  $t$  is  $\bar{s}_t - \bar{p}_t$ . We assume that  $\bar{s}_t - \bar{p}_t$  follows a random walk

$$(\bar{s}_{t+1} - \bar{p}_{t+1}) = (\bar{s}_t - \bar{p}_t) + e_{t+1} \quad (7)$$

with the shocks  $e_{t+1}$  specified exogenously.<sup>17</sup> We assume  $e_{t+1}$  is distributed i.i.d.  $N(0, \sigma_e^2)$  on the basis of the observed stochastic properties of exchange rate changes (see Section 5). Both the rational and anchored traders are assumed to know the stochastic properties of these real exchange rate shocks.

Equations (4) - (7) summarise the stochastic environment in the foreign exchange market. In Section 5, values for the parameters introduced,  $\lambda$ ,  $\theta$ ,  $\sigma_m$  and  $\sigma_e$ , are derived from data on interest differentials and floating exchange rates.

---

<sup>17</sup> Again, we assume both that these shocks are exogenous and that domestic real output is fixed to keep things simple. As an alternative, we could assume random productivity shocks which have an impact on real output and are the source of the real exchange rate shocks modelled in equation (7). With our focus on the stochastic environment in the foreign exchange market, we did not complicate the model in this way.

## 4.2 The Anchored Traders' Expectations

The anchored traders' expectation of the future exchange rate is given by

$$E_t^a(s_{t+1}) = f_t - \mathbf{b}[(s_t - p_t) - (\bar{s}_t - \bar{p}_t)], \quad \mathbf{b} > 0 \quad (8)$$

This equation embodies two behavioural assumptions:

- (i) the forward rate is an anchor for expectations of the one-period-ahead spot rate, but
- (ii) these traders also have a target level of the real exchange rate and if the rate is not at that level, they adjust to expect that it will return back towards that level over time. The parameter  $\beta$  quantifies by how much these traders' expectations adjust (from the anchor) when the exchange rate deviates from their current target level.

Defining the anchored traders' target level in terms of the *real* exchange rate ensures an appropriate market reaction to both real and nominal shocks as will be shown in Section 4.6.

The second behavioural assumption is crucial. Without it, these traders have no impact on the exchange rate. Thus, if anchored traders' expectations always equal the forward exchange rate, *regardless of the value of the current exchange rate*, and the market contains even a single rational risk-averse trader, she will drive the exchange rate to a Dornbusch overshooting equilibrium because the rest of the market has no reaction when the spot rate moves.<sup>18</sup>

## 4.3 The Traders' Asset - Demand Functions

With the wealth entrusted to them by investors, the traders choose the relative proportions to hold in the domestic and foreign interest-bearing assets.<sup>19</sup> Each

---

<sup>18</sup> It is easy to confirm that the solution of the model (equation (11)) is the same when  $\beta = 0$  as when there are no anchored traders in the market ( $\alpha = 0$ ).

<sup>19</sup> There are four assets in the model: money and the interest-bearing asset in each economy. Money is held by individuals in each economy because it provides transaction services.

period, each trader optimises with respect to the mean and variance of the end-of-period real wealth she manages. By assumption, both rational and anchored traders have the same coefficient of relative risk aversion,  $\gamma$ , and the same consumption bundle - which consists of a constant expenditure share  $g$  of foreign goods and  $(1 - g)$  of domestic goods. For trader  $k$ , if  $x_t^k$  is the proportion of period- $t$  wealth she holds in the foreign nominal asset, Appendix B shows that

$$x_t^k \approx g + [\mathbf{g}\mathbf{S}_e^2]^{-1} [i_t^* - i_t + E_t^k \Delta s_{t+1}], \quad k = a, r. \quad (9)$$

Each trader's demand for the foreign asset consists of two terms. The first term,  $g$ , is the minimum-variance portfolio. If a trader were infinitely risk-averse ( $\gamma = \infty$ ) she would hold this portfolio. The second term represents the 'speculative' portfolio. A higher (lower) expected excess return on the foreign asset induces the trader to hold more (less) of that asset than its share in the minimum-variance portfolio, to an extent limited by the trader's risk aversion and the variance of the return.

#### 4.4 The Supply of Domestic and Foreign Interest-Bearing Assets

To simplify the exposition, we describe the solution of the model for the special case in which the supply of domestic and foreign interest-bearing assets made available to be managed by the the traders exactly matches their minimum-variance portfolio: that is, of the total wealth managed by the traders, a constant proportion  $g$  is in the form of the foreign asset. A more general case, when there is no match, leads to only a small change to the basic story and is dealt with in Section 4.7.

#### 4.5 Determining the Spot Exchange Rate

With the anchored traders managing a proportion  $\alpha$  of the total market wealth, equating supply and demand for the foreign asset gives

$$g = \mathbf{a} \cdot x_t^a + (1 - \mathbf{a}) \cdot x_t^r \quad (10)$$

---

However, with the wealth entrusted to them by investors, the traders do not hold money because it is a dominated asset.

Equilibrium in the market for the foreign asset ensures equilibrium in the market for the domestic asset by Walras's Law. Equation (10) leads to a difference equation in  $d_j \equiv E_t^r (s_{t+j} - \bar{s}_{t+j})$ ,  $j \geq 0$ . Imposing the transversality condition that  $\lim_{j \rightarrow \infty} d_j = 0$  rules out rational bubbles and leads to the exchange rate change from  $t$  to  $t+1$ :

$$\mathbf{D}s_{t+1} = \mathbf{h}fd_t + (1 + \mathbf{q}\mathbf{h}/\mathbf{l})v_{t+1} + e_{t+1} \quad (11)$$

where  $\mathbf{h} \equiv \mathbf{h}(\mathbf{a}) = 1 - \mathbf{a}\mathbf{b}(1 + \mathbf{q}) / (1 + \mathbf{a}(\mathbf{b}\mathbf{q} - 1))$  and  $fd_t = (i - i^*)_t$ . (Details of the derivation are available from the authors on request.) From equations (8) and (11) we can derive the mean of market participants' exchange rate expectations

$$\begin{aligned} & \mathbf{a} E_t^a \mathbf{D}s_{t+1} + (1 - \mathbf{a}) E_t^r \mathbf{D}s_{t+1} \\ &= \mathbf{a} \left\{ fd_t - \mathbf{b}[(s_t - p_t) - (\bar{s}_t - \bar{p}_t)] \right\} + (1 - \mathbf{a})\mathbf{h}fd_t \\ &= fd_t \end{aligned} \quad (12)$$

It is a striking property of the model that, regardless of parameter values, the mean of market participants' expectations of the exchange rate change is equal to the forward discount. We discuss this result in Section 4.8.

#### 4.6 Market Reaction to Real and Nominal Shocks

Now is a convenient time to show that the model implies an appropriate market reaction to both real and nominal shocks. As can be confirmed from (11), whatever the proportion of anchored traders in the market, the sole effect of a *real* exchange rate shock,  $e_{t+1}$ , is to move the spot exchange rate  $\Delta s_{t+1}$  by the amount of the shock. This is the appropriate market reaction since, by assumption, both rational and anchored traders understand the (permanent) nature of real exchange rate shocks.

Now consider the effect of a nominal money shock when the market contains only anchored traders. In this case, there will never be a Dornbusch-style overshooting adjustment because the anchored traders are always unaware of the excess return to be earned on the foreign asset. With only anchored traders in the market and in the absence of further shocks, the time-evolution of the exchange rate in response to a

nominal shock  $v_{t+1}$  is  $s_{t+t} = s_t + v_{t+1} \left[ 1 - (1 - 1/q)^{t-1} \right]$ ,  $t \geq 1$  (from (6) and (11)). This time-evolution is simply the "anchored" path in Figure 2. There is no initial jump ( $\Delta s_{t+1} = 0$ ) and the exchange rate adjusts gradually to fully reflect the nominal shock (very gradually, given the numerical estimates of  $\theta$  from Section 5).

#### 4.7 Relative Asset Supplies Not Equal to the Traders' Minimum-Variance Portfolio

Here we report the solution of the model for the more general case in which the proportion of the total wealth managed by the traders in the form of the foreign asset is  $g + \mathbf{k}$ ,  $0 \leq g + \mathbf{k} \leq 1$ , (rather than  $g$ ) where  $g$  is again the proportion of foreign assets in the traders' minimum-variance portfolio and  $\mathbf{k}$  is an arbitrary constant. As shown in Appendix C, the effect of this generalisation is simply to add constants to the right hand sides of equations (11) and (12). The solutions for the exchange rate change and the mean of market participants' exchange rate expectations become

$$\mathbf{D}s_{t+1} = \mathbf{h}x + \mathbf{h}fd_t + (1 + \mathbf{q}\mathbf{h}/\mathbf{l})v_{t+1} + e_{t+1} \quad (13)$$

and 
$$\mathbf{a}E_t^a \mathbf{D}s_{t+1} + (1 - \mathbf{a})E_t^r \mathbf{D}s_{t+1} = (1 - \mathbf{a})x + fd_t \quad (14)$$

where  $x = \mathbf{k}g\mathbf{s}_e^2 / (1 - \mathbf{a})$ .

#### 4.8 Discussion

It is now clear that our explanation of the puzzles described in Section 2 survives the presence of risk-averse rational traders in the market. Equations (13) and (14) are of the same form as (1) and (2), so the model's predictions can be easily compared with the empirical results. In the standard regression for testing the bias of the forward discount ((1), (13)), the model coefficient on the forward discount is  $\mathbf{h}(\mathbf{a}) = 1 - (\mathbf{a}\mathbf{b}(1 + \mathbf{q})) / (1 + \mathbf{a}(\mathbf{b}\mathbf{q} - 1))$ . In a fully-rational market, this coefficient is one ( $\mathbf{h}(0) = 1$ ) and, up to a constant, the forward discount is an unbiased estimate of the future exchange rate change. However, as the proportion of anchored traders in the market rises,  $\eta(\alpha)$  falls monotonically (since  $\mathbf{h}(\mathbf{a}) / \mathbf{a} < 0$ ) taking the value  $\mathbf{h}(1) = -1 / \mathbf{q}$  when the market is composed entirely of anchored traders.

With both types of trader in the market, the anchored traders make systematic mistakes about the future time path of the exchange rate and the rational traders exploit these mistakes. But because the rational traders are risk-averse, they reduce, but do not eliminate the deviations of the exchange rate from an efficient-market path. Provided there are some anchored traders in the market ( $\alpha > 0$ ) the forward discount is a systematically biased estimate of the future exchange rate change and the direction of bias is that consistently observed empirically.

Turning now to market expectations, there are two groups with distinctly different expectations. The rational traders have unbiased expectations while the anchored traders' expectations are adjusted from their anchor - the forward exchange rate. But, whatever the proportion of anchored traders, the weighted mean of the traders' expected exchange rate change moves one-for-one with the forward discount (from (14)). Thus, up to a constant, uncovered interest parity holds for average market expectations in the model, despite the fact that no-one in the market has those expectations! Of course, this result for average market expectations accords well with the survey results in Section 2.

Thus, the model can explain both the stylised facts highlighted in Section 2 provided there are some anchored traders in the market in equilibrium. But an important question remains: is it plausible that a sufficient proportion of anchored traders survive in the long-run to generate values of  $\eta$  similar to those consistently observed in foreign exchange markets? The next section addresses this question.

## 5. EMPIRICAL RESULTS

### 5.1 Calibrating the Model

We derive empirical results for the special case of the model and hence the exchange rate evolves according to equation (11).<sup>20</sup> We require values for five parameters:  $\theta$ ,  $\lambda$ ,  $\sigma_m$ ,  $\sigma_e$  and  $\beta$ . Estimates for the first four come from real-world data.  $1/\theta$  is the rate at which goods-prices and the interest differential adjust to

---

<sup>20</sup> As will soon be clear, the size of the unpredictable real exchange rate shocks,  $\sigma_e$ , determines our empirical results. But these shocks are the same in both cases of the model, so results for the general case should be very similar to those reported below. However, deriving results for the general case requires specifying values for two more parameters ( $\gamma$  and  $\kappa$ ).

money supply shocks and is estimated from the time-series properties of 3-month interest differentials between pairs of the countries US, UK, (West) Germany and Japan. Details of the interest rates used are in the Data Appendix. The AR(1) equation describing the evolution of  $(i - i^*)_t$  (equation (6)) is estimated by OLS leading to the estimates for  $\theta$  shown in Table 1 (p 36).<sup>21</sup> To cover the range of results in Table 1, we generate numerical results with  $\theta = 6$  periods and  $\theta = 50$  periods.<sup>22</sup>

Estimates of the semi-elasticity of money demand with respect to the interest rate,  $\lambda$ , are derived from money demand functions estimated for the G-7 by Fair (1987). Details are presented in Appendix D and the resulting parameter values are shown in Table 2. When  $\theta = 6$  periods (50 periods), the derived value of  $\lambda$  implies that a 1% *increase* in the nominal money supply leads to contemporaneous *fall* in the domestic interest rate of 2.0% p.a. (0.7% p.a.).

The standard deviation of nominal money shocks,  $\sigma_m$ , is set so that the standard deviation of the interest differential,  $\sigma_{(i-i^*)}$ , equals 1.9% p.a. (the standard deviation of the Japan – Germany interest differential from Table 1, chosen because these two countries had comparable inflation rates over the 1980s). The AR(1) specification for  $(i - i^*)_t$  implies that  $\sigma_m = \mathbf{I} \sigma_{(i-i^*)} \sqrt{2 / \mathbf{q} - 1 / \mathbf{q}^2}$ , which allows  $\sigma_m$  to be determined for given  $\theta$  and  $\lambda$ . Again, the results are shown in Table 2.

The standard deviation of long-run real exchange rate shocks,  $\sigma_e$ , is set so that the model generates an appropriate amount of exchange rate volatility. Baille and Bollerslev (1989) examine the stochastic properties of the nominal exchange rates of France, Italy, Japan, Switzerland, UK and (West) Germany against the US dollar,

---

<sup>21</sup> A constant is included in the OLS regression to allow for different inflation rates between countries. Augmented Dickey-Fuller tests (Appendix D) imply, at least for the Japan–Germany and Japan–US interest differentials, that the hypothesis of a unit root can be rejected with some confidence. An alternative method for estimating  $\theta$  using the autocovariance of the interest differentials gives similar results.

<sup>22</sup>  $\theta = 50$  periods is chosen because the OLS estimates of  $\theta$  are biased downward.  $\theta = 6$  ( $\theta = 50$ ) implies that the half-life for goods-price adjustment is 3.5 months (32 months) which probably spans the plausible range for goods-price stickiness.

1980–85. For 4-week periods, these exchange rates are well modelled by the process

$$Ds_{t+1} = b_j + e_{t+1} \quad e_{t+1} \sim i.i.d. N\left(0, \mathbf{s}_{Ds_j}^2\right)$$

where  $b_j$  and  $\mathbf{s}_{\Delta s_j}$  are country-specific constants and  $\mathbf{s}_{\Delta s_j}$  ranges from 0.029 to 0.037 with a mean over the six exchange rates of 0.034. Choosing  $\sigma_e = 0.031$  for the model ensures that model values of  $\mathbf{s}_{\Delta s}$  always fall within the range found by Baille and Bollerslev.<sup>23</sup>

We present results for two values of  $\beta$ . With the goods market in long-run equilibrium ( $p_t = \bar{p}_t$ ), the impact response to a money shock  $v_{t+1}$  is  $\Delta s_{t+1} = 0$  with only anchored traders in the market and  $Ds_{t+1} = v_{t+1}(1 + \mathbf{q}/\mathbf{I})$  with only rational traders. We use values for  $\beta$  derived from the market's impact response to this shock when managed wealth is divided equally between the traders. When  $\alpha = 1/2$ , if the spot exchange rate immediately jumps halfway to the value it would take were the market fully rational ( $Ds_{t+1} = v_{t+1}(1 + \mathbf{q}/\mathbf{I})/2$ ), then from equation (11),

$$b = 1 / \mathbf{q}$$

and we describe the anchored traders as “weakly anchored”. Alternatively, when  $\alpha = 1/2$ , if the spot exchange rate immediately jumps only one-tenth of the way to the value it would take were the market fully rational,  $\Delta s_{t+1} = v_{t+1}(1 + \theta/\lambda)/10$ , then

$$b = 9 / \mathbf{q}$$

and we describe the anchored traders as “strongly anchored”.

---

<sup>23</sup> Exchange rate volatility in the model is dominated by real exchange rate shocks, but it also depends weakly on nominal shocks. Equations (11) or (13) imply that:

$$\text{Var}(\Delta s_{t+1}) \equiv \sigma_{\Delta s}^2 = \sigma_e^2 + \eta^2 \sigma_{i-i^*}^2 + (1 + \theta\eta/\lambda)^2 \sigma_m^2. \text{ With } \sigma_e = 0.031 \text{ and } \theta = 6 (\theta = 50), \sigma_{\Delta s}$$

ranges from 0.031 (0.031) when  $\alpha = 1$  to 0.033 (0.037) when  $\alpha = 0$ .

## 5.2 Endogenous Determination of the Proportion of Anchored Traders

With a proportion  $\alpha$  of market wealth managed by anchored traders, define  $R^k(\mathbf{a}, n)$  as the gross real return trader  $k$  earns over  $n$  periods on the portfolio she manages. Further, define  $Z(\mathbf{a}, n) \equiv R^a(\mathbf{a}, n) - R^r(\mathbf{a}, n)$  as the excess real return earned by anchored traders over  $n$  periods. Then, as demonstrated in Appendix E,

$$Z(\mathbf{a}, n) \approx K \sum_{j=1}^n (i - i^*)_j \left[ (\mathbf{h} - 1)(i - i^*)_j + e_{j+1} + (1 + \mathbf{q}\mathbf{h}/1) v_{j+1} \right] \quad (15)$$

where  $K = [1 - \mathbf{h} + \mathbf{b}\mathbf{l}(1 + \mathbf{q}\mathbf{h}/1)] / [\mathbf{g}\mathbf{s}_e^2] > 0$ .

For given  $\alpha$ , we use (15) to derive estimates of  $p(\mathbf{a}, n) \equiv \Pr[Z(\mathbf{a}, n) > 0]$ , the probability that the anchored traders' chosen portfolio outperforms the rational traders' portfolio over the assumed horizon of investors,  $n$ . From these estimates, we also derive estimates of the proportion of anchored traders in the market in the long-run. We assume that investors follow a simple decision rule. Every  $n$  periods, they compare the relative performance of the two types of traders over the past  $n$  periods. When type- $k$  traders ( $k = a$  or  $r$ ) had the better performance, investors increase the proportion of wealth entrusted to them by a constant  $q$ , subject to the constraint that the share of market wealth entrusted to either type of trader never falls below zero.<sup>24</sup> Then the evolution of  $\alpha$  follows a Markov chain

$$\mathbf{a}_{T+1} = \begin{cases} \min(1, \mathbf{a}_T + q) & \text{with probability } p(\mathbf{a}_T, n) \\ \max(0, \mathbf{a}_T - q) & \text{with probability } 1 - p(\mathbf{a}_T, n) \end{cases} \quad (16)$$

where the subscript  $T$  refers to successive  $n$ -period time-intervals.<sup>25</sup>

---

<sup>24</sup> We assume that both types of traders always have at least an infinitesimal market presence so that investors can always observe their relative performance. One way to motivate this assumption is to imagine that new anchored and rational traders enter the market every period, but only when they are successful do they attract a non-negligible fraction of market wealth.

<sup>25</sup> Incorporating this Markov chain for  $\alpha_T$  into the rational traders expectations in Section 4 would add little of substance but greatly complicate the algebra. Our decision rule for investors is similar to the 'Imitation Based on Realised Returns' rule of De Long et. al.

We can now derive the long-run stationary probability distribution of  $\alpha$ . We assume  $\mathbf{a}_0 = 0$  and  $q \in \{0.1, 0.3, 0.5, 1.0\}$ . Then there are at most eleven states for the Markov chain,  $\alpha^i = i/10$ ,  $i = 0, \dots, 10$ , and at all times,  $\alpha_T$  must take one of the values  $\alpha^i$ . The stationary probability distribution for  $\alpha$  is defined by the probabilities  $p^i$

$$p^i = \lim_{T \rightarrow \infty} \Pr(\mathbf{a}_T = \mathbf{a}^i), \quad i=0, \dots, 10$$

From this probability distribution, we derive the long-run average proportion of anchored traders in the foreign exchange market,  $\bar{\alpha}$ , given by

$$\bar{\alpha} = \sum_i p^i \mathbf{a}^i$$

and the long-run average coefficient on the forward discount in equation (11),  $\bar{\eta}$ ,

$$\bar{\mathbf{h}} = \sum_i p^i \mathbf{h}(\mathbf{a}^i)$$

### 5.3 Results and Discussion

Figure 3 shows empirical estimates of  $p(\alpha, n)$  for one set of parameter values ( $\theta = 6$  periods; strongly anchored traders) and for three assumed horizons for investors

---

(1990a). As an alternative, the investors' decision rule could be based on the traders' relative realised *utilities* rather than their realised *returns*. This alternative assumption would reduce the proportion of anchored traders in equilibrium but would not change the qualitative nature of our results. De Long et. al. present reasons for preferring a decision rule based on realised returns: "we find it plausible that many investors attribute the higher return of an investment strategy to the market timing skills of its practitioners and not to its greater risk. This consideration may be particularly important when we ask whether individuals change their own investment strategies that have just earned them a high return. When people imitate investment strategies, they appear to focus on standard metrics such as returns relative to market averages and do not correct for ex ante risk" (p. 724). Shiller (1992) presents supporting evidence (p. 70–74).

(one, three and ten years).<sup>26</sup> Table 3 reports long-run averages of the proportion of anchored traders in the market,  $\bar{\alpha}$ , and the coefficient on the forward discount in the standard test of forward discount bias,  $\bar{\eta}$ , derived for different assumed values of the Markov chain parameter,  $q$ , in equation (16).

For all sets of parameter values,  $p(0,n) \approx 0.5$  and  $p(\alpha,n)$  falls monotonically as  $\alpha$  increases.<sup>27</sup> But it is noteworthy that even over long horizons, the chance that the anchored traders' portfolio outperforms the rational traders portfolio is not so small. An example makes the point. With  $\theta = 6$  and market wealth divided equally between rational and strongly anchored traders,  $\eta$  takes the value  $\eta \approx -0.9$ . Thus, the forward discount is a very significantly biased estimate of the future exchange rate change. Despite this, over a three year (ten year) horizon, the portfolio chosen by the anchored traders (who use the forward discount as the anchor for their expectations of the exchange rate *change*) gives a higher return than the rational portfolio 31% (16%) of the time. This example highlights the general point that unpredictable exchange rate movements are so big that only a small advantage is imparted to those (rational) traders who understand the true relationship between interest rates and the exchange rate.

The results in Table 3 imply, over a wide range of parameter values, that a significant proportion of anchored traders survive in the long-run and that they induce substantial bias in the forward discount. The largest value of  $\bar{\eta}$  in the Table is  $\bar{\eta} = 0.63$  while for strongly anchored traders and shorter investor horizons, the value of  $\bar{\eta}$  is close to zero or negative. As reported in Section 2, the empirically estimated coefficient on the forward discount in the corresponding regression (equation (1)) is often negative. For the range of parameter values investigated, while the model does not often generate negative values of  $\bar{\eta}$ , it always generates strong forward discount bias in the direction consistently observed empirically.

While one might quibble with the specification chosen for the anchored traders' expectations, a strong message emerges from these empirical results. If rational agents in the foreign exchange market are risk-averse or liquidity-constrained, the

---

<sup>26</sup> Each year contains thirteen 4-week periods, and for each  $(\alpha, n)$  pair, 10,000 realisations of the random variable  $Z(\alpha,n)$  are used to generate results.

<sup>27</sup> Despite the fact that  $E Z(0,n) = 0$ , in all cases  $p(0,n) > 0.5$  because the random variable  $Z(0,n)$  is slightly skew when  $n > 1$ .

rational expectations solution for the exchange rate (allowing for risk-aversion) is a fragile one. Unpredictable exchange rate movements are so big that there are no strong grounds for presuming that agents with quite different expectations will be driven from the market. This may help explain why economists have had such difficulty modelling the short-run behaviour of exchange rates.

#### 5.4 Comparison with Other Work

We begin this sub-section by showing that the model generates several of the empirical regularities described in the literature. McCallum (1992) presents a useful summary of these regularities with eight OLS regressions using over twelve years of monthly data for \$/DM, \$/£ and \$/¥ exchange rates. For the first seven regressions, Table 4 compares his \$/DM results (which are representative) with results derived from a simulation of the model. The first regression in the table is the standard test for forward discount bias and, while the model produces a negative coefficient on the forward discount, it is significantly less negative than McCallum's coefficient. As discussed earlier, for the range of parameter values investigated, the model does not generate large negative values of this coefficient ( $\bar{\eta}$ ).

We now explain the *model* results for the remaining regressions in the table. The model results for regressions (2) and (3) are a consequence of the fact that  $s$  and  $f$  are co-integrated I(1) variables with co-integrating vector,  $s_t = f_t$ . This explains both the estimated slope coefficient and the very high regression  $R^2$ . Further, in the model,

$$Df_t = (\mathbf{h} - \mathbf{1}/\mathbf{q})fd_{t-1} + (1 + [\mathbf{q}\mathbf{h} - \mathbf{1}]/\mathbf{1})v_t + e_t$$

while 
$$Ds_t = \mathbf{h}fd_{t-1} + (1 + \mathbf{q}\mathbf{h}/\mathbf{1})v_t + e_t$$

Since  $e_t$  is much larger than all the other terms on the right-hand-side of either of these equations, a regression of  $\Delta s_t$  on  $\Delta f_t$  is rather like regressing  $e_t$  on itself, which explains the model results for regression (4). Lagging the right-hand-side variable in regression (4) generates regression (5), and the relationship disappears because, for the model, (5) is rather like regressing  $e_t$  on  $e_{t-1}$ . Similar logic explains the model results for regressions (6) and (7). Regression (6) is like regressing  $e_t + e_{t-1}$  on  $e_{t-1}$ ; which explains both the slope coefficient and the fact that  $R^2 \approx 1/2$ . Regression (7)

is like regressing  $e_t + e_{t-1} + e_{t-2}$  on  $e_{t-1} + e_{t-2}$ ; which again explains the slope coefficient and the fact that  $R^2 \approx 2/3$ . As is clear from the table, there is a very close fit between the model results and all the empirical results reported by McCallum.

The last OLS regression reported by McCallum is

$$s_t = a_0 + a_1 f_{t-1} + a_2 s_{t-1} + \text{error}_t$$

For each of his three exchange rate datasets, he shows that  $a_1 \approx 1 - a_2 \approx b$ , where  $b$  is the coefficient on the forward discount in the standard test for forward discount bias (regression (1) in Table 4). Equations (3) and (13) imply for the model that

$$s_t = c_0 + c_1 f_{t-1} + c_2 s_{t-1} + (1 + \mathbf{qh}/I)v_t + e_t$$

where  $c_1 \equiv 1 - c_2 \equiv \eta$ , and  $\eta$  is the model coefficient on the forward discount in the standard test for forward discount bias (equation (13)). Thus, again, McCallum's empirical results are reproduced by the model.

We conclude this sub-section with a potentially serious shortcoming of our assumed money supply process. We assume the log-level of the money supply follows an ARIMA(0,1,0) process while Lyons (1990) points out that quarterly log-*changes* in the US money supply are approximately ARIMA(0,1,1). Lyons' observation implies that a current shock to the money *stock* leads rational agents to adjust their estimates of future money *growth*. So a money shock has both a direct effect on the exchange rate as well as an indirect growth-rate effect. Our assumed money supply process ignores the indirect growth-rate effect because it assumes that shocks to the money stock are independent of each other.

There are two relevant comments. Firstly, for realistic parameter values, the importance of the indirect growth-rate effect is unclear. If money supply growth rate innovations are truly permanent, the indirect effect is about twice as large as the direct effect (Lyons' Table 5). But if they are long-lived but temporary, as seems more plausible, his Table 3 suggests that the indirect effect is *much* less important. Secondly, recall our observation that only a small advantage is imparted to those (rational) agents who understand the true relationship between interest rates and the exchange rate. This key observation explains the long-run survival of a significant

proportion of anchored traders. While Lyons' evidence represents a potential challenge to this observation, other empirical evidence (footnote 4) is strongly supportive of it. Note in particular the Meese and Rogoff (1983) finding that knowledge of the *actual* future money supply hardly helps at all to explain the level of the future exchange rate for horizons up to a year. To summarise, Lyons' point is an important conceptual one, but the balance of evidence strongly suggests that it does not overturn the key assumption on which our results depend – that only a small advantage is imparted to those who understand the true relationship between interest rates and the exchange rate.

## 6. CONCLUSION

Why should explanations outside the "rational" paradigm be considered? And if they are, by what criteria can we distinguish between the almost endless possible types of irrational behaviour?

There are at least two answers to the first question. Firstly, for the foreign exchange market, explanations which rely on the full rationality of market participants have been remarkably unsuccessful in explaining the short-run behaviour of exchange rates. And secondly, as has been shown in other markets, small deviations from rationality can make a substantive difference to economic equilibria (Akerlof and Yellen (1985), Mankiw (1985), Cochrane (1989)).

In response to the second question, there *are* ways of assessing the plausibility of different types of irrational behaviour. As well as drawing on psychological evidence on people's observed behaviour, in the case of foreign exchange, a further check is provided by the actual responses of market participants.

Aside from the assumption of goods-price stickiness, our model of the foreign exchange market relies on two small deviations from full rationality. The first deviation is based on well-documented evidence that "anchoring" is a common and systematic behavioural pattern. In the foreign exchange market, the forward rate fits very closely psychologists' definition of an anchor for exchange rate expectations. As we stress, compared to the range of possible exchange rate outcomes, being anchored to the forward rate induces only a small bias to exchange rate expectations.

The second deviation from full rationality which we invoke is the assumption that investors compare the relative performance of anchored and rational traders over a finite horizon. (We also assume that investors compare realised returns rather than realised utilities, but, as discussed, this distinction is not critical.) In this case as well, we do not require the deviation from full rationality to be a large one. As we have shown, the bias of the forward discount can be substantial even when investors use quite a long horizon (say, ten years) over which to make their investment decisions.

The message from this paper is that, together, these small deviations from full rationality can explain two well-established and enduring foreign exchange market puzzles.

## APPENDIX A: REDUCED FORMS FOR THE STOCHASTIC ENVIRONMENT

We assume a standard domestic LM curve,

$$m_t - p_t = \phi \bar{y} - \lambda i_t, \quad \phi, \lambda > 0,$$

where  $\bar{y}$  is fixed real output and the other variables are defined in the text. As discussed, demand,  $D_t$ , is a simplified version of the Dornbusch (1976) form

$$\ln(D_t) = \xi + \delta \bar{y} - \sigma i_t, \quad \delta, \sigma > 0,$$

and, following Dornbusch, goods prices adjust slowly to excess demand. Thus, the change in the log price of domestic goods,  $\Delta p_{t+1}$ , is

$$\Delta p_{t+1} = \pi \ln(D_t/Y_t) = \pi [\xi + (\delta - 1)\bar{y} - \sigma i_t], \quad \pi > 0. \quad (\text{A1})$$

With no price stickiness,  $p_t = \bar{p}_t$ . In the special case of the model described in the text, the arbitrage condition in the foreign exchange market (equation (10)) implies that  $i = i^*$  in a long-run with the domestic money supply fixed. This condition implies that  $\bar{p}_t = m_t + \lambda i^* - \phi \bar{y}$ , and hence

$$p_t - \bar{p}_t = \lambda (i - i^*)_t. \quad (\text{A2})$$

In this long-run,  $p_t = \bar{p}_t$ ,  $\Delta p = 0$ , and hence from (A1),

$$\xi + (\delta - 1)\bar{y} = \sigma i^*. \quad (\text{A3})$$

Equation (A3) is the constraint imposed between exogenous variables described in the text. It implies that (A1) may be rewritten as

$$\Delta p_{t+1} = -\lambda(i - i^*)_t/\theta \quad \text{where } \theta \equiv \lambda/\pi\sigma > 0,$$

which is equation (5) in the text. Finally, combining (A2) with (4) and (5) gives (6).

## APPENDIX B: THE TRADERS' ASSET - DEMAND FUNCTIONS

Define  $W_t$  as the real wealth a trader manages in period  $t$ ,  $x_t$  as the proportion of  $W_t$  held in the foreign nominal asset,  $r_{t+1}^d$  and  $r_{t+1}^f$  as the real returns on the domestic and foreign assets and  $z_{t+1} \equiv r_{t+1}^f - r_{t+1}^d$  as the real excess return on the foreign asset. Following Frankel and Engle (1984), real wealth in period  $t+1$ ,  $W_{t+1}$  is

$$W_{t+1} = W_t [1 + x_t z_{t+1} + r_{t+1}^d] \quad (\text{B1})$$

By assumption, traders maximise a function of the expected value and variance of end-of-period real wealth,  $F(E_t W_{t+1}, V_t W_{t+1})$ . Differentiating  $F(\bullet, \bullet)$  with respect to  $x_t$  and introducing the identity of traders with the subscript  $k$  gives<sup>28</sup>

$$x_t^k = -(V_t^k z_{t+1})^{-1} \text{cov}_t^k(z_{t+1}, r_{t+1}^d) + (\gamma V_t^k z_{t+1})^{-1} E_t^k z_{t+1}, \quad k = a, r \quad (\text{B2})$$

where  $\gamma \equiv -2W_t F_2 / F_1$  is the coefficient of relative risk aversion evaluated at  $W_t$ , common to both types of traders.

The real domestic return  $r_{t+1}^d$  and the excess foreign return  $z_{t+1}$  are given by

$$r_{t+1}^d = (1 + i_t) P_t^C / P_{t+1}^C - 1 \quad \text{and} \quad (\text{B3})$$

$$z_{t+1} = [(1 + i^*) (S_{t+1}/S_t) - (1 + i_t)] P_t^C / P_{t+1}^C, \quad (\text{B4})$$

where  $P_t^C$  is the price index, measured in domestic currency, for the traders' consumption basket. Each trader spends the fraction  $g$  of her consumption expenditure on foreign goods, so  $P_t^C$  is given by  $P_t^C = (P^* S_t)^g (P_t)^{1-g}$ . Then,<sup>29</sup>

$$P_t^C / P_{t+1}^C \approx 1 - g \Delta s_{t+1} - (1 - g) \Delta p_{t+1} \quad (\text{B5})$$

<sup>28</sup> Since the anchored traders' behaviour is determined by their own subjective expectations rather than by true mathematical expectations,  $V_t^a z_{t+1}$  should be interpreted as  $E_t^a [z_{t+1} - E_t^a(z_{t+1})]^2$ . A similar comment applies to  $\text{cov}_t^a(z_{t+1}, r_{t+1}^d)$ . As we shall see however, this subtlety makes no important difference to the analysis.

<sup>29</sup> The parameter values derived in Section 5 imply  $\Delta p_{t+1} \approx (i - i^*)_t \ll \Delta s_{t+1} \ll 1$ , which justifies the first-order Taylor series approximations used to derive (B5) - (B7).

and hence, from (B3) and (B4),

$$r_{t+1}^d \approx i_t - g \Delta s_{t+1} - (1 - g) \Delta p_{t+1}, \quad \text{and} \quad (\text{B6})$$

$$z_{t+1} \approx i^* - i_t + \Delta s_{t+1}. \quad (\text{B7})$$

Equation (B7) implies that  $V_t^k z_{t+1} \approx V_t^k (\Delta s_{t+1})$ . Since  $\Delta p_{t+1} \ll \Delta s_{t+1}$  it follows that  $\text{cov}_t^k(z_{t+1}, r_{t+1}^d) \approx -g V_t^k (\Delta s_{t+1})$ . Finally, the variance of exchange rate changes is closely approximated by  $\sigma_e^2$ , that is,  $V_t^k (\Delta s_{t+1}) \approx \sigma_e^2$  where  $k = r$  or  $a$ .<sup>30</sup> Substituting these approximations into the asset demand functions (B2) leads directly to equation (9) in the text.<sup>31</sup>

---

<sup>30</sup> The rational traders' estimate of the variance of exchange rate changes is larger than the anchored traders' estimate (because the former understand that nominal money shocks contribute to this variance). But the effect is small. For almost all the results we report  $1 \leq V_t^r (\Delta s_{t+1}) / \sigma_e^2 \leq 1.1$ . Hence, this approximation is a good one.

<sup>31</sup> We note here a refinement which slightly modifies the analysis but makes no important difference to the results. Krugman (1981) points out that (B5) - (B7) are not quite correct. The variance of exchange rate changes is so large that *second-order* Taylor expansions should be used for all the  $S_{t+1}/S_t$  terms. This refinement implies (Frankel (1983)) that the first term in equation (9),  $g$ , should be replaced by  $g - (g - 1/2)/\gamma$ . Implementing this refinement leads to a minimal change to the model. To be precise, the special case of the model introduced in Section 4.4 now applies when the proportion of foreign assets managed by traders is  $g - (g - 1/2)/\gamma$ . Assuming any other proportion of foreign assets leads to the more general form of the model described in Section 4.7.

### APPENDIX C: ASSET SUPPLIES NOT EQUAL TO THE TRADERS' MINIMUM-VARIANCE PORTFOLIO

In this case, the international arbitrage condition, equation (10), becomes

$$g + \kappa = g + [i^* - i_t + \alpha E_t^a(\Delta s_{t+1}) + (1 - \alpha) E_t^r(\Delta s_{t+1})] / \gamma \sigma_e^2. \quad (C1)$$

In the absence of domestic money shocks, the definitions of  $\bar{p}_t$  and  $\bar{s}_t$  imply  $p_t = \bar{p}_t$  and  $s_t = \bar{s}_t$ . Then, with anchored traders' expectations again given by equation (8),  $E_t^a(\Delta s_{t+1}) = i_t - i^*$  while  $E_t^r(\Delta s_{t+1}) = 0$ . To satisfy (C1) requires

$$i^* = i_t + x, \quad (C2)$$

where  $x = \kappa \gamma \sigma_e^2 / (1 - \alpha)$ . The excess return on the foreign asset,  $x$ , is a risk premium required in the aggregate by the traders to increase their holdings of the foreign asset from  $g$  to  $g + \kappa$ . Reintroducing domestic money shocks implies time-evolutions for the interest differential and domestic prices which satisfy (C3) and (C4):

$$(i_{t+1} - i^* + x) = (i_t - i^* + x) [1 - 1/\theta] - v_{t+1}/\lambda, \quad (C3)$$

$$\Delta p_{t+1} = -\lambda(i_t - i^* + x)/\theta. \quad (C4)$$

Equations (C3) and (C4) replace (5) and (6) in the text. It is now straightforward, if tedious, to solve the model and derive equations (13) and (14).

**APPENDIX D: AUGMENTED DICKEY-FULLER TESTS ON 3-MONTH  
NOMINAL INTEREST DIFFERENTIALS**

Country Pair	Number of autoregressive lags	ADF-statistic
UK-US ('79-'91)	20	- 2.28
Ger-US ('79-'91)	20	- 1.99
Jap-UK ('81-'91)	16	- 2.11
Jap-Ger ('81-'91)	8	- 3.02
Jap-US ('81-'91)	19	- 2.62

The ADF tests are conducted on the weekly data, assuming a constant and no time trend. The null hypothesis of a unit root is rejected when the ADF statistic is more negative than the critical value. For a sample size of 500, 1%, 5% and 10% critical values are - 3.44, - 2.87 and - 2.57 respectively. We include 20 autoregressive lags on the differenced interest differential and sequentially reduce the number of lags when the last lag has a t-statistic less than 2 in absolute value.

**Deriving values for  $\mathbf{l}$  from Fair (1987)**

Using quarterly data, Fair estimates a money demand function of the form

$$\begin{aligned} \log(M_t/(POP_t P_t)) &= a_1 + a_2 \log(Y_t/POP_t) + a_3 i_t \\ &+ a_4 \log(M_{t-1}/(POP_{t-1} P_{t-1})) + a_5 \log(M_{t-1}/(POP_{t-1} P_t)) \end{aligned} \quad (D1)$$

where  $POP_t$  is the population in quarter  $t$ . The parameter values we use for (D1) are the average of Fair's estimates for Canada, France, Germany, Japan and Italy (US and UK are excluded because Fair chooses a different specification for the US and finds (D1) mis-specified for the UK). Assuming  $Y_t$  and  $POP_t$  are fixed, we consider a once-off 1% shock to the nominal money supply at the beginning of a quarter. Given  $\theta$ , assuming prices evolve according to our model ( $\Delta p_{\tau+1} = (\bar{p}_\tau - p_\tau)/\theta$ , with  $\tau$  measured in 4-week periods) the chosen value of  $\lambda$  minimises the mean squared difference between the simulated interest rate path in our model and in Fair's estimated model when the two simulations are compared over 3 years.

## APPENDIX E: COMPARING THE TRADERS' PERFORMANCE

Starting with real wealth,  $W_t$ , in period  $t$  and choosing new portfolio allocations each period, a trader's real wealth in period  $t + n$  is, from equation (B1),

$$\begin{aligned} W_{t+n} &= W_t \prod_{j=t}^{t+n-1} \left[ 1 + x_j z_{j+1} + r_{j+1}^d \right] \\ &\approx W_t \left[ 1 + \sum_{j=t}^{t+n-1} \left\{ x_j z_{j+1} + r_{j+1}^d \right\} \right] \end{aligned}$$

since  $x_j z_{j+1} + r_{j+1}^d \ll 1$  for all  $j$ . Using our first-order Taylor expansions for  $x_j$  and  $z_{j+1}$  (equations (9) and (B7)), the difference between the anchored and rational traders' real wealth after  $n$  periods is

$$W_{t+n}^a - W_{t+n}^r \approx \frac{W_t}{\mathbf{g} \mathbf{s}_e^2} \sum_{j=t}^{t+n-1} \left( i_j^* - i_j + \mathbf{D} s_{j+1} \right) \left( E_j^a - E_j^r \right) \mathbf{D} s_{j+1}$$

Evaluating  $\left( E_j^a - E_j^r \right) \mathbf{D} s_{j+1}$  and recognising that  $Z(\mathbf{a}, n) \equiv \left( W_{t+n}^a - W_{t+n}^r \right) / W_t$  leads to equation (15) in the text.<sup>32</sup>

---

<sup>32</sup> The Krugman (1981) refinement discussed in the previous footnote adds extra terms to both  $x_j$  and  $z_{j+1}$ . The extra terms in  $x_j$  cancel when the difference  $(x_j^a - x_j^r)$  is formed. The (small) extra term in  $z_{j+1}$  is  $(1/2 - g)(\Delta s_{j+1})^2$ . Over the range of possible values for  $g$  ( $0 \leq g \leq 1$ ) we have established empirically that this extra term makes negligible difference to the only quantity we care about:  $p(\alpha, n) \equiv \Pr[Z(\alpha, n) > 0]$ . Note finally that all these extra terms are zero when foreign and domestic goods enter equally in the traders' consumption basket, i.e., when  $g = 1/2$ .

## APPENDIX F: REAL EXCHANGE RATE SHOCKS ARE AR(1)

The empirical results reported by (for example) Frankel and Meese (1987) suggest modelling real exchange rate shocks as a stationary AR(1) process rather than a random walk and hence replacing equation (7) by

$$(\bar{s}_{t+1} - \bar{p}_{t+1}) = \mathbf{r}(\bar{s}_t - \bar{p}_t) + e_{t+1}, \quad 0 < \mathbf{r} < 1 \quad (7')$$

For the special case of the model when the supply of domestic and foreign interest-bearing assets managed by the traders matches their minimum-variance portfolio, the "no-bubble" solution for the exchange rate change  $\Delta s_{t+1}$  becomes

$$\mathbf{D}s_{t+1} = \mathbf{h}fd_t + (1 + \mathbf{q}\mathbf{h}/1)v_{t+1} + \mathbf{c}[(\mathbf{r} - 1)(\bar{s}_t - \bar{p}_t) + e_{t+1}] \quad (11')$$

where  $\mathbf{c} = (\mathbf{a}\mathbf{b}) / (\mathbf{a}\mathbf{b} + (1 - \mathbf{a})(1 - \mathbf{r}))$ . There are two things of note about equation (11'). Firstly, real exchange rate shocks are uncorrelated with the forward discount by assumption. Hence, for given  $\alpha$ , since the coefficient on the forward discount is the same in equations (11) and (11'), the bias of the forward discount is also the same. In this respect, modelling real exchange rate shocks as a stationary AR(1) process makes no difference to the results.

Secondly, and by contrast, note that in the absence of anchored traders, the real exchange rate shocks have no impact on the exchange rate ( $\alpha = 0 \Rightarrow \chi = 0$ ). In setting the spot exchange rate, rational agents in the foreign exchange market simply ignore stationary shocks to the long-run real exchange rate. Given our empirical estimates of  $\sigma_m$ , this implies (counterfactually) very little exchange rate volatility when  $\alpha = 0$ . Thus, to explain the observed volatility of exchange rates within the framework of the model assuming (7') rather than (7), it would be necessary to invoke some auxiliary assumption (like, perhaps, imposing some myopia on the rational foreign exchange traders).

**DATA APPENDIX**

To calibrate the model, we use weekly observations of these interest rate series:

USA	3 month Treasury Bill rate, 2 November 1979 - 15 March 1991.
UK	3 month Treasury Bill rate, 2 November 1979 - 15 March 1991.
Germany	3 month Interbank rate, 2 November 1979 - 15 March 1991.
Japan	3 month Gensaki rate, 6 February 1981 - 15 March 1991.

All data are from the International Department of the Reserve Bank of Australia, with the exception of the Gensaki rate from 22 April 1988 to 15 March 1991, which was supplied by the Sydney Office of the Mitsubishi Bank of Australia.<sup>33</sup>

---

<sup>33</sup> The Gensaki rate is chosen as it provided the longest available consistent weekly interest rate series. A Gensaki is a bond transaction with a repurchase agreement. Gensaki transactions accounted for more than half of the Japanese bond market volume in the second half of the 1970s. However, during the 1980s the Gensaki volume has varied between ten and thirty per cent of total bond trading volume (Japanese Securities Research Institute (1990)).

## TABLES AND FIGURES

**Table 1: Interest Differentials and Estimates of  $\mathbf{q}$**

Country Pair	Mean Interest Differential (% p.a)	Standard Deviation of Interest Differ. (% p.a)	$\theta$ (4-week periods)
UK-US ('79-'91)	2.9	2.8	12.3
Ger-US ('79-'91)	-1.9	1.8	6.9
Jap-UK ('81-'91)	-5.6	2.0	12.9
Jap-Ger ('81-'91)	-0.9	1.9	26.3
Jap-US ('81-'91)	-2.7	2.2	17.4

**Table 2: Values of the Parameters in the Model**

$\theta$ (periods)	$\lambda$ (periods)	$\sigma_m$ (per period)	$\sigma_e$ (per period)	$\beta$ (per period)
6	6.4	0.0052	0.031	0.17 (weakly anchored) 1.50 (strongly anchored)
50	18.2	0.0053	0.031	0.02 (weakly anchored) 0.18 (strongly anchored)

**Table 3: Model Results**

**Long-run averages of the proportion of anchored traders,  $\bar{\mathbf{a}}$  and the coefficient on the forward discount,  $\bar{\mathbf{h}}$ , from the regression  $\Delta s_{t+1} = \bar{\eta} \text{fd}_t + \varepsilon_{t+1}$**

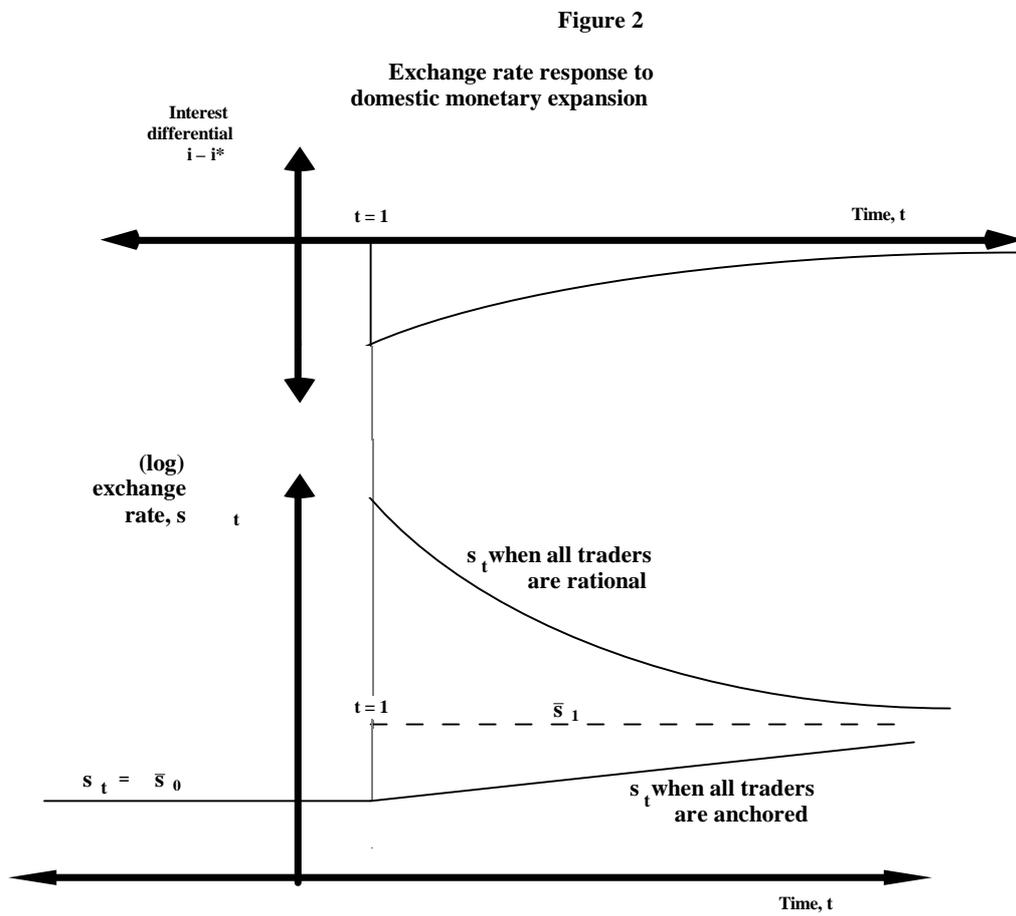
$\theta$	Investors' horizon (Years)	Markov chain parameter $q$	All anchored traders are weakly anchored ( $\beta = 1/\theta$ )		All anchored traders are strongly anchored ( $\beta = 9/\theta$ )	
			$\bar{\alpha}$	$\bar{\eta}$	$\bar{\alpha}$	$\bar{\eta}$
6 periods (24 weeks)	One	0.1	0.37	0.23	0.22	-0.13
		0.3	0.42	0.14	0.33	-0.17
		0.5	0.44	0.09	0.40	-0.13
		1.0	0.45	0.06	0.45	0.06
	Three	0.1	0.27	0.45	0.14	0.11
		0.3	0.35	0.28	0.25	0.02
		0.5	0.39	0.19	0.34	0.00
		1.0	0.42	0.13	0.42	0.13
	Ten	0.1	0.18	0.63	0.08	0.37
		0.3	0.26	0.46	0.17	0.24
		0.5	0.31	0.35	0.25	0.20
		1.0	0.37	0.23	0.37	0.23
50 periods (200 weeks)	One	0.1	0.51	0.31	0.34	0.09
		0.3	0.48	0.34	0.41	0.11
		0.5	0.50	0.31	0.46	0.18
		1.0	0.50	0.32	0.50	0.32
	Three	0.1	0.43	0.41	0.24	0.21
		0.3	0.45	0.39	0.35	0.19
		0.5	0.48	0.35	0.41	0.24
		1.0	0.47	0.36	0.47	0.36
	Ten	0.1	0.31	0.58	0.13	0.41
		0.3	0.37	0.50	0.25	0.34
		0.5	0.41	0.44	0.33	0.34
		1.0	0.43	0.41	0.43	0.41

**Table 4: Comparison with McCallum's (1992) Results**

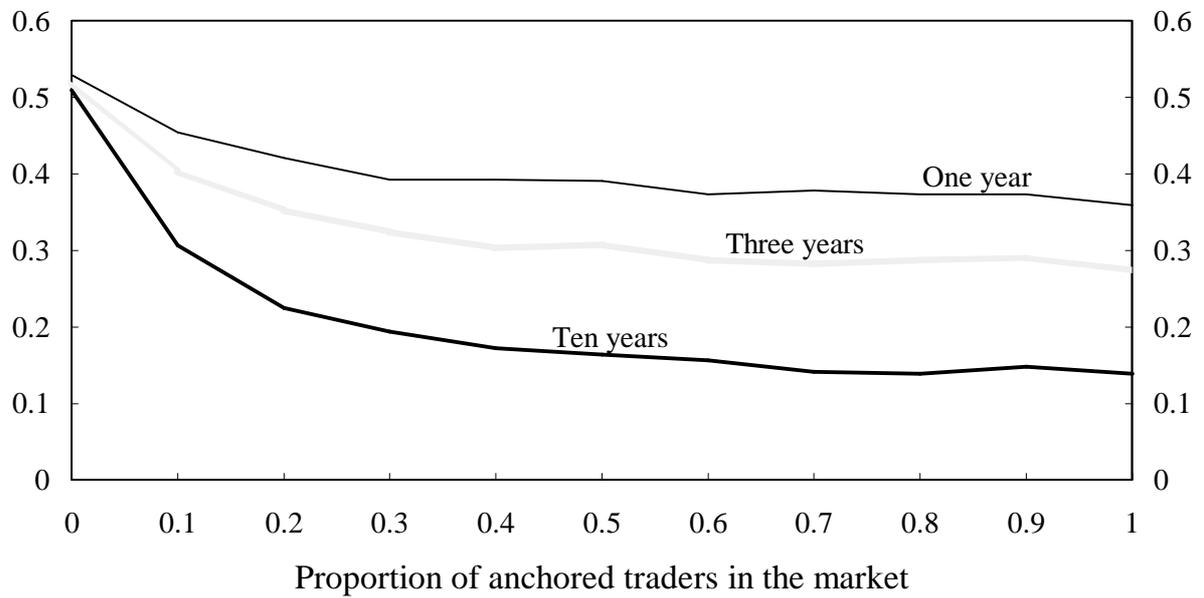
Variables (Regression No.)		OLS estimates (OLS std.errors)		R <sup>2</sup>	S E	D-W
		Constant	Slope			
$s_t - s_{t-1}$ on $f_{t-1} - s_{t-1}$ (1)	Data	-0.016 (0.006)	-4.30 (1.70)	0.041	0.035	2.19
	Model	0.001 (0.001)	-0.11 (0.46)	0.000	0.032	2.05
$s_t$ on $f_t$ (2)	Data	-0.003 (0.001)	1.001 (0.001)	1.000	0.002	0.37
	Model	0.000 (0.000)	1.000 (0.000)	1.000	0.002	0.31
$s_t$ on $f_{t-1}$ (3)	Data	-0.009 (0.012)	0.990 (0.016)	0.963	0.036	2.05
	Model	0.002 (0.001)	0.998 (0.002)	0.995	0.032	2.03
$\Delta s_t$ on $\Delta f_t$ (4)	Data	0.000 (0.000)	1.002 (0.002)	0.999	0.001	1.60
	Model	0.000 (0.000)	1.002 (0.001)	0.999	0.001	2.23
$\Delta s_t$ on $\Delta f_{t-1}$ (5)	Data	0.002 (0.003)	-0.063 (0.082)	0.004	0.036	1.96
	Model	0.001 (0.001)	-0.023 (0.022)	0.001	0.032	2.00
$s_t - s_{t-2}$ on $f_{t-1} - s_{t-2}$ (6)	Data	-0.001 (0.003)	0.940 (0.083)	0.461	0.036	1.93
	Model	0.001 (0.001)	0.977 (0.022)	0.487	0.032	1.99
$s_t - s_{t-3}$ on $f_{t-1} - s_{t-3}$ (7)	Data	-0.002 (0.003)	1.054 (0.061)	0.670	0.036	2.18
	Model	0.001 (0.001)	1.003 (0.016)	0.661	0.032	2.04

Data are McCallum's results for \$/DM, monthly 1978:1 to 1990:7. Model results are based on a 2000 period simulation assuming investors' horizon is one year,  $q = 0.1$ ,  $\theta = 6$  and strongly anchored traders ( $\beta = 9/\theta$ ).

**Figure 2: Exchange Rate Response to Domestic Monetary Expansion**



**Figure 3: Probability that Anchored Portfolio Outperforms Rational Portfolio over Investors' Horizon (Strongly Anchored Traders:  $\mathbf{q} = 6$ ).**



**REFERENCES**

**Akerlof, G. A. and J.L. Yellen (1985)**, "Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria?", *American Economic Review*, 75, 708-20.

**Baillie, R. T. and T. Bollerslev (1989)**, "The Message in Daily Exchange Rates: A Conditional-Variance Tale", *Journal of Business and Economic Statistics*, 7, 297 - 305.

**Baillie, R.T. and T. Bollerslev (1990)**, "A Multivariate Generalised Arch Approach to Modeling Risk Premia in Forward Foreign Exchange Markets", *Journal of International Money and Finance*, 9, 309 - 324.

**Baldwin, R. E. (1990)**, "Re-Interpreting the Failure of Foreign Exchange Market Efficiency Tests: Small Transaction Costs, Big Hysteresis Bands", NBER Working Paper No. 3319.

**Black, F. (1986)**, "Noise", *Journal of Finance*, XLI, 703-738.

**Campbell, J. Y. and R. H. Clarida (1987)**, "The Dollar and Real Interest Rates", *Carnegie-Rochester Conference Series on Public Policy*, 27, 103-140.

**Campbell, J.Y. and A.S. Kyle (1993)**, "Smart Money, Noise Trading and Stock Price Behaviour", *Review of Economics and Statistics*, 60, 1-34.

**Clarida, R. H. and M. P. Taylor (1993)**, "The Term Structure of Forward Exchange Premia and the Forecastability of Spot Exchange Rates: Correcting the Errors", Manuscript, January.

**Cochrane, J. H. (1989)**, "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives", *American Economic Review*, 79, 319-37.

**Cumby, R. E. (1988)**, "Is it Risk? Explaining Deviations from Uncovered Interest Parity." *Journal of Monetary Economics*, 22, 279-299.

**Cumby, R. E. and M. Obstfeld (1984)**, "International Interest Rate and Price Level Linkages under Flexible Exchange Rates", in J. Bilson and R. Marston (eds.), *Exchange Rate Theory and Practice*, University of Chicago Press, Chicago.

**De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990a)**, "Noise Trader Risk in Financial Markets", *Journal of Political Economy*, 98, 703-738.

**De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990b)**, "Positive Feedback Investment Strategies and Destabilising Rational Speculation", *Journal of Finance*, XLV, 379-95.

**De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1991)**, "The Survival of Noise Traders in Financial Markets", *Journal of Business*, 64, 1-19.

**Dornbusch, R. (1976)**, "Expectations and Exchange Rate Dynamics", *Journal of Political Economy*, 84, 1161-1176.

**Dornbusch, R. (1980)**, "Exchange Rate Economics: Where Do We Stand?", *Brookings Papers on Economic Activity*, 2, 143-85.

**Engel, C. (1992)**, "On the Foreign Exchange Risk Premium in a General Equilibrium Model", *Journal of International Economics*, 32, 305-319.

**Fair, R. C. (1987)**, "International Evidence on the Demand for Money", *Review of Economics and Statistics*, 69, 473-480.

**Frankel, J. A. (1983)**, "Estimation of Portfolio-Balance Functions that are Mean-Variance Optimising", *European Economic Review*, 23, 315-327.

**Frankel, J. A. (1985)**, "The Dazzling Dollar", *Brookings Papers on Economic Activity*, 1, 199 - 217.

**Frankel, J.A. (1988)**, "Recent Estimates of Time-Variation in the Conditional Variance and in the Exchange Risk Premium", *Journal of International Money and Finance*, 7, 115-125.

**Frankel, J. A. and M. Chinn (1991)**, "Exchange Rate Expectations and the Risk Premium: Tests for a Cross-Section Of 17 Currencies", NBER Working Paper No. 3806.

**Frankel, J. A. and C. M. Engel (1984)**, "Do Asset-Demand Functions Optimise Over the Mean and Variance of Real Returns? A Six-Currency Test", *Journal of International Economics*, 17, 309 - 323.

**Frankel, J. A. and K. A. Froot (1990a)**, "Chartists, Fundamentalists, and the Demand for Dollars", in A.S. Courakis and M.P. Taylor (eds), *Private Behaviour and Government Policy in Interdependent Economies*, Clarendon Press, Oxford.

**Frankel, J. A. and K. A. Froot (1990b)**, "Chartists, Fundamentalists, and Trading in the Foreign Exchange Market", *American Economic Review*, 80, 181-85.

**Frankel, J. A. and R. Meese (1987)**, "Are Exchange Rates Excessively Variable?", NBER Macroeconomics Annual, 117-162.

**Friedman, M. (1953)**, "The Case for Flexible Exchange Rates", in his *Essays In Positive Economics*, University Of Chicago Press, Chicago.

**Froot, K. A. (1990)**, "On the Efficiency of Foreign Exchange Markets", manuscript, MIT.

**Froot, K. A. and J. A. Frankel (1989)**, "Forward Discount Bias: Is it an Exchange Risk Premium?", *Quarterly Journal of Economics*, 53, 139 - 161.

**Froot, K. A. and R. H. Thaler (1990)**, "Anomalies: Foreign Exchange", *Journal of Economic Perspectives*, 4, 179-92.

**Goodhart, C. (1988)**, "The Foreign Exchange Market: a Random Walk with a Dragging Anchor", *Economica*, 55, 437 - 460.

**Hodrick, R. J. (1987)**, *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets*, Fundamentals of Pure and Applied Economics, 24, Harwood Academic Publishers.

**Ito, T. (1990)**, "Foreign Exchange Rate Expectations: Micro Survey Data", *American Economic Review*, 80, 434-449.

**Japan Securities Research Institute (1990)**, *Securities Market in Japan 1990*, Tokyo, Chapter 4.

**Krasker, W. S. (1980)**, "The Peso Problem in Testing the Efficiency of Forward Exchange Markets", *Journal of Monetary Economics*, 6, 269-76.

**Krugman, P. (1981)**, "Consumption Preferences, Asset Demands and Distribution Effects in International Financial Markets", NBER Working Paper 651.

**Kyle, A.S. (1985)**, "Continuous Auctions and Insider Trading", *Econometrica*, 53, 1315-34.

**LeRoy, S.F. and R. D. Porter (1981)**, "The Present Value Relation: Tests Based on Variance Bounds", *Econometrica*, 49, 555-74.

**Lewis, K. K. (1989)**, "Changing Beliefs and Systematic Rational Forecast Errors with Evidence from Foreign Exchange", *American Economic Review*, 79, 621-636.

**Lyons, R. K. (1990)**, "Whence Exchange Rate Overshooting: Money Stock or Flow?", *Journal of International Economics*, 29, 369-84.

**Mackay, C. (1841)**, *Memoirs of Extraordinary Popular Delusions*, Bentley, London.

**Mankiw, N. G. (1985)**, "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly", *Quarterly Journal of Economics*, C, 529-37.

**McCallum, B. T. (1992)**, "A Reconsideration of the Uncovered Interest Parity Relationship", NBER Working Paper 4113.

**Meese, R. A., and K. Rogoff (1983)**, "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?", *Journal of International Economics*, 14, 3-24.

**Northcraft, G.B. and M.A. Neale (1987)**, "Experts, Amateurs, and Real Estate: An Anchoring-and-Adjustment Perspective on Property Pricing Decisions", *Organisational Behaviour and Human Processes*, 39, 84-97.

**Nurkse, R. (1944)**, *International Currency Experience*, League of Nations, Geneva.

**Obstfeld, M. (1985)**, "Floating Exchange Rates: Experience and Prospects", *Brookings Papers on Economic Activity*, 2, 369-450.

**Rogoff, K. (1979)**, "Essays on Expectations and Exchange-Rate Volatility", Phd Dissertation, MIT.

**Shiller, R. J. (1989)**, *Market Volatility*, The MIT Press, Cambridge, Mass.

**Shiller, R. J. (1992)**, "Who's Minding the Store?", in *The Report of the Twentieth Century Fund Task Force on Market Speculation and Corporate Governance*, Twentieth Century Fund Press.

**Smith, J. and D. W. R. Gruen (1989)**, "A Random Walk Around the \$A: Expectations, Risk, Interest Rates and Consequences for External Imbalance", Reserve Bank of Australia Research Discussion Paper 8906.

**Summers, L.H. (1986)**, "Does the Stock Market Rationally Reflect Fundamental Value?", *Journal of Finance*, XLI, 591-601.

**Takagi, S. (1991)**, "Exchange Rate Expectations: A Survey of Survey Studies", *IMF Staff Papers*, 38, 156-183.

**Tversky, A. and D. Kahneman (1974)**, "Judgement Under Uncertainty: Heuristics and Biases", *Science*, 185, 1124-1131.

**Wright, W. F. and U. Anderson (1989)**, "Effects of Situation Familiarity and Financial Incentives on Use of the Anchoring and Adjustment Heuristic for Probability Assessment", *Organisational Behaviour and Human Processes*, 44, 68-82.