THE INTERTEMPORAL GOVERNMENT BUDGET CONSTRAINT AND TESTS FOR BUBBLES

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Research Discussion Paper
8809
November 1988

Research Department
Reserve Bank of Australia

We would like to thank Warwick McKibbin, Dirk Morris, Malcolm Edey, Rob Trevor and participants at the 16th Australian Conference of Economists for helpful discussions. The views expressed are those of the authors and should not be interpreted as views of the Reserve Bank of Australia.
ABSTRACT

The methodology for the testing of bubbles in asset prices has recently been applied to testing the sustainability of government debt accumulation. In particular, Hamilton and Flavin (1986) and MacDonald and Speight (1987) use the methodology developed by Flood and Garber (1980) in an attempt to identify a period of bubble financing of the budget deficit for the US and UK respectively. MacDonald and Speight and Trehan and Walsh (1988) also use recently developed cointegration methods in an alternative test of the hypothesis of unsustainable financing.

This paper uses the above methods to test for bubble financing of the fiscal deficit for Australia. We develop the method to allow for the effects of income growth on the sustainability of deficits and critically appraise some of the methods used in previous studies and suggest improvements. Our finding is that over the period 1953/54 to 1986/87 there is no evidence of unsustainability of government debt. The analysis suggests that instead, seignorage was used to pay for sustained fiscal deficits, and that the overall level of debt as a ratio of GDP fell over the period as a result of strong GDP growth and inflation.
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1. Introduction

Much attention has recently been focussed upon the appropriate stance of fiscal policy. Many macroeconomic analysts and policy makers, both in Australia and overseas, have argued that protracted fiscal deficits do not aid economic performance and might adversely affect future real economic activity by crowding out private investment expenditures and generating inflation. In addition, concern has recently been expressed about the sustainability of trends in the growth of public debt and this has led some governments to embark upon programmes of fiscal restraint in order to reduce their levels of outstanding debt. As Blanchard, Dornbusch and Buiter (1986) point out, an interesting question in this regard concerns the optimal path of debt stabilisation because it is far from clear that the optimal policy involves the fastest possible adjustment.

The purpose of this paper is to examine the appropriateness of fiscal stance in relation to the government’s intertemporal budget constraint. The latter relates the sustainable growth of government debt to the non-interest fiscal deficit, the growth of nominal output and the service costs of outstanding debt. Specifically, the government’s intertemporal budget constraint provides the sustainable
limits to the growth of government debt given the performance of the macroeconomy. Although this constraint must hold in the long run, there is ample scope for governments to exceed it over short periods of time by engaging in so-called bubble financing. Tests for the existence of bubble financing have been reported by Hamilton and Flavin (1986), Hakkio and Rush (1987) and Trehan and Walsh (1988) for the US and by MacDonald and Speight (1987) for the UK. On close examination, however, a number of problems exist with the method in some of these studies and this paper presents the corrected tests. We have developed the framework to allow for the effects of income growth on debt sustainability and applied the tests to Australian data over the period 1953/54 to 1986/87. Amongst the main findings are the lack of evidence of bubble financing in Australia together with an indication of the historical importance of monetisation for the financing of non-interest fiscal deficits.

Section 2 presents the framework of the government’s intertemporal budget constraint and explains how it constrains the sustainable growth of debt. Section 3 generalises the tests of bubble financing utilising the cointegration methodology of Engle and Granger (1987) and the approach of Hamilton and Flavin (1986) to allow for income growth. The tests are applied to the data in Section 4. The final section summarises the paper and draws conclusions.
2. The Intertemporal Government Budget Constraint

In order to examine the relationship which exists between the government's fiscal stance and the performance of the macroeconomy, the appropriate framework is the government's intertemporal budget constraint. This can be written either in nominal terms and/or as a proportion of GDP and it establishes the link which exists between the prevailing level of government indebtedness and the future debt servicing requirements.

Table 1 shows the nominal and deflated budget deficits together with the level of outstanding debt in Australia over the period 1953/54 to 1986/87. Figure 1 shows that although nominal debt outstanding has increased dramatically, the deflated series has tended towards historically low levels in recent years.

Consider first the nominal version of the government's intertemporal budget constraint which is written as equation (1).

\[
B_{t+1} = (1 + i_t)(B_t - S_t)
\]

where \( B \) denotes the level of debt prevailing at the start of the period (denoted in subscripts), \( S \) denotes the 'primary' surplus (i.e. the deficit net of interest payments on outstanding debt) plus seignorage and \( i_t \) denotes the nominal rate of interest which prevails between periods \( t \) and \( t+1 \). It is assumed that all debt issued matures at the end of the year. Equation (1) can be rewritten as
If we start at time \( t=0 \) and solve equation (1)' recursively forward in time, we obtain equation (2)

\[
B_0 = S_0 + \frac{1}{(1+i_0)} + \frac{1}{[(1+i_0)(1+i_1)]} S_1 + \frac{1}{[(1+i_0)...(1+i_{t-1})]} S_t + \frac{1}{[(1+i_0)...(1+i_{t-1})] B_{t+1}} \tag{2}
\]

which yields the intertemporal budget constraint by utilising the transversality condition that the level of outstanding nominal debt does not permanently grow at a faster rate than the nominal interest rate, i.e.,

\[
\lim_{t \to \infty} \frac{1}{[(1+i_0)...(1+i_{t-1})]} B_{t+1} = 0 \tag{3}
\]

By imposing this transversality condition (3) in equation (2) and denoting the nominal discount factor by \( R_t = 1/(1+i_0)...(1+i_{t-1}) \), we can simplify the nominal version of the government’s intertemporal budget constraint to equation (2)’

\[
B_0 = \sum R_t S_t \tag{2}’
\]

which states simply that the present value of debt servicing obligations must be equal to the prevailing level of outstanding debt.
It is pertinent to note that the public sector borrowing requirement constitutes an inappropriate indicator of the government's fiscal stance. The inclusion of public trading enterprises in the measure of the deficit is misleading as these corporations resemble and act as private enterprises. From the perspective of the budget constraint, only the subsidies paid to and receipts received from these enterprises are relevant. Accordingly, the Australian government's fiscal deficit measure which is employed in this paper is obtained by aggregating the Commonwealth and State and Local authorities' revenues and outlays net of public trading enterprises.

As Blanchard, Dornbusch and Buiter (1986) point out it is illuminating to express the government's intertemporal budget constraint as a proportion of GDP. By noting that the growth rate of nominal output \((Y_{t+1} - Y_t)/Y_t\) is equal to the sum of real output growth \((y_t)\) plus inflation \((p_t)\),

\[
Y_{t+1} = (1+y_t)(1+p_t)Y_t
\]

and we use this to deflate the nominal budget constraint of equation (1)' to get

\[
b_t = s_t + [(1+y_t)(1+p_t)/(1+i_t)] b_{t+1}
\]

where \(b_t = B_t/Y_t\) and \(s_t = S_t/Y_t\).
Proceeding as before, the real version of equation (2)' is equal to

$$b_0 = \sum \delta_t s_t$$

(6)

where:

$$\delta_t = \left(\frac{(1+y_0)(1+p_0)/(1+i_0) \ldots (1+y_{t-1})(1+p_{t-1})/(1+i_{t-1})}{l+y} \right)$$

This expression can be simplified to (6)' by writing

$$\delta = (1+y)(1+p)/(1+i)$$

where \(y, p\) and \(i\) denote respectively the long term average growth of real output, the long term average inflation rate and the long term interest rate (the analysis henceforth uses these long term averages with the bar dropped from equations);

$$b_0 = \sum \delta_t s_t$$

(6)'

The interesting aspect of this version of the government's intertemporal budget constraint is that it demonstrates how higher growth of nominal output (i.e. either higher real output growth and/or higher inflation) reduces the burden of debt repayments. As long as the long term average growth in nominal output exceeds the long term average nominal interest rate, the government is not constrained by the amount of debt it can service. Because this is not generally the case, however, governments are faced with a constraint which will be binding if the growth of debt equals \((1+i)/(1+y)(1+p)\) which implies that on average new issues of bonds are restricted in their amount to cover interest payments on the current level of outstanding debt.
It is generally the case in real economic circumstances that the rate of growth of government bonds diverges from the upper limit implied by the intertemporal budget constraint. There is not necessarily a mechanism to prevent governments from issuing debt in such quantities that its future growth exceeds the sustainable rate which is implied by nominal output growth and nominal interest rates, at least over a short period of time. If this occurs, the government is said to be engaging in bubble financing of its fiscal deficit. There is an adjustment mechanism, however, which causes the constraint to apply over time. When government debt is imperfectly substitutable for other private sector financial assets, the price of bonds must fall in order to induce portfolio balancers to take up the debt. The resulting higher debt servicing costs are also reflective of the increased incentive which is required to offset investors’ uncertainty about whether the government will be able to honour its future debt obligations.
3. Testing For Bubble Financing

In this section, we examine two tests for the existence of bubble financing in Australia over the period 1953/54 to 1986/87. The first test employs the cointegration method pioneered by Granger (1981) and Engle and Granger (1987) which has been utilised by MacDonald and Speight (1987) to examine the British case and Hakkio and Rush (1987) and Trehan and Walsh (1988) for the US. The second test investigates the price level bubble approach pioneered by Flood and Garber (1980) and adapted to examine the intertemporal government budget constraint for the US by Hamilton and Flavin (1986).

Cointegration can be defined as follows. If the components of a vector $x_t$ have a stationary invertible ARMA representation after differencing $d$ times, then $x_t$ is said to be integrated of order $d$, i.e. $x_t \sim I(d)$. Variables integrated of the same order are said to be cointegrated of order $d,b$, if there exists a linear combination of these variables such that $z_t = a'x_{nt}$ is integrated of order $d-b$ where $d,b>0$. In the special case where $d=b$, $z_t$ is stationary. This implies that the general trend in each series is the same; there is a common path followed by the two series which, although it may not hold at all points, is returned to time and time again.

The first of these tests is derived from equation (1). This can be rearranged to give the following equation:

$$g_t - (t_t + \Delta m_t) = b_t - \frac{(1+i)/(1+y)(1+p)}{b_{t-1}}$$

(7)
where the lower case letters \( g, t, m, \) and \( b \) refer respectively to the
government spending, taxation, the money base and debt outstanding
deflated by GDP. Seignorage (financing the deficit by expansion of
the money base)\(^1\) is included as the government can simply issue money
to pay its bills.

When the constraint is binding, the change in the level of bonds must
be stationary. This is proved formally for the nominal case in
Trehan and Walsh (1988)\(^2\). The point is seen by rearranging equation
(7) in the form

\[
\Delta b_t = g_t - (t_t + \Delta m_t) + \frac{(1+i)/(1+y)(1+p) - 1}{b_{t-1}}
\] (7')

which is approximately (and exactly in continuous
time) equal to

\[
\Delta b_t = g_t - (t_t + \Delta m_t) + (i - y - n)b_{t-1}
\] (7'')

The last term here is simply the interest payments on debt deflated
by GDP growth and inflation. For debt not to be explosive the right
hand side of the above equation must be stationary (it may have a
negative deterministic component over a sample if debt is being
repaid). This differs from the related equation in Trehan and Walsh
(1988) as it allows for the possibility that inflation and/or

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\(^1\) In this paper seignorage refers to financing the deficit by
issuing money (inflation tax is treated separately). Even though the
government may issue enough debt to finance a deficit, if this debt
is taken up by the central bank then the effect is as if the central
bank printed the money.

\(^2\) The proof is lengthy and will not be reproduced here.
economic growth may finance the deficit. Trehan and Walsh (1988) point out that the requirement that the deficit excluding interest payments be stationary for sustainability (as tested in Hakkio and Rush (1987) and MacDonald and Speight (1987)) is not sufficient; the deficit including interest rates must be stationary.

There is a special case when these two hypotheses will be identical. If the deficit excluding interest rates is stationary (as in the null hypothesis of Hakkio and Rush (1987)) equation (7)' reduces to

\[ \Delta b_t = (i-y+p)t_{-1} + \text{a stationary term} \] (8)

The change in the bond series will be stationary if \( y+p \) is greater than \( i \). This corresponds to a root of the bond series of \( 1+i-y-p \) which is less than one. The bond series and thus the deficit including as well as excluding the deficit is stationary. This shows that the tests for a deficit including and excluding interest payments are equivalent in this case.

In general, however, the right hand side of equation (7)'' will be stationary if the sum of the trends in each of the variables cancel. This allows for the possibility that the deficit excluding interest payments to be non stationary and still satisfy the constraint if it is cointegrated with interest payments.

One test of the above hypothesis is to test the right hand side of equation (7)' for stationarity. This is equivalent to imposing a
cointegrating relationship (with a cointegrating vector of \((1 -1 -1)\)) between the government spending (inclusive of interest), taxation and seignorage variables.

Previous researchers have variously tested for cointegration between the variables including and excluding interest payments. Trehan and Walsh (1988) employ U.S. data to test both the restricted model and the unrestricted model when interest payments are included in the deficit. In each case they accept the hypothesis of a stationary deficit although in the unconstrained estimation the coefficient for their seignorage variable is not close to unity\(^3\). Hakkio and Rush (1987) and MacDonald and Speight (1987) both test the unrestricted model employing the deficit excluding interest payments for U.S. and U.K. data respectively. Both accept the hypothesis of cointegration with coefficients close to unity. From the argument from Trehan and Walsh (1988) regarding inclusion of interest rates, and our argument relating the two hypotheses under the special condition of income growth and inflation outpacing interest rates, the two sets of results are invalid (as they do not allow for income growth the special condition cannot hold).

It is important to notice that if the primary deficit is financed by money creation, it is likely that the hypothesis of cointegration will be accepted. This is not an unlikely finding for Australia as it is only recently that the government has been able to determine

\(^3\) The likely cause of this is that their seignorage variable is stationary. Their test for non stationarity accepts at the 10% level of significance. However, further tests using this variable show that its addition to a stationary variable results in a stationary variable. This is a contradiction to the original finding.
the amount of debt that it issues. Prior to this, the institutional arrangements for issuing debt in Australia have involved the government setting the price while allowing the private sector to take up as much or as little as desired. That which was not purchased by the public resulted in increases in the holdings of debt by the Reserve Bank. Seignorage therefore became the residual financing instrument of the primary deficit in Australia over the sample period analysed in this paper. Figure 2 illustrates this point. Both the $g_t$ (excluding interest payments) and $(t_t + \Delta m_t)$ series follow very similar paths.

The second test is based upon Hamilton and Flavin's (1986) adaptation of Flood and Garber's (1980) test for hyper-inflationary bubbles to examine whether governments have resorted to bubble financing of their fiscal deficits. The test is derived from equation (2) above and reproduced below as equation (9).

\[
b_t = \sum_{i=t+1}^{N} \delta^{i-t} s_i + B_N (1+y)^N (1+p)^N (1+i)^N (1+i)^t/(1+y)^t (1+p)^t \tag{9}
\]

This specification shows us that the level of debt outstanding at time $t$ is equal to the sum of all future surpluses plus the present value of debt outstanding at time $t=N$. Taking expectations of the right hand side of this equation; the current level of debt is equal to agents expectations of future surpluses and their expectation of the amount of debt that will not be repatriated. In particular if

\[
\lim_{N \to \infty} B_N (1+y)^N (1+p)^N/(1+i)^N = 0 \tag{10}
\]
the present value of bonds held at time $N$ is zero and the constraint holds. This does not preclude a positive holding of bonds in the limit (i.e. $B_N$ need not tend to zero as long as its present value does) which means that the government can continue to roll over debt so long as the debt is not increasing at a rate greater than $(1+i)/(1+y)(1+p)$. The test is based on the existence of this term.

If the limit of this is zero the term drops out of the equation (9) while if it is not, the bubble term will remain. Hamilton and Flavin (1986) show that if the limit approaches a constant then the alternative hypothesis is that $N = s$. With a significant value for $A_0$ indicating that investors do not expect that the government will be able in the limit to repatriate all its outstanding debt.

It can be seen that the first term in equation (9)' cannot be tested because we cannot measure the expected future path of the government surplus, both Hamilton and Flavin (1986) and MacDonald and Speight (1987) assume that current trends in the surplus will continue into the future. Whilst this does not constitute the full information set which investors consider in making expectations of future surpluses, this extrapolation is consistent with our test insofar as we are testing the hypothesis that current trends in government spending and financing decisions imply that the level of debt is too high. The expected surplus term can therefore appropriately be proxied in the

\[
D_t = \sum_{i=t+1}^{N} \delta^{i-t} S_i + A_0 \delta^{-t} \tag{9}'
\]
present context by utilising an autoregressive representation of past surpluses. The expectations term was substituted out in the method of Hansen and Sargent (1981). For a lag length of one

\[ b_t = k_1 + a_0 \delta^{-t} + \frac{a_1 b}{1-a_1} s_t \]  

This equation is estimated jointly with the autoregression of \( s_t \)

\[ s_t = k_2 + a_1 s_{t-1} \]  

The bubble term in equation (11) is:

\[ A_0 \cdot 1/\delta^t \]

where \( A_0 \) is the coefficient to be estimated and \( t \) is a time trend. Considering the variable \( (1/\delta^t) \) where \( \delta \) is constant, the variable is a deterministic term integrated of order greater than one. The intuition of this result is that for \( (1+i)>(1+y)(1+p) \), (which must be the case if there is a constraint as in Hamilton and Flavin (1986)), then this term trends upwards over time. This term will pick up any deterministic trend in the bond series which is not accounted for by the surplus (including seignorage). The bond series in Hamilton and Flavin (1986) is stationary, which precludes the finding of a significant coefficient on the bubble term.

\[ ^4 \text{Initially three lags were specified. Use of likelihood ratio tests for the full system resulted in acceptance of a lag length of one.} \]
If \( i < y + p \), the bubble term asymptotes to zero (the specification no longer tests for a bubble). The limit term can approach any value as \( 1/\delta^t \) goes to zero as \( t \) increases. Thus a finding of a significant \( A_0 \) here is compatible with the hypothesis that the constraint is holding.

Once non-stationarity of the bond series is established, we can no longer make use of the usually applied \( t \)-statistic on the bubble term as it is no longer \( t \) distributed (Durlauf and Phillips (1986)), a point which is overlooked by Macdonald and Speight (1986). An alternative to testing by the above method is necessary to determine whether or not the bubble term is correctly included in the model. Hall (1986) examines the inclusion of a variable in a cointegrating equation on the strength that its inclusion is necessary for white noise residuals to be obtained. Thus the method of testing is to test the restricted equation's residuals for stationarity (i.e. no bubble term). If the residuals remain stationary when the term is excluded then it is not significant. This can be applied by noting that when the series for bonds is integrated at a higher order than the fundamentals, then the residuals of the estimated equation (excluding the bubble term) will be non-stationary. If the inclusion of the bubble term results in an equation with stationary residuals then the bubble term is found to be correctly included and significant.

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5 These authors did not report any diagnostic checking of their final system of equations. What is likely is that the price level effects contained in the bond series is appearing in the equation through a significant deterministic trend (i.e. the bubble term is significant). This may explain why they obtain results in conflict with their (possibly incorrect) cointegration results.
4. Empirical Evidence

A description of the data including sources is included in Appendix 1. For our sample, the average nominal interest rate is exceeded by the rate of growth of real GDP plus average inflation. This means that in the first test use of the deficit including and excluding interest payments are equivalent. Also, in the second test a significant "bubble term" is not a finding of debt unsustainability but instead is compatible with the hypothesis that debt is being repatriated over the period.

a. The Cointegration Test

We apply the cointegration test described in the previous section to test the hypothesis that government spending (inclusive and exclusive of interest payments), taxation and monetary policy have been conducted in such a way that bonds are growing on an unstable path. It is first necessary to determine that both series are integrated of an equal order greater than one. An examination of the graphs for the series under consideration shows that each of the series has trended upwards with a large jump in the early seventies. This trend in the data indicates that it is likely that the series are non-stationary.

The results of Dickey-Fuller tests (Dickey and Fuller (1979) and (1981)), Perron and Phillips (1987) and Stock and Watson (1986) are given in Table 2. The government spending series (including and excluding interest payments) and the constructed taxation plus the

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6 All results refer to data deflated by GDP.
seignorage series appear to be non-stationary around a linear time trend, i.e., there appears to be both a deterministic component and a stochastic unit root. The deterministic components of the series are not removed as it is the time path of the series we are interested in, and it is the equivalence of the non-stationary components between the series (both deterministic and stochastic) which is to be tested. The first differences of all series were tested for non-stationarity in the same manner as above. All tests reject non-stationarity indicating that all series under consideration are integrated of order one. We can now proceed with the cointegration tests.

The results for the cointegration test for the equivalence of the unit roots in the two series are shown on Table 3. The tests for cointegration are those proposed by Engle and Granger (1987), namely, their cointegrating regression Durbin Watson test statistic (CRDW) and the Dickey-Fuller statistic (DF). Both methods amount to the testing of the residuals of the cointegrating equation (equation (9)) for non-stationarity, the first uses the method of Bhargarva (1986) and the second uses the method of Dickey and Fuller. The null hypothesis of non cointegration of the series is accepted if non-stationarity of the residuals is accepted. The critical values for these tests are given in Engle and Yoo (1987) for the case of fifty observations. It has been shown that tests for non-stationarity such as these have low power when the alternative hypothesis is that rho is close to (but less than) one. The test of Stock and Watson (1986) is also applied. Phillips (1987) and Phillips and Perron (1986) have proposed non-parametric tests for non-stationarity which have
greater power than those considered by Engle and Granger (1987) and these tests have been applied here.

Both the DF test and the CRDW tests indicate that the hypothesis of no cointegration can be rejected at the 95 per cent significance level for both constructions of government spending. These findings are backed up by the PP test. The cointegrating coefficient ($\alpha$) is, at 0.934, close to the hypothesised value of unity whilst the constant is insignificantly different from zero. Dropping the insignificant constant also yields coefficients close to one. We know that our estimate for $\alpha$ is biased, although Banerjee, Dolado, Hendry and Smith (1986) have shown that this bias is proportional to the inverse of the $R^2$ for the cointegrating regression. With a high $R^2$ of 0.89 this bias is unlikely to be a problem here. The Stock and Watson (1986) tests both accept the alternate hypothesis of only one unit root at the 95% significance level.

Unfortunately, the standard errors calculated along with the regression results cannot be used to test the statistical significance of the departure of our estimate of the cointegrating coefficient from unity as the usually constructed $t$ statistics are not $t$ distributed. This problem is not insurmountable. One method, applied in Bewley and Elliott (1987), is to estimate an error correction model for a range of values of $\alpha$ and choose the alpha which maximises the log of the likelihood of the system. The plot of

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7 The constant here has a significant interpretation. This is the mean deficit over the period (a small error comes from $\alpha$ not being exactly one). A significantly positive constant with stationarity of the residuals and an $\alpha$ of one implies a sustainable deficit over the sample period.
the log of likelihoods can be used to calculate critical bounds for the estimate of $\alpha$. An alternative method which was foreshadowed in section III and applied here is to impose the hypothesised value for $\alpha$ and test the constructed variable for stationarity. If this variable (i.e. the primary deficit plus seignorage) is stationary, then we can accept the hypothesised value as a true cointegrating coefficient.

Tests for the stationarity of the deficit are given in Table 4. It can be seen that the null hypothesis of non-stationarity is rejected in both the Dickey Fuller tests and the more powerful Phillips test. The Stock and Watson (1986) tests indicate that for the series excluding interest payments stationarity can be rejected at the 99% level of significance whilst the series including interest payments rejects the null hypothesis at the 90% level of significance. On this evidence we accept that government spending and the taxation plus seignorage variables are cointegrated with a cointegrating coefficient of unity. As predicted, both specifications of the deficit (including and excluding interest payments) yield the same result.

b. The Restricted Flood Garber Test

The bubble test of Hamilton and Flavin (1986) has also been estimated. As mentioned before, the average nominal interest rate is exceeded by the rate of growth of real GDP plus average inflation so the bubble term is not a test for a rational bubble, i.e. a significant bubble term is compatible with the repatriation of debt
over time. The true steady state interest rates should be used here, but data limitations confine us to the long run average rate over the sample period of our study.

The results of the estimation of equations (11) and (12) are presented in Table 5. For lags of the deficit term greater than one, nonlinear estimation techniques are necessary as there are cross equation restrictions which are nonlinear. Estimates for more than one lag of the deficit term show that the extra lags are insignificant (likelihood ratio tests were employed). The first point to note from these tests, given that we expect that the bond series and the bubble term are non-stationary, is that the equations should be considered in the framework of the cointegration literature. The low Durbin Watson coefficient of 0.17 shows that in fact the bubble term is not fully accounting for the non-stationarity in the bond series. An analysis of the residuals shows a strong negative deterministic trend which implies that the negative trend in the bubble term does not fully explain the determination of the level of bonds. This could be due to either a mis-specification of the bubble term or mis-specification of the proxy for expected deficits. The coefficients given in Table 5 are biased due to both non-stationarity and the mis-specification; very little trust can be put into their interpretation. It is interesting to note, however, that the bubble term is positive in sign. This is further, although very weak, evidence that inflation tax and GDP growth have contributed over the period to the sustainability of continuous primary government deficits.
5. Conclusion

The purpose of this paper has been to examine the sustainability of government debt over the period 1953/54 to 1986/87. To explore this question, we have used two tests adapted from the literature on 'bubbles'. Our study has generalised the previous approaches to allow for the effects of income growth and demonstrates that the methods of Trehan and Walsh (1988) and previous practitioners, based on cointegration, are the same under certain conditions.

The finding that the two series, government spending and taxation plus seignorage, are cointegrated with a cointegrating coefficient of unity leads us to accept that governments have not attempted to pursue unsustainable fiscal deficits for any lengthy period between 1953 and 1987. In other words we cannot consider the historical level of government debt to be at levels which are unsustainable.

An important implication of the cointegration results is that we cannot reject the hypothesis that governments have used seignorage to finance primary deficits. Evidence from our second test indicates that this has played a significant role in reducing the real value of government debt up until recently. This is compatible with the hypothesis that seignorage rather than debt financing has been the residual for financing fiscal deficits.
APPENDIX 1: DATA SOURCES

Government Spending and Taxation

The series for Commonwealth, State and Local spending are from the ABS publication "Government Financial Estimates" (5501.0). The Public Trading Enterprise sector is subtracted (excepting for subsidies paid and receipts received). The nominal series is used.

Money

The series is the money base as published in the Reserve of Australia Occasional Paper 4B and RBA Bulletins (various).

Debt

This series is Commonwealth securities on issue from RBA Occasional Paper 8A (Norton and Garmston (1984))(Table 2.21) less RBA holdings of securities (Table 2.23a)

Interest Rates

The series chosen was the rate on two year government bonds as reported in the RBA Bulletins (various).

GNP

This data was obtained from the ABS publication "Quarterly estimates of National Income and Expenditure" (5206.0)
REFERENCES


Figure 1

SURPLUS vs DEBT OUTSTANDING

Graph 1: Nominal

Graph 2: Deflated by GDP

Note that the definition of a surplus here is the surplus excluding interest payments as employed in the paper (i.e. includes state and local surpluses excluding public trading enterprises and including seignorage).
GOVERNMENT SPENDING vs TAXATION PLUS SEIGNIORAGE

Graph 3: Nominal

Graph 4: Deflated by GDP
TABLE 1: GOVERNMENT DEFICITS AND DEBT OUTSTANDING - 1955 TO 1987

<table>
<thead>
<tr>
<th>Year</th>
<th>Deficit</th>
<th>Deficit % of GDP</th>
<th>Debt Outstanding</th>
<th>Debt Outstanding % of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1955</td>
<td>110</td>
<td>1.14</td>
<td>7007</td>
<td>72.89</td>
</tr>
<tr>
<td>1956</td>
<td>159</td>
<td>1.53</td>
<td>7248</td>
<td>69.59</td>
</tr>
<tr>
<td>1957</td>
<td>46</td>
<td>0.41</td>
<td>7511</td>
<td>66.21</td>
</tr>
<tr>
<td>1958</td>
<td>46</td>
<td>0.40</td>
<td>7475</td>
<td>64.45</td>
</tr>
<tr>
<td>1959</td>
<td>271</td>
<td>2.17</td>
<td>7634</td>
<td>61.25</td>
</tr>
<tr>
<td>1960</td>
<td>234</td>
<td>1.70</td>
<td>7694</td>
<td>55.97</td>
</tr>
<tr>
<td>1961</td>
<td>32</td>
<td>0.22</td>
<td>7871</td>
<td>53.85</td>
</tr>
<tr>
<td>1962</td>
<td>358</td>
<td>2.39</td>
<td>8156</td>
<td>54.49</td>
</tr>
<tr>
<td>1963</td>
<td>415</td>
<td>2.56</td>
<td>8646</td>
<td>53.43</td>
</tr>
<tr>
<td>1964</td>
<td>419</td>
<td>2.33</td>
<td>9188</td>
<td>51.13</td>
</tr>
<tr>
<td>1965</td>
<td>183</td>
<td>0.93</td>
<td>9420</td>
<td>47.65</td>
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<tr>
<td>1966</td>
<td>255</td>
<td>1.23</td>
<td>9947</td>
<td>47.89</td>
</tr>
<tr>
<td>1967</td>
<td>552</td>
<td>2.41</td>
<td>10394</td>
<td>45.45</td>
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<tr>
<td>1968</td>
<td>643</td>
<td>2.63</td>
<td>10876</td>
<td>44.55</td>
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<td>1969</td>
<td>386</td>
<td>1.40</td>
<td>11667</td>
<td>42.33</td>
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<tr>
<td>1970</td>
<td>191</td>
<td>0.63</td>
<td>12015</td>
<td>39.34</td>
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<td>1971</td>
<td>10</td>
<td>0.03</td>
<td>12582</td>
<td>37.29</td>
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<td>1972</td>
<td>134</td>
<td>0.36</td>
<td>13534</td>
<td>35.92</td>
</tr>
<tr>
<td>1973</td>
<td>696</td>
<td>1.62</td>
<td>14133</td>
<td>32.94</td>
</tr>
<tr>
<td>1974</td>
<td>281</td>
<td>0.55</td>
<td>13863</td>
<td>26.99</td>
</tr>
<tr>
<td>1975</td>
<td>2552</td>
<td>4.13</td>
<td>16494</td>
<td>26.70</td>
</tr>
<tr>
<td>1976</td>
<td>3576</td>
<td>4.90</td>
<td>17599</td>
<td>24.17</td>
</tr>
<tr>
<td>1977</td>
<td>2881</td>
<td>3.23</td>
<td>19051</td>
<td>22.91</td>
</tr>
<tr>
<td>1978</td>
<td>3283</td>
<td>3.63</td>
<td>22531</td>
<td>24.94</td>
</tr>
<tr>
<td>1979</td>
<td>3426</td>
<td>3.35</td>
<td>25889</td>
<td>25.34</td>
</tr>
<tr>
<td>1980</td>
<td>1989</td>
<td>1.73</td>
<td>27566</td>
<td>24.02</td>
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<tr>
<td>1981</td>
<td>1080</td>
<td>0.83</td>
<td>27856</td>
<td>21.29</td>
</tr>
<tr>
<td>1982</td>
<td>553</td>
<td>0.37</td>
<td>28697</td>
<td>19.40</td>
</tr>
<tr>
<td>1983</td>
<td>4448</td>
<td>2.77</td>
<td>35504</td>
<td>22.08</td>
</tr>
<tr>
<td>1984</td>
<td>7932</td>
<td>4.40</td>
<td>43972</td>
<td>24.38</td>
</tr>
<tr>
<td>1985</td>
<td>6720</td>
<td>3.35</td>
<td>49169</td>
<td>24.51</td>
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<tr>
<td>1986</td>
<td>5726</td>
<td>2.56</td>
<td>54783</td>
<td>24.54</td>
</tr>
<tr>
<td>1987</td>
<td>2716</td>
<td>1.10</td>
<td>60631</td>
<td>24.60</td>
</tr>
</tbody>
</table>

Figures for the Deficit (Commonwealth government only, unadjusted for Public Trading Enterprises and interest payments) and Debt outstanding are from tables 2.14 and 2.23 from Occasional Paper No. 8A published by the RBA. RBA holdings of debt are not included in debt outstanding. Note that this measure of the deficit is that usually considered when fiscal policy is discussed and not the definition of the deficit used in this study.
**TABLE 2: TESTS FOR STATIONARITY**

<table>
<thead>
<tr>
<th></th>
<th>Govt. Exp. Excluding Interest Payments</th>
<th>Govt. Exp. Including Interest Payments</th>
<th>Taxation plus Seignorage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Trend</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-statistic</td>
<td>2.13</td>
<td>2.29</td>
<td>2.25</td>
</tr>
<tr>
<td>DF</td>
<td>-2.38</td>
<td>-2.50</td>
<td>-2.35</td>
</tr>
<tr>
<td>ADF</td>
<td>-1.98</td>
<td>-2.00</td>
<td>-1.76</td>
</tr>
<tr>
<td>Adjusted DF</td>
<td>-2.26</td>
<td>-2.37</td>
<td>-2.25</td>
</tr>
<tr>
<td>Perron and Phillips (PP)</td>
<td>2.36</td>
<td>2.59</td>
<td>2.41</td>
</tr>
<tr>
<td>Stock and Watson (SW)</td>
<td>-10.49</td>
<td>-10.93</td>
<td>-10.17</td>
</tr>
</tbody>
</table>

The null hypothesis of non-stationarity is rejected at the 95 per cent significance level if the DF or ADF statistic is greater than -3.60. The critical values for the PP and SW tests (95 per cent) are 7.24 and -21.7, respectively.
TABLE 3: COINTEGRATION TESTS

<table>
<thead>
<tr>
<th></th>
<th>Govt Exp Excluding Interest Payments</th>
<th>Govt Exp Including Interest Payments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Not Constant</td>
</tr>
<tr>
<td>Constant</td>
<td>0.026</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>(1.11)</td>
<td></td>
</tr>
<tr>
<td>Alpha</td>
<td>0.934</td>
<td>0.998</td>
</tr>
<tr>
<td></td>
<td>(15.59)</td>
<td>(114.27)</td>
</tr>
<tr>
<td>CRDW</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>ADF</td>
<td>-2.34</td>
<td>-3.94</td>
</tr>
<tr>
<td>R2</td>
<td>0.89</td>
<td>0.997</td>
</tr>
<tr>
<td>Q3</td>
<td>7.36</td>
<td>7.75</td>
</tr>
<tr>
<td>PP</td>
<td>6.27</td>
<td>6.63</td>
</tr>
<tr>
<td>SW</td>
<td>-21.56</td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis of no cointegration is rejected at the 95% significance level if the CRDW exceeds 0.386. The critical value for the ADF statistic is -3.37. The Critical value for the SW statistic is -17.5 (95%). The Q3 and PP critical values for the cointegrating case are unknown and excluded here as an extra guide.
### TABLE 4: TESTS FOR STATIONARITY

<table>
<thead>
<tr>
<th>Test</th>
<th>Deficit Excluding Interest Payments</th>
<th>Deficit Including Interest Payments</th>
</tr>
</thead>
<tbody>
<tr>
<td>DF</td>
<td>-4.00</td>
<td>-4.27</td>
</tr>
<tr>
<td>Adjusted DF</td>
<td>-3.82</td>
<td>-4.16</td>
</tr>
<tr>
<td>( \hat{\alpha}_3 )</td>
<td>7.75</td>
<td>9.13</td>
</tr>
<tr>
<td>Perron and Phillips (PP)</td>
<td>6.61</td>
<td>7.71</td>
</tr>
<tr>
<td>Stock and Watson (SW)</td>
<td>-21.67</td>
<td>-23.54</td>
</tr>
</tbody>
</table>

The null hypothesis of non-stationarity is accepted at the 95 per cent significance level if the DF statistics are greater than -3.00. The critical values for the \( \hat{\alpha}_3 \) and PP tests are 7.24 (95%) and 5.91 (90%) and for the SW test is -14.1 (95%).
### TABLE 5: RESTRICTED FLOOD GARBER TEST

**Equation (11)**

$$b_t = k_1 + \lambda_0 w^t + (a_1 b)/(1-a_1 b) S_t$$

**Equation (12)**

$$S_t = k_2 + a_1 S_{t-1}$$

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0$</td>
<td>1.33</td>
<td>(7.08)</td>
</tr>
<tr>
<td>$k_1$</td>
<td>0.26</td>
<td>(17.07)</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.0007</td>
<td>(0.17)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.33</td>
<td>(1.07)</td>
</tr>
<tr>
<td>$b$</td>
<td>-4.75</td>
<td>(-0.12)</td>
</tr>
</tbody>
</table>

(t statistics in brackets)

Log of the likelihood 120.451

**Durbin Watson Statistics**

- Equation 1 0.17
- Equation 2 1.75