

THE EFFECTIVENESS OF FISCAL POLICY  
IN AN ECONOMY WITH ANTICIPATORY WAGE CONTRACTS

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## ABSTRACT

This paper studies an economy where the labour market does not necessarily clear because real wages are sticky. Wages are fully indexed to prices, and are optimally adjusted over time in response to steady state deviations of output and of inflation. Inflation deviations will arise if wage setters make contracts that take into account long term forces, fulfilling a "social contract". When fiscal policy drives a wedge between long run and current inflation, it can have supply-side effects.

If goods markets always clear and future inflation is perfectly foreseen, fiscal policy can have real supply-side effects. With only money finance, such effectiveness exists only if the economy has suffered from an exogenous supply shock. If debt shares in the finance of fiscal deficits, real effects can be created even if the system is in steady state. The direction of the effects depend upon the extent to which fiscal policy is endogenised on account of the implied debt service. A fiscally cautious government will create net output gains after an expansion.

If goods markets do not clear, and the price level (but not inflation) is sticky, supply-side fiscal policy can only operate in a Classical Unemployment region. In this region, fiscal expansion only affects inflation, and output is perpetually stuck.

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1. Introduction

One of the key tenets of Keynesian fundamentalism is that fiscal pump-priming can stimulate demand-determined output. Resistance to the faith has focussed theoretically on the assumption of non-clearing goods and labour markets, and practically on the fear, hardened by bitter experience, that the gains have always seemed shortlived and that inflation has followed early in the wake.

The simplistic belief in non-clearing markets has been seriously disturbed by the emergence of New Classical macroeconomics but, while concessions may be made about goods markets, there are few who might budge on the issue of non-clearing labour markets. From an empirical viewpoint, Artus (1984) has established that the slowdown of productivity and employment in all major non-American<sup>1</sup> OECD economies in the late 1970s and early 1980s has been due to supply shocks which have not been accommodated by warranted falls in real wages. This malaise would not appear to recommend fiscal pump-priming - indeed quite the opposite. A widespread preference has been for the pursuit of disinflation, hoping that the harsh stick of unemployment will beat real wages down. According to this view, tight demand management and a period of unemployment is vital to weaken inflationary expectations and unrealistic behaviour in labour markets. Labour legislation, particularly in the United Kingdom, has been used to increase wage responsiveness to unemployment.

This paper takes up these issues to establish whether there is a case for fiscal expansion in an economy suffering from real wage stickiness and having clearing/non-clearing goods markets. Real wage stickiness is achieved by assuming perfect indexation of wages to prices. Throughout, output is assumed to be supply-determined and so there is no scope for conventional Keynesian demand management. However fiscal policy can have an influence on current inflation and on core or steady-state inflation.<sup>2</sup> Outside steady state, these two are not necessarily the same. All of the real effectiveness results in this paper come from this wedge. Fiscal policy derives its supply-side

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1. Sachs (1983) presents evidence of a real wage gap in the United States and Japan, but cannot attribute much of this to their unemployment. However, he also provides support for the hypothesis that real wage gap is the main problem for unemployment in the European economies.
  2. Buiter and Miller (1981) used core inflation in their Phillips curve.

leverage, outside steady state, from its ability to influence this "inflation gap". In this paper the gap only emerges when the goods market always clears and expectations of price inflation are formed rationally. With price inertia, no gap will be created.

The inflation gap is a critical element in the wage adjustment equation. In conventional models with 'expectations-augmented Phillips curves', only expected current inflation appears. But it is now well-established that these models have very unsatisfactory properties when perfect foresight or rational expectations is assumed. In particular the stable solutions to these models are typically only backward-looking in time, thus totally ignoring relevant future information which may be foreseen. Mussa (1981) discusses the problem in detail, and develops a microeconomic foundation of price adjustment<sup>3</sup> that rationalises the presence of 'equilibrium' rather than current, inflation in the wage adjustment equation. The essence of the analysis is that individual agents suffer a fixed cost for each price adjustment, and yet suffer integral costs that depend upon the deviation of their own price from the 'equilibrium' price. Search theory models also justify the use of 'equilibrium' inflation in the wage adjustment equation, since the costly gathering of information about the 'equilibrium' price is the foundation of that model.

The difficulty is to agree on the meaning of the 'equilibrium' price. Mussa (1981) gives a succinct definition that suggests the difficulty: "The 'equilibrium price' reflects both the play of market forces and the relative strength and skill of bargainers in any price negotiation". If only market forces were considered, the equilibrium price would be the rationally expected aggregate price over the life of the contract. The models of Taylor (1979) Phelps (1978) and Calvo (1983) solve this problem giving wage behaviour that is both forward and backward looking, and delivering policy effectiveness. If we permit bargaining forces, an additional source of policy effectiveness may be adduced. For example, political pressures may force wage setters to take account of core or underlying indicators of the rate of inflation (or in an open economy, competitiveness). Anticipatory wage contracts will then optimally apply a discount for core inflation and the 'equilibrium' price would be the core price which in this paper grows at the same rate as nominal money per capita. The

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3. See also, Sheshinski and Weiss (1977).

analysis is particularly relevant for economies in which the trade union movement has a "social contract" with the ruling political party. Such a phenomenon exists or has recently existed in a number of OECD countries, e.g. United Kingdom, Australia, Sweden, Norway.

If there is no government debt, fiscal policy can only have real effects if an output gap exists and prices are flexible. Fiscal pump-priming can speed up the process of adjustment of real wages and output, provided core inflation is below a critical rate defined by the maximum sustainable deficit. On the fiscal effectiveness side of the knife-edge, stagflation is a natural outcome.

The introduction of partial government debt financing allows for fiscal effectiveness, even if the economy is initially in steady state. A debt gap is created which will be matched by an inflation gap provided the goods market clears. Again, this is dependent on core inflation being below a critical rate. However, the issue of debt finance creates problems of future debt service, and stability of this process may require endogeneity of fiscal policy (see Christ (1979) Sheen (1987)). Over-cautious governments will make sure that the fiscal deficit adjusts down by more than the implied debt service. This is an important factor in the behaviour of output and inflation after a shock to the exogenous fiscal component. Below the critical inflation rate, an over-cautious government will create net output gains after a fiscal expansion.

If goods prices are sticky, no inflation gap can exist and there are no forces to reduce Classical Unemployment associated with excessive real wages. With wages fully indexed, and inflation unchanging, there are no incentives through the "social contract" to lower real wages; Classical Unemployment persists. This is consistent with the insider/outsider view of labour markets whereby wage setters do not concern themselves with the disenfranchised unemployed.

In Section 2, the underlying behavioural equations and identities of the paper are discussed. Sections 3 and 4 deal with a model of price flexibility and sticky real wages. In Section 3, fiscal deficits are financed only by money while Section 4 introduces debt. Section 5 discusses the implications of sticky prices and Section 6 presents some conclusions.

## 2. The Underlying Model

### (a) Private Sector Behaviour

Let output be produced by identical firms employing constant capital and variable labour with a constant returns to scale production function.

"Effective capital" is assumed to grow at the labour force growth rate,  $n$ , and in a steady state, output per capita is constant. Effective capital per capita is normalised to unity; since steady state output per capita depends only on this magnitude, it also must be unity. Firms are assumed to be able to sell all their output. Defining the logarithm of output per capita as  $y$  and of real wages as  $w$ , profit maximisation conditional on the real wage gives output as a negative function of the real wage or

$$y = c_0 - \frac{(1-\rho)}{\rho} w \quad (1)$$

where  $c_0$  is a constant, and  $\rho$  is the share of profits in output. The Walrasian steady state value of the logarithm of output,  $\hat{y}$ , is 0, and so the steady state output gap is defined to be  $y$ .

Output may be greater than its steady state (or natural) rate,  $\hat{y}$  if the real wage falls below  $\hat{w}$ . Firms can attract labour beyond the natural rate (say, in the form of overtime), although such an occurrence is only temporary because the real wage will subsequently be bid upwards.

Perfect indexation of wages to goods prices is assumed. Representative wage-setters make periodical nominal wage contracts, suffering two costs: a fixed cost per contract; and a cost over the life of the contract if the indexed wage deviates from that associated with the system in a full employment steady state. This means that the latter cost will be provoked if actual (expected) inflation differs from steady state inflation (over the life of the contract); as the gap between these increases, so the real wage will be bid down. In aggregate, the rate of change in real wages depends upon the output gap and upon the inflation gap,  $\pi - \hat{\pi}$ . In the Appendix the following equation is derived

$$\dot{w} = \phi'(y - \hat{y}) - \eta'(\pi - \hat{\pi}) \quad (2)$$

where  $w(0) = w_0$ . Perfect indexation implies that discrete jumps in the price level are always matched by nominal wage jumps - hence the real wage is predetermined. The existence of  $\hat{\pi}$ , or, what has been called "core inflation" (see Buiter and Miller (1982)) in the above Phillips curve is a critical element for the results in this paper. If  $\eta'$  were 0, then output would only depend upon the predetermined real wage, and there would no further interest in the inflation-output tradeoff. The parameter,  $\phi'$ , which measures the sensitivity of wage change to unemployed resources is shown in the Appendix to depend on the inflation gap; this aspect is ignored at minor cost in all but the final section.

Differentiating (1) and substituting into (2) gives the equation which will drive output

$$\dot{y} = \phi(\hat{y}-y) + \eta(\pi-\hat{\pi}) \quad (3)$$

where  $\phi \equiv -\frac{1-\rho}{\rho}\phi'$        $\eta \equiv -\frac{1-\rho}{\rho}\eta'$

If inflation is currently above its steady state level (and expected to eventually regress to that level), then nominal wages will rise at a slower rate than inflation and output will be stimulated. With output set by firms facing predetermined real wages, aggregate demand management only has a bearing insofar as it can influence the inflation gap and hence the real wage.

Output is sold to households and to government. No investment is permitted. The share of government expenditure in output is given at a point in time, and so the goods market equilibrium condition effectively solves for household expenditure. The price level adjusts instantaneously to ensure that this condition is met. Alternatively, and equivalently in this model, the price level will maintain continuous equilibrium in the money market.

Households own shares in the "effective capital" of firms. With effective capital per capita normalised to unity, the real rate of return,  $r$ , is defined by profits, which varies with output, i.e.  $r = \rho e^y$ . Linearising about the steady state, where  $\hat{y}=0$  gives

$$r \approx \rho(1+y) \quad (4)$$

A market and a price for capital is excluded deliberately to retain algebraic manageability of the model. It may be acceptable to think of the shares as being instantaneous, short term assets that mature instantly and are re-issued.

In Section 4, real short term government debt is introduced into the model. The rate of interest on this debt will be equalised to the rate of return on effective capital. The real rate of interest is seen to be positively related to output in (4).

Households also hold real money balances, defined in logarithms as  $m$ . The stock of nominal money balances is predetermined at a point in time by past issues by the government. The price level is free to adjust instantaneously to enable aggregate households to reach their desired real money balances. Assuming perfect foresight of inflation the demand for real money balances depends negatively on the nominal interest rate,  $r+\pi$ , and positively on output.

$$m^d = \ell_0 - \ell_1 (r + \pi) + \ell'_y y$$

When  $m^d = m$  is imposed in Sections 3 and 4, we can generally describe money market equilibrium by (using (4))

$$m = -\ell_1 \pi + \ell'_y y \quad (5)$$

where  $\ell_0 - \rho \ell_1$  is normalised to 0 and  $\ell'_y \equiv \ell'_y - \rho \ell_1$  is assumed positive

A useful definition for this paper is the inflation or interest elasticity of the demand for money,  $\xi$

$$\xi \equiv -(\hat{\pi} + n) \frac{\partial m}{\partial \pi} > 0 \quad (6)$$

(b) The Government's Budget Constraint

The first model to be analysed (in Section 3) excludes government debt and assumes that the per capita fiscal deficit,  $D_0$ , can only be financed by issuing

money. Linearised<sup>4</sup> in logarithms, about the steady state, this means that

$$\dot{m} \approx D_0 e^{-\hat{m}} (1 + \hat{m} - m) - (\pi + n) \quad (7)$$

In the steady state,  $\dot{m} = 0$  and the deficit is financed by inflation taxes

$$D_0 = (\hat{\pi} + n) e^{\hat{m}} \quad (8)$$

The inflation tax is composed from a tax rate,  $\hat{\pi} + n$ , and a tax base,  $e^{\hat{m}}$ . As inflation increases, the rate rises at the expense of the base. The tax initially rises reaching a maximum when the inflation elasticity of demand for money reaches unity. Thus the maximum sustainable deficit,  $D_0^C$ , is obtained at the point when  $\xi$  reaches unity, or, from (6) and (8), at the critical steady state inflation rate.

$$\hat{\pi}^C = \frac{1}{\xi_1} - n \quad (9)$$

Using (8) in (7) gives

$$\dot{m} \approx (\hat{\pi} + n)(1 + \hat{m} - m) - (\pi + n) \quad (7')$$

In this simple model, there is no need to distinguish between fiscal and monetary policy makers. Domination of one over the other is irrelevant.

The second model (of Section 4) includes real or indexed short term government<sup>5</sup> debt,  $b$ , as another means of financing the deficit. To proceed,

4. Writing nominal money balances (not in logarithms) as  $M$ , population as  $L$

and prices as  $P$ , the budget constraint is  $\dot{M}/PL \equiv D_0$  or  $(\frac{\dot{M}}{PL}) / (\frac{M}{PL}) \equiv D_0 / (\frac{M}{PL}) - (\pi + n)$ . This can be written in terms of  $m$  as

$$\dot{m} \equiv D_0 e^{-m} - (\pi + n)$$

which can be linearised using a Taylor's expansion on  $D_0 e^{-m}$ , i.e.

$$D_0 e^{-m} \approx (D_0 e^{-\hat{m}}) - (m - \hat{m}) D_0 e^{-\hat{m}} = D_0 e^{-\hat{m}} (1 + \hat{m} - m)$$

5. The use of real rather than nominal debt is an unimportant assumption. All the results in the paper go through. An additional source for the base of the inflation tax would be obtained with nominal debt.

we now have to introduce assumptions about the relationship between the fiscal and monetary authorities. The fiscal deficit will be assumed to comprise endogenous and exogenous elements. The endogenous elements arise in part because government spending and taxation vary with output. The extent to which they vary may be a contributor to the stability of the equilibrium of the system. The issue of debt implies higher debt service costs in the future and therefore further issues. This potentially destabilising force can be mitigated if the issue of debt can raise output and reduce the deficit. However, if steady state debt must rise, taxes on steady state output will be irrelevant for ensuring the stability and existence of this equilibrium.

The fiscal authorities will be obliged to introduce fiscal rules which respond to the level of outstanding debt. The total deficit,  $D$ , inclusive of debt service is given by

$$D = D_0 - te^y - \theta e^b + re^b$$

Remembering that  $y$  and  $b$  are in logarithms,  $t$  and  $\theta$  represent the linearly endogenous elements of fiscal policy.

This total deficit has to be financed by money and debt. Just as with fiscal policy, monetary policy shall be endogenous, except for an extreme parameter value. That parameter is  $\alpha$  which gives the share of the deficit financed by money. If  $\alpha = 0$  monetary growth may be exogenously set at a non-zero value. The effectiveness of fiscal policy will be seen to depend upon its dominance over monetary policy -  $\alpha > 0$ . Hence  $\alpha D$  is financed by the real per capita value of nominal money issue, and  $(1-\alpha)D$  by per capita real debt issue

$$\dot{m} = \alpha D e^{-m} - (\pi+n) \quad (10)$$

$$\dot{b} = (1-\alpha) D e^{-b} - n \quad (11)$$

In steady state,  $\dot{m} = 0$ ,  $\dot{b} = 0$  and

$$(\hat{\pi}+n)e^{\hat{m}} = \alpha \hat{D} \quad (12)$$

$$n e^{\hat{b}} = (1-\alpha)\hat{D} \quad (13)$$

where, noting that  $\hat{r} = \rho$  from (4)

$$\hat{D} = D_0 - t + (\rho-\theta)e^{\hat{b}} \quad (14)$$

Linearising<sup>6</sup> (10) about the steady state, using (12)-(14), gives

$$\dot{\hat{m}} = (\hat{\pi}+n)(1+\hat{m}-m) + \alpha e^{-\hat{m}}(\rho e^{\hat{b}}-t)y + \alpha e^{\hat{b}-\hat{m}}(\rho-\theta)(b-\hat{b}) - (\pi+n) \quad (10)'$$

Similarly linearising (11) about the steady state gives

$$\begin{aligned} \dot{\hat{b}} &= n(1+\hat{b}-b) + (1-\alpha)e^{-\hat{b}}(\rho e^{\hat{b}}-t)y - n \\ &= n(\hat{b}-b) + (1-\alpha)e^{-\hat{b}}(\rho e^{\hat{b}}-t)y \end{aligned} \quad (11)'$$

From (11)', there appears to be no problem about stability of debt issue. Indeed the partial derivative of  $\dot{\hat{b}}$  with respect to  $b$  is simply  $-n$ . This is mere appearance. The process of expressing the model in logarithms has implicitly enforced the assumption that

$$n + (1-\alpha)(\theta-\rho) > 0 \quad (15)$$

The existence of the steady state in (13) and (14) requires positive  $(D_0-t)/n+(1-\alpha)(\theta-\rho)$  because  $e^{\hat{b}}$  cannot be negative. Assuming net deficits,  $D_0-t > 0$ , in steady states, (15) is a necessary condition for existence.

$$\text{i.e. } \theta > \rho - \frac{n}{1-\alpha}$$

A sufficient condition is that  $\theta > \rho$ . If a government seeks to ensure that its fiscal deficit improves more than the steady state debt service worsens, then that government will be called FISCALLY CAUTIOUS. If it utilises the freedom conferred by growth and sets  $\theta$  so that it lies between  $\rho$  and  $\rho \cdot n/(1-\alpha)$ , it will not be considered cautious.

6. The linearisation of  $De^{-m} = D_0e^{-m}-te^{Y-m}+(\rho e^Y-\theta)e^{b-m}$  utilises the approximation

$$\begin{aligned} e^{z-w} &\approx e^{\hat{z}-\hat{w}}(1-\hat{z}+\hat{w}+z-w). \text{ Hence } De^{-m} \approx \{(D_0-t+(\rho-\theta)e^{\hat{b}})(1+\hat{m}-m) \\ &+(\rho e^{\hat{b}}-t)y \\ &+(\rho-\theta)e^{\hat{b}}(b-\hat{b})\}e^{-\hat{m}}. \end{aligned}$$

The inflation tax finances a policy-determined share of the deficit,  $D_0 - t$ . From (13)-(14), that share is given by  $\alpha n / n + (1 - \alpha)(\theta - \rho)$ . The inflation tax is maximised where  $\xi$  is unity, and so the critical inflation rate remains (9). The maximum sustainable exogenous deficit,  $D_0^C$ , with money and debt financing will be greater than in the case of just money financing because the inflation tax ceiling only restrains that portion of the deficit financed by money.  $D_0^C$  will also be greater as  $\theta$  and  $t$ , the endogenous components of fiscal policy, increase.

### 3 A Model of Money Finance and Flexible Prices

This model is basically the Cagan model (see Sargent and Wallace (1973)) with endogenous output and is completely specified by (3), (5) and (7'). Prices are flexible and their changes perfectly foreseen. By Walras Law, we are entitled to drop one market; in this paper, the goods market is not specified and hence the distinction between government expenditure and taxes is irrelevant. Real money balances are not predetermined because the price level is free to jump to ensure continuous money market equilibrium. Equivalently, given the assumption of perfect foresight about inflation, the equilibrium inflation rate is also determined in this market. Output is predetermined because the nominal wage is assumed to be perfectly indexed to discrete and continuous changes in the price level.

The linear dependence between  $m$ ,  $y$  and  $\pi$  is evident in equation (5). Any two of these three can be chosen to describe the fundamental dynamics. I analyse the model in terms of inflation and output. Using (6) one can express inflation changes as

$$\dot{\pi} = \frac{-1}{l_i} \dot{m} + \frac{l_y}{l_i} \dot{y} \quad (15)$$

Using (3), (7') and (15), and introducing the definition of  $\xi$  from (6) gives the following system expressed as deviations from steady state:

$$\begin{bmatrix} \dot{\pi} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \frac{(1 - \xi + \eta l_y)}{l_i} & \frac{l_y (\hat{\pi} + n - \phi)}{l_i} \\ n & -\phi \end{bmatrix} \begin{bmatrix} \pi - \hat{\pi} \\ y - \hat{y} \end{bmatrix} \quad (16)$$

It is necessary to add two boundary conditions to (16) to permit a solution. With  $y$  predetermined and  $\pi$  not, the conditions for a two point boundary value solution are:

$$\lim_{t \rightarrow \infty} \pi(t) \rightarrow \hat{\pi}$$

$$\lim_{t \rightarrow 0} y(t) \rightarrow y_0$$

I shall consider the properties of the model when initial output is in steady state (i.e. zero) and when it is in depressed (i.e. less than zero).

This type of problem is extremely well-known (see Buiter (1983) for a general statement) and it is a well accepted procedure to seek the unique convergent saddlepath solution. Without the first boundary condition, there are typically infinite possible solutions, termed speculative bubbles, where the jump variable ( $\pi$  in our case) takes on any value but that required to reach the saddlepath solution. One can justify the obsession with saddlepath solutions on the basis of underlying preferences if the asset involved is essential. An asset is essential if disutility becomes infinitely large as the value of the asset declines to zero. Speculative hyperinflations were ruled out by Brock (1974) on this basis. If the essential property cannot be accepted, Obstfeld and Rogoff (1984) show that a believable government guaranteed floor on an asset's value will prevent bubbles. Hyperdeflations can immediately be ruled out because eventually the real value of the asset must eventually exceed output, an impossible outcome under perfect foresight.

To find the saddlepath, we first have to establish whether one exists. In our two equation system, it is necessary and sufficient that the real parts of the two eigenvalues of the transition matrix in (16) be opposite to sign. One must be positive, the "unstable" eigenvalue  $S^+$ , and the other negative, the stable eigenvalue  $S^-$ . In general, one must have as many stable eigenvalues as predetermined variables. The product of the eigenvalues must equal the determinant of the transition matrix. The determinant of the matrix in (16) is

$$\Delta = -\frac{1}{\lambda_1} \left[ (1-\xi)\phi + \frac{\eta\lambda}{\lambda_1} Y\xi \right] \quad (17)$$

and we need it to be negative. If  $\eta=0$ , then a necessary condition is that  $\xi < 1$ . It is too strong if  $\eta > 0$ . This is worth noting because it is a common feature of inflation tax models that the inflation elasticity of money demand must be less than unity; this means that an increase in inflation will raise the overall tax take. If output is responsive to inflation, ( $\eta > 0$ ), then the higher output will raise money demand and hence the tax base. The direct inflation effect on the tax may then be negative, if the indirect effect via output is big enough to more than compensate. The indirect effect can also be boosted further if one allows the deficit to move anticyclically; this channel will be present in the next model. From (9), the maximum sustainable deficit is achieved as  $\xi$  approaches unity from below. Henceforth it shall be assumed that  $\xi \leq 1$ , in which case the saddlepath condition,  $\Delta < 0$ , must be met.

Another property is that the sum of the eigenvalues equals the trace of the transition matrix. For the matrix in (15), we have

$$\text{Trace} = \frac{(1-\xi)}{\lambda_1} - \phi + \frac{\eta \lambda_1}{\lambda_1} \quad (18)$$

If  $\eta=0$ , the transition matrix becomes triangular and the eigenvalues,  $S^+$  and  $S^-$  are immediately available from the diagonal. With obvious notation we get

$$S^+[0] = \frac{(1-\xi)}{\lambda_1} \quad ; \quad S^-[0] = -\phi$$

For  $\eta > 0$ , it is apparent from (16) and (18) that  $\Delta[\eta] > \Delta[0]$  and  $\text{Trace}[\eta] > \text{Trace}[0]$

Since  $S^+ = (\text{Trace} + (\text{Trace}^2 - 4\Delta)^{1/2})/2$ , it is simple to show that  $dS^+/d\eta > 0$ .

Hence  $S^+[\eta] > S^+[0] = \frac{(1-\xi)}{\lambda_1}$ . This result will be very useful at a later stage. We shall need to know that

$$S^+[\eta] - \frac{(1-\xi)}{\lambda_1} > 0 \quad (19)$$

In our linear differential equation system, the saddlepath implies a unique linear convergent relationship between the output gap,  $y(t)$ , and the inflation

gap  $(\pi(t) - \hat{\pi})$ . To be in that relationship, the non-predetermined variable,  $\pi$ , is available to jump appropriately after any shock. A restriction associated with the boundary condition has to be applied to the general solution if we are to confine ourselves to the saddlepath. The restriction is that we have to rule out that part of the general solution which involves the unstable eigenvalue  $s^+$ . Associated with any eigenvalue is a normalised row eigenvector,  $v'$  such that

$$v' [SI - A] = [0] \quad (20)$$

where A is the transition matrix. For unanticipated shocks, the restriction to rule out the unstable eigenvalue involves the unstable eigenvector,  $v^u$  in the following way.<sup>7</sup>

7. Consider the second order differential equation system

$$\begin{bmatrix} \dot{\pi}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \pi(t) - \hat{\pi} \\ y(t) - \hat{y} \end{bmatrix}$$

If there is only one predetermined variable and one stable eigenvalue, the convergent solution will be of the form

$$\pi(t) - \hat{\pi} = w_{11} e^{s^- t}$$

$$y(t) - \hat{y} = w_{12} e^{s^- t}$$

Differentiation of this gives

$$\begin{bmatrix} \dot{\pi}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} s^- & 0 \\ 0 & s^- \end{bmatrix} \begin{bmatrix} \pi(t) - \hat{\pi} \\ y(t) - \hat{y} \end{bmatrix}$$

Since this solution has to be identical to the above, we get

$$\frac{\pi(t) - \hat{\pi}}{y(t) - \hat{y}} = \frac{w_{11}}{w_{12}} = \frac{-a_{12}}{-s^- + a_{11}} = v^u$$

Since  $s^- + s^+ = \text{trace } A = a_{11} + a_{22}$

$v^u = \frac{-a_{12}}{s^+ - a_{22}}$  which is a solution which would emerge from

$$[-1, v^u] [s^+ I - A] = [0, 0].$$

$$0 = -(\pi(t) - \hat{\pi}) + v_1^u Y(t) \quad (21)$$

To determine  $v_1^u$ , we solve (20)<sup>8</sup> to give (21) at  $t=0$ .

$$\text{Hence } v_1^u = \frac{\lambda_Y(\phi - (\hat{\pi} + n)) / \lambda_i}{s^+ + \phi} \quad (22)$$

whose sign simply depends upon  $\phi - (\hat{\pi} + n)$ . Thus inflation and output will be positively correlated over time if  $\phi > \hat{\pi} + n$ .

The parameter  $\phi$  measures the responsiveness of real wages to unemployed resources. If this responsiveness is low, and the underlying core inflation is high, then stagflation (or its converse) will be experienced whereby rising unemployment will be associated with rising inflation. The switch to the stagflation mode occurs at core rates of inflation that exceed  $\phi - n$ . This was observed in many industrialised countries in the 1970's - average inflation rates were at a historical high and underlying growth,  $n$ , was falling.

The reason for this condition on  $\phi$  is simple. If per capita output is languishing below 0, it must rise in the future if the system is convergent. But future rising output and current low output impacts on the direction of change of inflation,  $\dot{\pi}$ , through (15). Money demand will be rising with output in the future, and hence  $\dot{\pi}$  can be positive. But the inflation tax base will current be low due to output, causing the supply of real money balances to increase thus reducing  $\dot{\pi}$ . The greater is  $\phi$ , the faster will  $\dot{y}$  grow and the greater  $\dot{m}^d$ ; the greater is the inflation tax rate,  $\hat{\pi} + n$ , the greater is  $\dot{m}^s$ . If the former exceeds the latter,  $\dot{\pi}$  will be positive, and because the saddlepath is a line,  $\pi(t) \leq \hat{\pi}$  if  $y(t) \leq 0$ .

8. Taking the unstable eigenvalues, (20) becomes

$$[-1, v_1^u] \begin{bmatrix} s^+ - \frac{(1-\xi)}{\lambda_i} & \frac{\lambda_Y}{\lambda_i} \\ \frac{\eta}{\lambda_i} & s^+ + \phi - \frac{\eta \lambda_Y}{\lambda_i} \end{bmatrix} = [0, 0]$$

Given  $y(0) - y_0$  and  $\pi(0)$  from (21), the evolution of  $\pi$  and  $y$  over time, is described by

$$\begin{aligned}\pi(t) - \hat{\pi} &= v_1^u y(0) \cdot e^{\bar{s} t} \\ y(t) &= y(0) \cdot e^{\bar{s} t}\end{aligned}\tag{23}$$

One can easily consider fiscal policy changes using the solutions in (21)' and (23). A fiscal expansion (an increase in  $D_0$ ) leads to a steady state increase in inflation and a decrease in real money balances if the interest elasticity of money demand is less than one. From (5), (6) and (8) we get

$$\frac{d\hat{m}}{dD_0} = \frac{-\xi}{D_0(1-\xi)} < 0\tag{24}$$

$$\frac{d\hat{\pi}}{dD_0} = \frac{1}{e^{\hat{m}}(1-\xi)} > 0$$

If the policy were instituted in a period of full employment,  $y(0) = \hat{y} = 0$ , it can be seen from (21) that  $\pi(0)$  would equal  $\hat{\pi}$ ; the inflation rate would merely jump to the new steady state level. Nominal wages would follow suit exactly, and there would be no need for output to move at all.

The interesting question to ask concerns the transitory relationship between inflation and output after a supply shock. For example, if energy were a factor of production, an oil price increase would constitute a negative supply shock that would require a fall in real wages. Assuming the shock to have taken place sometime back, and that real wages are not yet down by enough, output would be below potential,  $y(0) < 0$ . For a given fiscal deficit, the post-shock level of steady state inflation would be higher than the pre-shock level (see (5) and (8)). With sticky real wages the current inflation rate will have overshoot the steady state value if  $\phi < \hat{\pi} + n$ .

The final matter of interest in this simple model is whether fiscal policy can play any role in helping the economy out of the transitory recession of output. There is, of course, no direct demand management role, because the output gap has been posed as a real wage or supply-side problem. The only way fiscal policy can work is by influencing the inflation gap and thereby real wages. From (3),  $\dot{y}$  will be greater if  $\pi - \hat{\pi}$  can be raised. Fiscal expansion does raise steady state inflation, but with output below steady state, it is not immediately obvious how it will affect  $\pi - \hat{\pi}$ .

I can show that fiscal expansion will speed up output adjustment, up to a point, whether or not there is stagflation. Beyond this point, the saddlepath property of the model disappears, and both eigenvalues become negative. Since the model is then convergent from any starting point (of  $\pi$  and  $y$ ), and not just on the saddlepath line, it is sensible to conjecture that the rate of inflation will immediately collapse to its steady state value; only  $\phi$  will then matter for output adjustment. However, the point at which the saddlepath disappears turns out to be beyond the maximum sustainable deficit, defined where the inflation elasticity is unity.

To prove the above, one only need to know what happens to the eigenvalues, when  $D_0$ , and thus  $\hat{\pi}$  (see (24)), is increased. Remembering that  $\xi$  depends on  $\hat{\pi}$ , the total differentiation of  $\Delta$  (see (17)), which equals  $S^+ \cdot S^-$ , gives

$$S^+ \frac{dS^-}{d\hat{\pi}} + \frac{S^- dS^+}{d\hat{\pi}} = \phi$$

Differentiating the trace of the matrix in (16) gives

$$\frac{dS^+}{d\hat{\pi}} + \frac{dS^-}{d\hat{\pi}} = \phi$$

and so the solutions are

$$\frac{dS^-}{d\hat{\pi}} = \frac{\phi(1-S^-)}{S^- - S^+} < 0 \quad (25)$$

$$\frac{dS^+}{d\hat{\pi}} = \frac{\phi(1+S^+)}{S^- - S^+} < 0 \quad (25)$$

Now only the stable eigenvalue,  $S^-$ , is relevant for the solution to  $y$  in the saddlepath zone (see (23)); since it increases in absolute size (given  $S^+ > 0$ ) fiscal pump-priming is an effective mechanism. But the positive, unstable eigenvalue also decreases eventually reaching a critical point after which it changes sign. That point is where  $S^+ = 0$  and  $\Delta = 0$  or from (17) where

$$\hat{\pi} = (1 + \eta \ell_y / \phi \ell_i) / \ell_i - n \quad (27)$$

Comparing (27) to (9), it is evident that the saddlepath disappears after the point of the maximum sustainable deficit.

In Figure 1, the relationship between  $\pi$ ,  $y$ ,  $\hat{\pi}$  and  $|S^-|$  is shown.

Between points A and B,  $\phi > \hat{\pi} + n$ , and the non-stagflation case is observed.

At B,  $\phi = \hat{\pi} + n$  and  $v_1^u = 0$ , (see (22)); inflation will have no influence on output. Between B and C, stagflation appears with inflation above its

steady state but set to fall as output improves. In Figure 1a, the two

possible saddlepaths (from (22)) are shown as BC and AB. A fiscal expansion

shifts and tilts them, causing the inflation gap to increase i.e.  $g_1 < g_0$ ,

$h_1 > h_0$ . The intuitive reason why  $\pi$  increases more than  $\hat{\pi}$  in either

zone is that the increased nominal demand coming from the fiscal expansion can be cleared only by a change in inflation in the short run, but over time will be eliminated by adjustment in both output and inflation. In the stagflation zone, BC, fiscal pump-priming or declines in the underlying population and capital augmenting growth rate,  $n$ , must eventually engender a collapse of the system.

#### 4 A Model of Debt and Money Finance with Flexible Prices

The previous model permitted only money finance of fiscal deficits. This meant that fiscal policy would have real effects only when output had suffered from an exogenous supply shock. When the government can also issue debt, fiscal policy can shock the real and the financial system out of a steady state.

The model can be reduced to three independent equations in  $\dot{m}$ ,  $\dot{\pi}$ ,  $\dot{y}$  and  $\dot{b}$ . Choosing the last three and summarising the model from (3)-(7), (10)'-(11)' and (12)-(14), we get

$$\begin{bmatrix} \dot{\pi} \\ \dot{y} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \frac{1}{l_1}(1-\xi+n l_y) & \frac{1}{l_1}[l_1(\hat{\pi}+n-\phi) - \frac{(\rho e^{\hat{b}-t})}{\hat{D}}] & \frac{-\alpha}{l_1}(\rho-\theta)e^{\hat{b}-\hat{m}} \\ n & -\phi & 0 \\ 0 & n \frac{(\rho e^{\hat{b}-t})}{\hat{D}} & -n \end{bmatrix} \begin{bmatrix} (\pi - \hat{\pi}) \\ (y - \hat{y}) \\ (b - \hat{b}) \end{bmatrix} \quad (28)$$

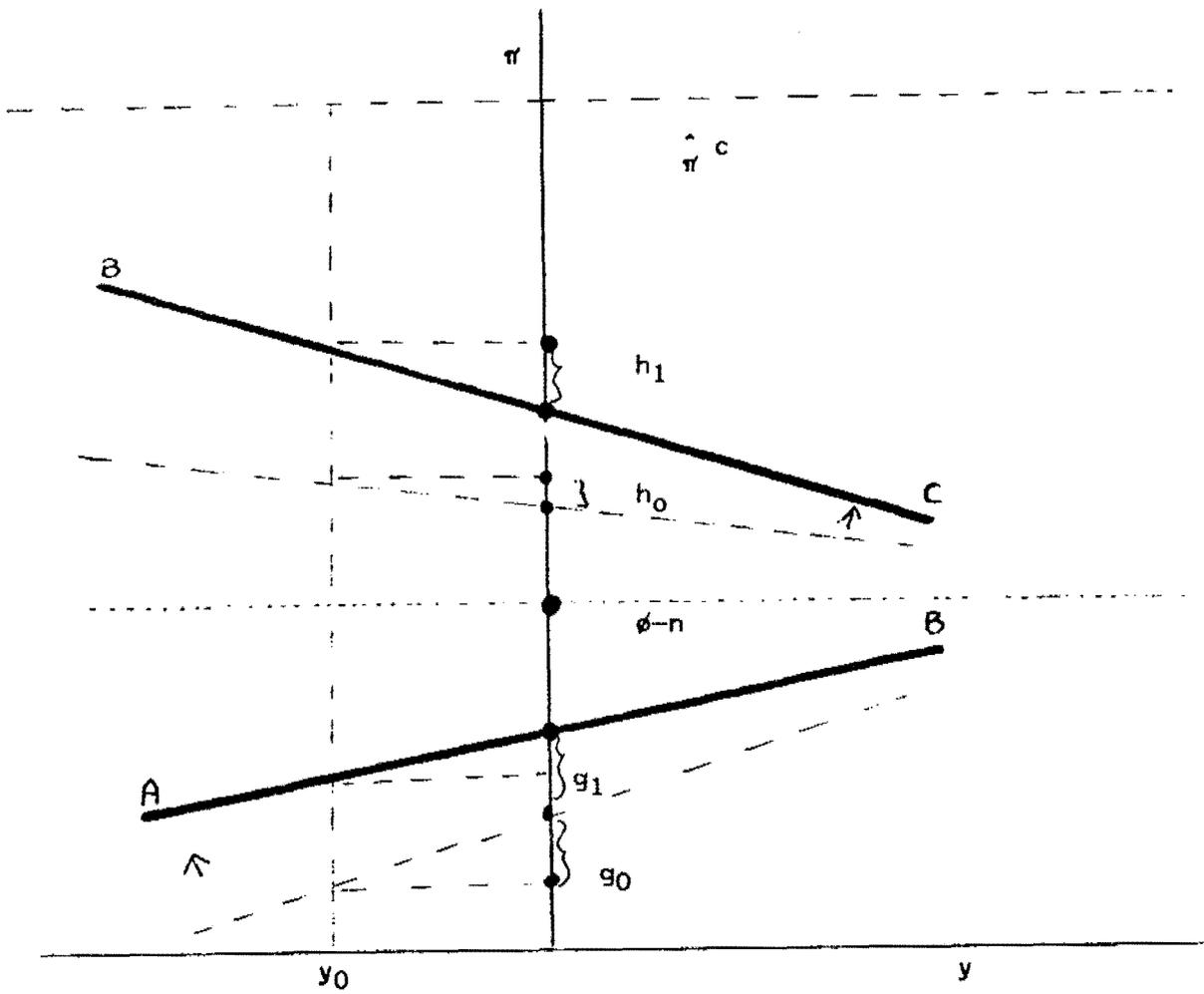


Figure 1a

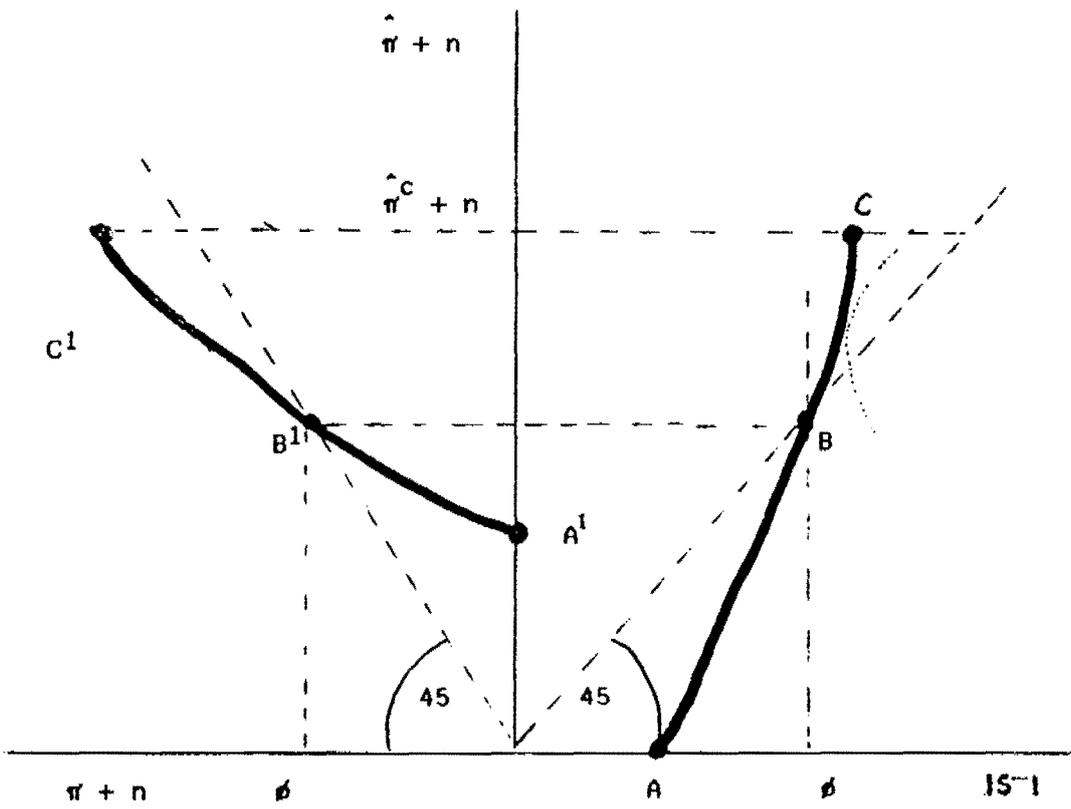


Figure 1b

with the boundary conditions

$$\lim_{t \rightarrow \infty} \pi(t) \rightarrow \hat{\pi}$$

$$\lim_{t \rightarrow 0} y(t) \rightarrow y_0$$

$$\lim_{t \rightarrow 0} b(t) \rightarrow b_0$$

Taking the determinant of the transition matrix in (28), one gets

$$\Delta = \frac{n}{k_1} [\phi(1-\xi) + \eta(\hat{\pi} + n)(k_y - \frac{(\rho e^{\hat{b}-t})(D_0 - t)}{\hat{D}^2})] \quad (29)$$

Since there are two predetermined variables,  $y$  and  $b$ , in this model, we need two stable eigenvalues (that have negative real parts),  $S_1^-$  and  $S_2^-$ , and one stable one,  $S^+$ . The product of these is  $\Delta$  and must be positive if a saddlepath is to exist.  $\Delta$  will be positive if  $t$  is sufficiently large. A sufficient condition is that  $D_0 > t > \rho e^{\hat{b}}$ , which means that the income tax rate must exceed the ratio of debt service to output. This condition will easily be met in a modern economy. The necessary condition is a highly non-linear equation in all the parameters of the model.

If the trace of the matrix is negative, then we can be sure that positive determinant did not come about from three positive eigenvalues. The trace is negative if  $\phi + n > (1 - \xi + \eta k_y) / k_1$ . Of course, this condition is not necessary, but assume that it holds.

A point on the saddlepath is given by a linear combination of the steady state deviations of the endogenous variables; the unstable eigenvector provides the weights with analogous notation to that in the previous section:

$$\pi(0) - \hat{\pi} = v_1^u (y(0) - \hat{y}) + v_2^u (b(0) - \hat{b}) \quad (30)$$

Consider the situation when  $y(0) = \hat{y} = 0$ ; the first term on the right hand side of (30) drops out and we get

$$\pi(0) - \hat{\pi} = v_2^u (b(0) - \hat{b}) \quad (30')$$

It is now apparent why fiscal policy can move the system from a steady state position. Government debt is predetermined in that it can only be altered over time; a change in the exogenous fiscal deficit will typically affect its steady state value creating a "debt gap". With money financing endogenous ( $\alpha \neq 0$ ), an associated inflation gap emerges which moves the real wage and then output. Notice that if the rate of money growth were exogenous ( $\alpha = 0$ ), the model would simplify dramatically.  $\Delta$  would become  $\eta\phi/\lambda_1$ , and  $v_2^u = 0$ ; fiscal policy would become neutral, although monetary policy would not. The issue of interest is the determination of the circumstances for which fiscal policy expansion creates net output losses or gains.

From (13) and (14), a fiscal expansion will raise  $\hat{b}$  thus creating a negative debt gap; but remember that the existence and stability of the equilibrium depends upon (15), or that

$$\theta > \rho - \frac{n}{1-\alpha}$$

If a government is overly cautious, because it ignores growth,  $n$ , adjusting fiscal policy by more than the implied debt service, then the total steady state deficit,  $\hat{D}$ , will in fact increase less than  $D_0$ . The opposite occurs if the government is not cautious. This will be the basic reason for the different output effects of the two different types of government. For a fiscally cautious government I shall show that  $v_2^u < 0$  and I shall conjecture that  $v_1^u > 0$ .

A negative debt gap will be associated with a positive inflation gap if the government is overly cautious. This causes output to start rising from its steady state. To show this, the sign of the eigenvalue,  $v_2^u$ , has to be established. As before:

$$[-1, v_1^u, v_2^u] [S^+ I - A] = [0, 0, 0]$$

where  $A$  is the transition matrix in (28). Considering only the third column in  $[S^+ I - A]$  gives

$$v_2^u = \alpha \frac{(\rho - \theta) e^{\hat{b} - \hat{m}}}{(s^+ + n) l_1} \quad (31)$$

Evidently, this is negative if  $\theta > \rho$ . So from (30), the fiscal-induced negative debt gap will be associated with a positive inflation gap, and from (3), this will induce output to rise. The intuition behind this is that fiscally cautious governments are expected to rein in endogenous fiscal instruments in the future as debt accumulates. Hence future nominal demand is expected to fall, and with it inflation. Thus  $\dot{\pi}(t) < 0$  for  $t > 0$ . Inflation must then overshoot its steady state value for this to be true. The converse occurs for governments that are not overly cautious and who set  $\theta$  so that  $\rho - n / (1 - \alpha) < \theta < \rho$ .

If the eigenvalues have no imaginary roots, cycles about the steady state cannot be experienced. In that case, it is manifestly clear that fiscally cautious governments will be able to create net output gains. The positive effects of the inflation gap will eventually be counteracted by the increasing output gap effect (via  $\phi$ ) on real wages. As in the previous section, the output gap will have an ambiguous influence on the inflation gap, via  $v_1^u$ . Previously the relationship simply depended on  $\phi - (\hat{\pi} + n)$  (see (26)). Now using the second column of A

$$v_1^u = \frac{1}{l_1 (s^+ + \phi)} [l_Y (\phi - (\hat{\pi} + n)) + \frac{(\rho e^{\hat{b} - t})}{\hat{D}} (\alpha \frac{n(\rho - \theta) e^{\hat{b} - \hat{m}}}{(s^+ + n)} - 1)] \quad (32)$$

As mentioned with reference to (29),  $\rho e^{\hat{b} - t}$  is expected to be negative. Hence, for a cautious government ( $\theta > \rho$ ), the likelihood of  $v_1^u > 0$  is definitely greater than in Section 3. If  $v_1^u > 0$  for a cautious government, then the initial positive inflation gap induced by the debt gap will initially be stimulated as the output gap becomes positive. When the output gap begins to fall later on, the inflation gap will follow suit. Hence stagflation (or its converse) would not be observed. This possible outcome is depicted in Figure 2a. If  $v_1^u < 0$  for a fiscally uncautious government, then the negative output gap would raise the negative inflation gap and so inflation would be observed to be rising throughout the adjustment period. In the initial phase, declining output and rising inflation - stagflation - would be observed. This outcome is shown in Figure 2b.

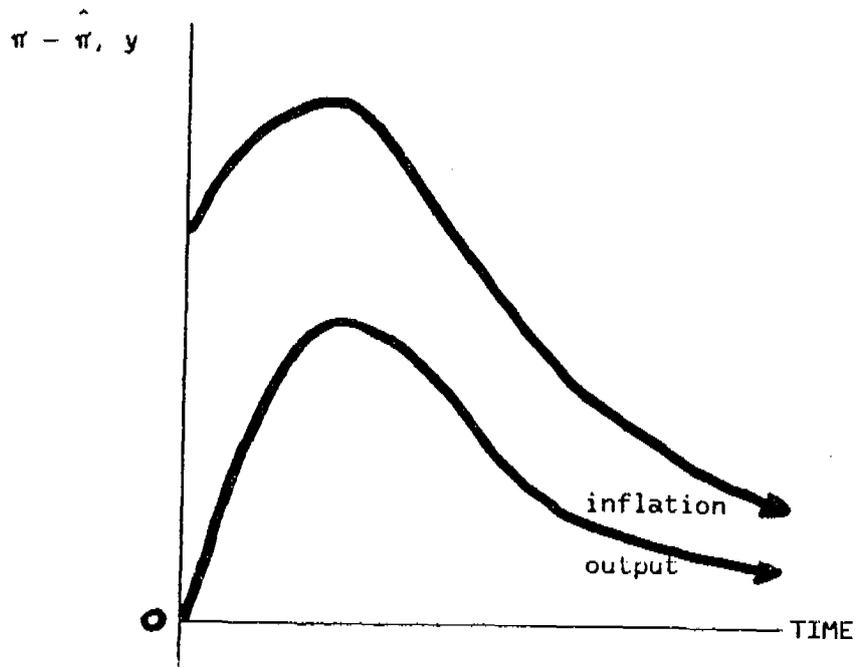


Figure 2a

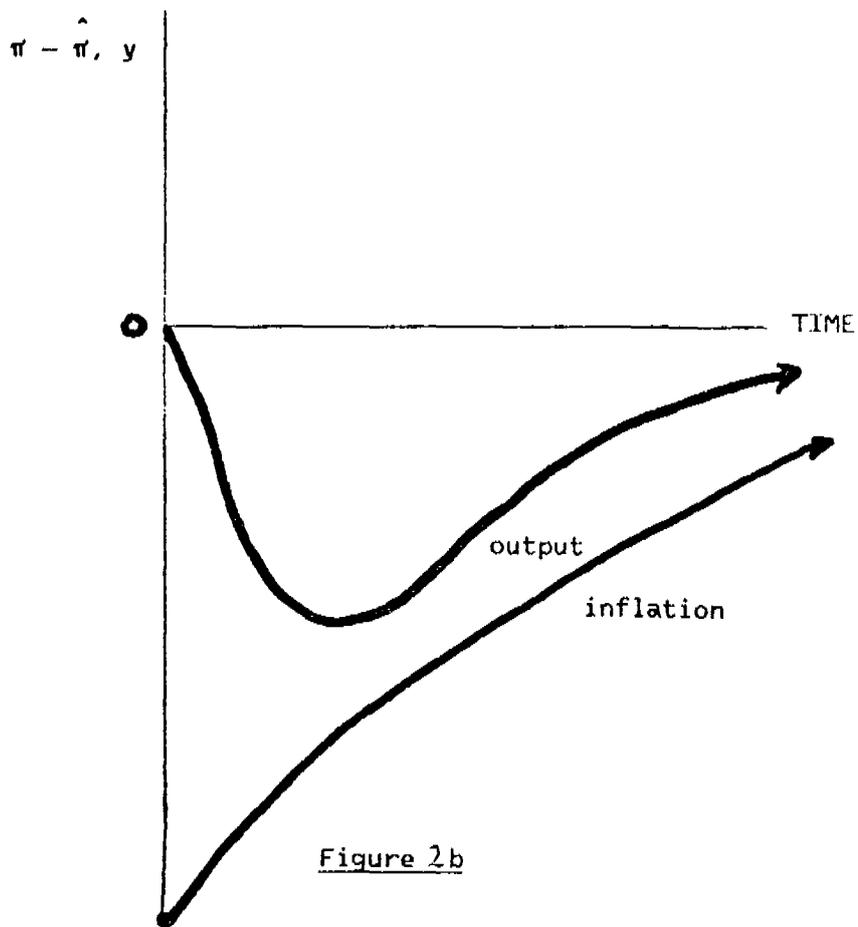


Figure 2b

The net output or gain between two steady states (0 and 1) can be easily obtained regardless of whether the dynamics are characterised by cycles. We need to find  $\int_0^{\infty} y(t) dt$ . To proceed, (28) can be manipulated to eliminate  $(b-\hat{b})$  and  $(\pi-\hat{\pi})$ , and to end up with an equation for  $y$  as a linear function of  $\dot{\pi}$ ,  $\dot{y}$  and  $\dot{b}$ . Integrating that over time, and noting that  $\int_0^{\infty} \dot{y}(t) dt = \hat{y}_1 - \hat{y}_0 = 0$ ,  $\int_0^{\infty} \dot{\pi}(t) dt = \hat{\pi}_1 - \hat{\pi}_0$ , and  $\int_0^{\infty} \dot{b}(t) dt = \hat{b}_1 - \hat{b}_0$  we get

$$\int_0^{\infty} y(t) dt = \frac{\eta\alpha}{\Delta} [(\hat{\pi}_1 - \hat{\pi}_0) + \frac{\alpha(\theta-\rho)e^{\hat{b}_1 - \hat{m}_1}}{\lambda_1} (\hat{b}_1 - \hat{b}_0)] \quad (33)$$

$$\text{for } \hat{\pi}_1 < \hat{\pi}^C$$

From the steady state equations, (12) and (13), a proportional increase in  $D_0$ , by say  $\delta$ , leads to a linearised increase in  $\hat{\pi}$  and  $\hat{b}$  of the form

$$\begin{aligned} (\hat{\pi}_1 - \hat{\pi}_0) &= \frac{\alpha\delta D_0}{e^{\hat{m}_1(1-\xi_1)}} \\ (\hat{b}_1 - \hat{b}_0) &= \frac{e^{-\hat{b}_1(1-\alpha)}\delta D_0}{\eta + (1-\alpha)(\theta-\rho)} \end{aligned} \quad (34)$$

Inserting (34) in (33) gives

$$\int_0^{\infty} y(t) dt = \frac{\eta\alpha\delta D_0}{\Delta} \left[ \frac{1}{e^{\hat{m}_1(1-\xi_1)}} + \frac{(1-\alpha)(\theta-\rho)e^{-\hat{m}_1}}{\lambda_1(\eta + (1-\alpha)(\theta-\rho))} \right] \quad (35)$$

Evidently, this is positive for a fiscally cautious government ( $\theta-\rho > 0$ ). For uncautious governments, a net loss is more likely as the share of money in deficit finance ( $\alpha$ ) decreases. If  $\eta$  or  $\alpha$  are 0, fiscal policy is seen to be neutral. Non-neutrality can be achieved if money financing depends on fiscal policy and if real wages are forward looking.

These results are true only up to a point. As in the simpler model, the critical factor is that the exogenous fiscal deficit is sustainable. Fiscal pump-priming would cause the system to explode if the underlying inflation rate pierced its critical level as defined in (9).

Consider now a supply shock such as an oil price increase if energy were specified as a factor of production. Steady state output falls, but current output falls further because real wages will, initially, be too high. If taxes were independent of output,  $t=0$ , then  $\hat{b}$  would be unaffected. Otherwise  $\hat{b}$  must rise after the supply shock because equilibrium income taxes will have fallen. Hence negative output and debt gaps will be immediately created. The effect on the inflation gap is again ambiguous because  $v_1^u$  cannot be signed. The only question that can be addressed is whether fiscal expansion can speed up the return of output to steady state. In the simple model without debt, the answer was in the affirmative and this was due to the change in  $v_1^u$ . Fiscal expansion now worsens the debt gap, and this improves the inflation gap for a fiscally cautious government (given positive  $S^+$ ). This direct effect speeds up output adjustment. Further  $v_2^u$  will increase in absolute size since the numerator in (31) is greater and the denominator will be smaller. The smaller denominator, as in the previous section, occurs because the stable eigenvalue decreases in size. Also  $v_1^u$  will decrease (eventually turning negative) thereby further magnifying the output correction effect.

A fiscally cautious government can use expansionary fiscal policy to raise the speed of adjustment of output after a supply shock, provided the deficit is sustainable and monetary policy is not exogenous. For an uncautious government, the effectiveness is reduced.

##### 5. Stickiness in Goods Prices

In the previous two sections, expansionary fiscal policy could have beneficial output effects in an economy with sticky real wages but flexible prices. If goods prices were also sluggish the dynamic version of the fix-price macroeconomic models would be applicable.

One generalisation disaggregates the goods market into a flexible price and a sluggish price sector. As the share of the flexible price sector declines, the results of the previous two sections would be moderated. The reason is that current inflation would become increasingly restrained in its excessive short run response to unanticipated fiscal shocks. In the limit, there would never be an inflation gap to exploit. This diminution of the supply-side effectiveness of fiscal policy would occur provided the economy remains in a regime where firms determine output and employment. In the fix-price modelling terminology, a Classical Unemployment (CU) regime would have to exist.

A necessary (but not sufficient) condition for CU is that the real wage exceeds its Walrasian steady state value ( $w > \hat{w}$ ) implying  $y^S < \hat{y} = 0$ , where  $y^S$  is output supply. In addition, to support CU for given  $w > \hat{w}$ , the Walrasian steady state gap of the price level cannot be (a) greater by an amount that would reduce real money balances below a floor denoted  $m_u(y)$  beyond which aggregate goods demand would be below supply creating Keynesian Unemployment (KU), or (b) less than an amount that would raise real money balances above a ceiling  $m_u(y)$  and reduce labour supply below demand, creating Repressed Inflation (see Barro and Grossman (1976)). CU will be assumed henceforth.

While traditional fiscal effectiveness results apply in a KU regime, they do not apply in a CU one; excess goods demand would be exacerbated by a fiscal expansion, simply adding to inflation with no feedback. To establish this, I shall use the Calvo (1983) model of price sluggishness, incorporating the possibility of steady state inflation.

Goods price contracts are stochastic and exponentially distributed, identically and independently, over the range of products. Any existing contract at  $t$  will last till  $t+\tau$  with probability  $\delta \exp(-\delta\tau)$ . Aggregating over contracts that fixed a price,  $V(s)$  at a previous date  $s$ , the current (predetermined) price level becomes

$$P(t) = \int_{-\infty}^t V(s) \delta \exp(-\delta(t-s)) ds$$

which is equivalent to

$$\dot{\pi}(t) = \dot{P}(t) = \delta(V(t) - P(t)) \quad (36)$$

If CU steady state inflation is non-zero,  $P = \tilde{\pi}$ , the (marginal) steady state contract price,  $\tilde{V}$ , exceeds the aggregate price,  $\tilde{P}$ , by the amount  $\tilde{\pi}/\delta$ , where  $1/\delta$  is the expected duration of the contract. The CU steady state (denoted by ' $\sim$ ') may or may not be identical to the Walrasian steady state.

By assumption, the contract price rationally accounts for future aggregate prices and for the deviation of aggregate output from perceived aggregate demand  $y - y^d$ . Perceived demand<sup>9</sup> is not necessarily realised because, in CU

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9. Throughout this paper, the specification of aggregate goods demand has been avoided. Perceived aggregate demand will typically depend on real money balances, inflation, real wages and the fiscal deficit.

it is always greater than or equal to supply. Except in full Walrasian equilibrium, it does not equal 0. Hence

$$V(t) = \int_t^{\infty} [P(s) + \gamma(y(s) - y^d(s))] \delta \exp(-\delta(s-t)) ds$$

which differentiated becomes

$$\dot{V} = \delta(V - P - \gamma(y - y^d)) \quad (37)$$

In steady state,  $\tilde{V} = \tilde{\pi}$ , and  $y - y^d = 0$ . (36) and (37) can be combined to give

$$\dot{\pi} = -\gamma\delta^2(y - y^d) \quad (38)$$

As current output rises, (for given  $V_t$  and  $y_t^d$ ) the change in contract prices falls according to  $-\gamma\delta$ . This causes a subsequent ( $\delta$ ) fall in inflation change. However, it is important to observe that both  $\pi$  and  $V$  are not historically predetermined, while  $P$  is.

Adding (38) to the simple model with just money finance<sup>10</sup> described in Section 2, we get the following system.

$$\begin{bmatrix} \dot{\pi} \\ \dot{y} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} 0 & -\gamma\delta^2 & 0 & \pi \\ \eta & -\phi & 0 & y \\ -1 & 0 & -(\tilde{\pi} + \eta) & m \end{bmatrix} + \begin{bmatrix} \gamma\delta^2 y^d \\ \phi\tilde{y} - \eta\tilde{\pi} \\ (\tilde{\pi} + \eta)(1 + \tilde{m}) - \eta \end{bmatrix} \quad (39)$$

where  $y \leq y^d \leq 0$ ;  $m_l(y) < m < m_u(y)$

The determinant of the transition matrix is  $-\gamma\delta^2\eta(\tilde{\pi} + \eta)$  and the trace is  $-(\phi + (\tilde{\pi} + \eta))$ . With only two predetermined variables,  $y$  and  $m$ , the sign of the determinant implies one or three negative (real parts of the) eigenvalues. Since  $\gamma\delta^2$  is relatively small, one would expect the latter. Hence a multiplicity of convergent solutions are possible. Since any value of  $\pi$  may be chosen, it is standard to posit that  $\pi$  (and hence  $V$ ) jumps immediately to its steady state value. An expected fiscal expansion will raise  $\tilde{\pi}$  and  $\pi$  by the same amount, and so will have no real effects.

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10. The general conclusions are not altered if debt finance is permitted.

This scenario involves a further insidious property. Since current and steady state inflation are always equal, output change is actually zero - in (3), the inflation gap is zero, but also  $\phi'$  is zero. The appendix demonstrates that the endogenous contract length,  $T$ , becomes infinite if there is no inflation gap and that  $\phi'$  is inversely related to  $T$ . Real wages are only bidden down if an inflation gap exists.

Thus output is stuck below full employment and by (38) we have a CU steady state displaying full inertia

$$\tilde{y} = \tilde{y}^d < 0$$

$$\tilde{\pi} = D_0 e^{-\tilde{m}} - n$$

$$\tilde{m} = -\lambda_1 \tilde{\pi} + \lambda_y \tilde{y} \quad (39)$$

Outside the CU steady state, real money balances need not be constant. The analogue of (7)' in this section becomes

$$\dot{m} = (\tilde{\pi} + n)(1 + \tilde{m} - m) - (\tilde{\pi} + n) = (\tilde{\pi} + n)(\tilde{m} - m)$$

$$\text{for } m_l(\tilde{y}) < m < m_u(\tilde{y})$$

and so real balance eventually converge to  $\tilde{m}$  where, typically  $\tilde{m} < \hat{m}$ . A continued rise in fiscal inflation will eventually push the system into a KU regime after real money balances fall below  $m_l(\tilde{y})$ .

The remedy in this underemployment equilibria is the standard Classical prescription - an enforced cut in the real product wage. An alternative solution (but more difficult than the model suggests) would be to engineer a fall in real money balances (via open market operations) so that the condition in the goods market switched from excess demand to supply; Keynesian fiscal demand management could then be used. Of course, if wage indexation was abandoned, the sticky real wage problem would disappear and convergence to the Walrasian equilibrium would return.

## 6 Conclusions

The effectiveness of fiscal policy, in an explicitly non-Keynesian setting, turns on its interaction with monetary policy and the subsequent gap that may

be created between current and core inflation. A surprising result of this paper is that the supply-side effectiveness can only exist in a model where goods markets always clear. The inflation gap can then be exploited to alter real wages and output; the vital mechanism is that wage setters suffer a politically-induced cost to the extent that their indexed wage contracts can not perfectly reflect longer term forces. If perfect indexation to long term forces were feasible, output would never deviate from steady state. Given the inefficiency, policy can be used to aid and complement optimal social wage contracts. The art of demand management in this paper is to manipulate the inflation gap to maximise the speed of adjustment of output back to steady state, without getting too close to the critical ceiling on core inflation.

Supply-side effectiveness is lost if goods and money markets do not clear instantaneously. In that case current inflation is no longer anchored by the need to maintain monetary equilibrium. With prices inflexible, but inflation free to take on any value, no inflation gap will emerge, wage setters will suffer no endogenous costs, and there will be no forces to push the system out of classical unemployment. Fiscal expansion will be merely inflationary.

Accepting the conjecture that the productivity slowdown in recent years was caused by supply shocks and inadequately responsive real wages, the result of output effectiveness of policy for flexible price models and of only inflation effectiveness for sticky price models may not mean that the policy recommendation is conditional on these two polar alternatives. Since goods markets can be broken down into flexible and sticky price sectors, a convex combination of the results of the two models may apply. It is an empirical question to decide upon the appropriate weights.<sup>11</sup>

An important caveat arises if we permit capital accumulation. The crowding out of investment by fiscal policy is a vital issue which will have an important bearing on the results in the current paper. In particular, even if fiscal expansion does speed up real wage and output adjustment, the possible crowding out of investment will reduce future productivity. The critical policy issue would be to determine an optimal balance between current and future output losses.

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11. Blundell-Wignall and Masson (1985) estimate a simultaneous equation model of the Federal Republic of Germany over the period 1973-1982. Their inflation equation assumes a breakdown between a flexible and a sticky price sector with the share of the latter estimated as 0.8 with a standard error of .05.

In the same way, opening the economy to trade will be an interesting extension. From the demand-determined model of output by Buiter and Miller (1981, 1982), we know that fiscal and monetary policy can have serious effects on output through real exchange rate effects. For the sticky real wage model, one may also expect fiscal expansion to cause over-appreciation of the real exchange rate. The model could be developed to allow the exchange rate depreciation gap into the Phillips curve. A positive gap can be created after fiscal expansion, replicating the paper's results. If an external debt crisis arose necessitating improvements in the current account, fiscal policy and the wage process would become complementary forces for correction.

Finally, this paper has been built on the strong assumption that inflation is not something to be feared; it is an intermediate target variable which, under particular circumstances, may be exploited to minimize the period of time that output is away from steady state. However there is no doubt that inflation enters the objective function of many democratic governments. One cost of inflation, amongst others, is that it is an index of the closeness of the fiscal deficit to its maximum sustainable level. In that case, the inflation-output trade off can be optimally determined by constructing social indifference curves of the speed of adjustment of output and core inflation, and equating the marginal rates of substitution and transformation.

APPENDIX

To establish the wage adjustment equation (2), I shall adapt the analysis described in Mussa (1981). A representative wage setter incurs a fixed cost,  $A$ , for each  $T$ -length contract that it makes. Over the life of the nominal wage contract, prices are rationally expected to change. There are two inflation concepts that are relevant - current inflation and the equilibrium inflation. In this paper the latter is interpreted as state inflation,  $\hat{\pi}$ . Assume that the nominal wage is perfectly indexed to current price movements - both discrete and continuous. After having build indexation into the nominal wage, the individual wage setter is assumed to suffer a quadratic cost over the life of the contract which depends upon the difference between the indexed nominal wage, deflated by the expected price if it were inflating at  $\hat{\pi}$ , and the full employment real wage,  $\hat{w}$ . Recognising that the frequency of adjustment is  $1/T$ , the problem of the  $i$ th agent is

$$\min_{W_i(\tau), T} \quad \frac{A}{T} + \int_0^T B[W_i(\tau) + \int_0^t \pi(\tau+s)ds - P(\tau) - \hat{\pi}t - \hat{w}]^2 dt$$

Approximating the indexation integral assuming near linearity in time, ( $\int_{t_1}^{t_2} f(s)ds \approx \frac{t_2-t_1}{2} (f(t_2) + f(t_1))$ ), the solutions for the optimal wage before inflation indexing and the optimal contract length are

$$W_i(\tau) = P(\tau) + \frac{T\hat{\pi}}{2} + \hat{w} - \frac{T}{2} \pi^*(\tau) \quad (A1)$$

$$T^4 = \frac{(4A)}{B} (\pi^*(\tau) - \hat{\pi})^{-2}$$

$$\text{where } \pi^*(\tau) = \frac{1}{2} (\pi(\tau) + \pi(\tau+T))$$

The optimal wage after indexing makes a mid-point correction for the change in core prices. The optimal contract length is seen to decrease with the inflation gap, since the cost of incorrect real wages are increased. As steady state is approached, the frequency of recontracting declines towards zero; in steady state, there is no need to renegotiate in finite time, indexation being sufficient. The endogenous contract length phenomenon is ignored in the dynamic analysis; basically, it stretches out the convergence process, by slowing adjustment speeds.

To aggregate, assume all wage setters to be identical, their indexed wages equispaced over the unit interval and ordered by the date of the most recent wage bargain. This gives the aggregate nominal wage after indexation as

$$W(t) = \frac{1}{T} \int_{t-T}^t [W_i(\tau) + \frac{(t-\tau)}{2} (\pi(t) + \pi(\tau))] d\tau \quad (A2)$$

Inserting (A1) in (A2), differentiating with respect to  $t$ , and making the approximation  $\dot{\pi}(t) \approx \frac{1}{t-\tau} (\pi(t) - \pi(\tau))$  gives

$$\dot{W}(t) = \frac{1}{T} [P(t) + \frac{T}{2} \hat{\pi}(t) - P(t-T) - \frac{T}{2} \hat{\pi}(t-T) - \frac{T}{2} (\pi^*(t) + \pi^*(t-T))] + \pi(t) \quad (A3)$$

Assuming near linearity in time of the integral in (A2), the aggregate nominal wage approximates as

$$W(t) \approx \frac{1}{2} [P(t) + \frac{T}{2} \hat{\pi}(t) + \hat{w} - \frac{T}{2} \pi^*(t) + P(t-T) + \frac{T}{2} \hat{\pi}(t-T) + \hat{w} + \frac{T}{2} \pi^*(t-T)] \quad (A4)$$

A(3) and A(4) together with the fact that  $w(t) = W(t) - p(t)$ , imply

$$\dot{w}(t) = \frac{2}{T} [\hat{w}(t) - w(t)] + \hat{\pi}(t) + \pi(t) - \pi^*(t) \quad (A5)$$

or, using (1), that

$$\dot{w}(t) = \frac{2}{T} \left( \frac{-\rho}{1-\rho} \right) (y(t) - y(t)) + (\hat{\pi}(t) - \pi^*(t)) \quad (A6)$$

Equation (2) in the text is derived from (A5) on the assumption that  $(\hat{\pi} - \pi^*)$  is always some fraction of  $(\hat{\pi} - \pi)$ . The contract length inflation gap will always be smaller than the steady state gap.

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