

# $r^*$ : Definition, Uses, Measurement, and Drivers

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“Causes, Challenges and Consequences of the Low Interest Rate Environment”

Disclaimer: The views expressed here do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System. Thanks to Katie Baker and Shlok Goyal for help in preparing this presentation.

## Why are (were?) interest rates low in the U.S. and globally?

- **Interest rates are low** because  $r^*$  is low—**for secular reasons**, not because of accommodative monetary policy
- $r^*$  **is low** in large part because of the **increasing premium for safety/liquidity** since the late 1990s —  $r$  for illiquid/less safe assets has declined much less

# Outline

- ①  $r^*$ : what is it, and what is it for?
- ②  $r^*$  as a benchmark for monetary policy
- ③ Measuring  $r^*$  for the US and **globally**—and determining its drivers
- ④ An  $r^*$  for financial stability:  $r^{**}$

## What is $r^*$ ?

- $r^*$  is the short term **real rate of return** in an **hypothetical economy without nominal (price and wage) rigidities**, but otherwise identical to the one we live in
  - The notion goes back to Wicksell (1898) and was fully integrated into New Keynesian DSGE models by Michael Woodford (2003). See the discussion in [“Why Are Interest Rates So Low?”](#), Liberty Street.
- In this economy monetary policy **cannot** affect real rates of return ...
- ... nor should it try to. The level of output ( $y^*$ ) is the optimal one – given the environment.

## What is $r^*$ for?

- 1) Provide **information on real rates of return – net of monetary policy**. (Positive)
  - What would the real rate be in a parallel universe where monetary policy had no effects?
  - If  $r$  is low because  $r^*$  is low, then accommodative monetary policy has little to do with this phenomenon.

## What is $r^*$ for?

2) Provide a **benchmark for setting the short term rate**. (Normative)

- Why? A simple New Keynesian economy can be described by an AD curve resulting from the interaction of monetary policy (the LM) and the IS relationship:

$$(y_t - y_t^*) = \sigma(r_t - r_t^*) + \mathbf{E}_t[(y_{t+1} - y_{t+1}^*)]$$

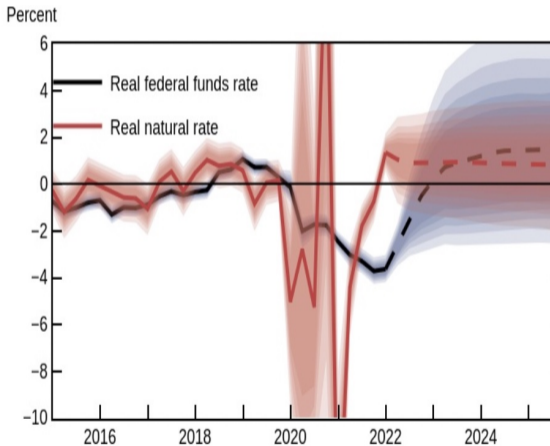
and by an AS curve

$$\pi_t = \kappa(y_t - y_t^*) + \beta \mathbf{E}_t[\pi_{t+1}]$$

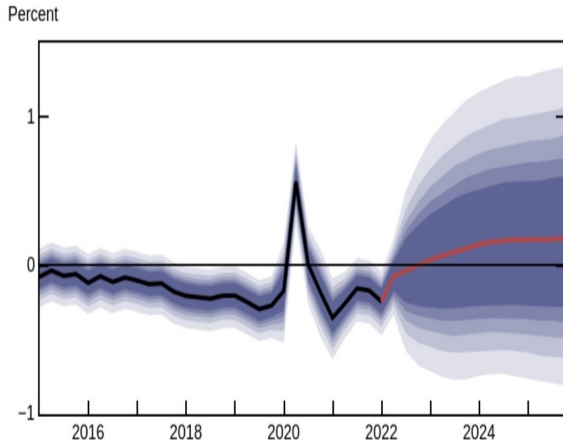
- In this simple economy setting  $r$  equal to  $r^*$  kills two birds with one stone (“divine coincidence”): 1) it closes the output gap ( $y = y^*$ ) and 2) stabilizes inflation ( $\pi = 0$ ). In more complex economies, where the divine coincidence breaks,  $r^*$  is no longer optimal — still, a helpful benchmark

## $r^*$ in practice

### Real Interest Rates



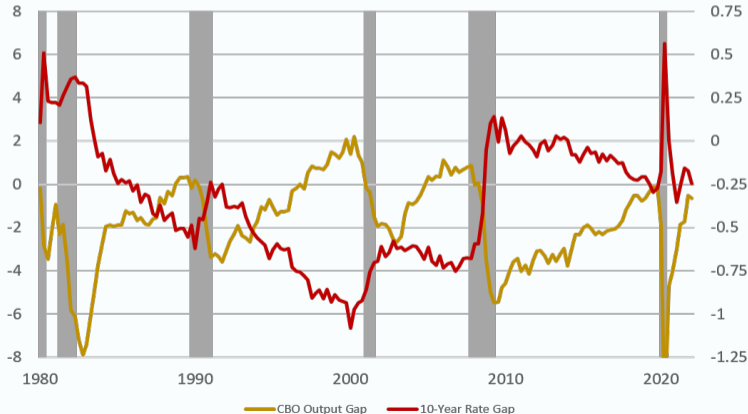
### Ten-Year Risk-Neutral Yield Gap



Projections from “The New York Fed DSGE Model Forecast—June 2022”, Liberty Street.

## The $r^*$ gap and business cycles in the US

- The gap between  $r$  and  $r^*$  is strongly countercyclical (negatively correlated with CBO output gap).
- This does not mean the monetary policy “caused” recessions and booms; only that it could have responded more forcefully – according to the model





# Measuring $r^*$

Two approaches:

① Use a **structural** model (DSGE):

- Advantage: well defined. Disadvantages: i) model-dependent/misspecification; ii) usually no low frequency

② Use a **reduced form** model:

- Define  $r^*$  as the **trend** in observed interest rates (under the maintained hypothesis that monetary policy does not affect the steady state)
- Advantage: flexible, captures low frequency. Disadvantages: short run  $r^*$  not defined
- Semi-structural models (Laubach-Williams): in between

## A reduced form approach to measuring $r^*$ in the U.S.

- “**Trendy VAR**”: Multi-variate unobserved component model:

$$y_t = \Lambda \bar{y}_t + \tilde{y}_t$$

where  $y_t$  are  $n \times 1$  observables,  $\bar{y}_t$  are the  $q \times 1$  trends ( $\Lambda$  is the matrix of loadings)

$$\bar{y}_t = \bar{y}_{t-1} + e_t$$

and the *stationary components*  $\tilde{y}_t$  follow an unrestricted VAR

$$\Phi(L)\tilde{y}_t = \varepsilon_t$$

- Del Negro et al., “Safety, Liquidity, and the Natural Rate of Interest”, Brookings, 2017

## Trends

## Observables (1960Q1-2022Q1)

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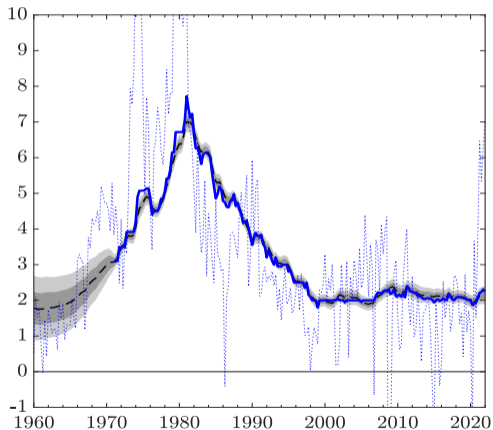
$\bar{\pi}_t$	Inflation	$\pi_t$		$+ \tilde{\pi}_t$
	Infl. Exp. (long run)	$\pi_t^e$	$= \bar{\pi}_t$	$\dots$
$\bar{r}_t$	T-bill rate	$R_{3M,t}$		$+ \tilde{R}_{3M,t}$
	T-bill Exp. (long run)	$R_t^e$	$= \bar{\pi}_t + \bar{r}_t$	$+ \tilde{R}_t^e$
	Long-run Treas.	$R_{20Y,t}$		$+ \bar{\pi}_t + \tilde{R}_{20Y,t}$

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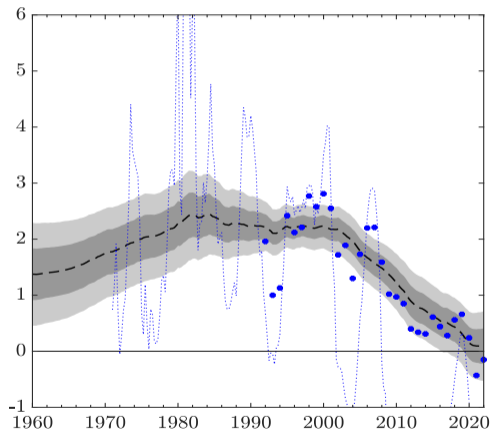
- $\bar{r}$ : trends in real rates (for safe/liquid assets such as Treasuries), which we construct from trends in the **level of the term structure**

# Measuring $r^*$

$\bar{\pi}_t$



$\bar{r}_t$



## Decomposing the trends in $r^*$

- Start from the pricing equation

$$1 = E_t [M_{t+1}(1 + r_t)(1 + CY_{t+1})]$$

⇒ Decompose  $\bar{r}_t = \bar{m}_t - \bar{cy}_t$

- $\bar{m}$ : trends in the (-log of the) stochastic discount factor, due to, say, trends in growth/**productivity** (eg,  $\bar{m} = \frac{1}{\sigma}\bar{g} + \bar{\beta}$ )
- $\bar{cy}$ : trends in the (log of the) **convenience yield** (Krishnamurthy & Vissing-Jorgensen, 2012, Greenwood, Hanson, Stein, 2015, Kyiotaki & Moore, 2012, ...)

# Trends

# Observables (1960Q1-2022Q1)

$\bar{\pi}_t$	Inflation	$\pi_t$		$+\tilde{\pi}_t$
	Infl. Exp. (long run)	$\pi_t^e$	$= \bar{\pi}_t$	$\dots$

Decompose  $\bar{r}_t = \bar{m}_t - \bar{c}y_t$

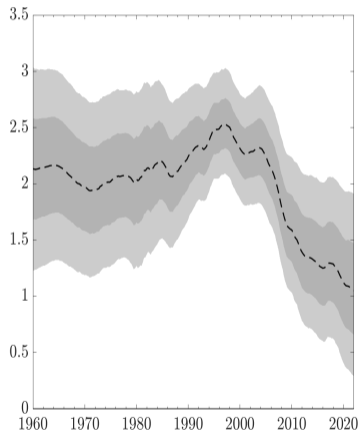
$\bar{r}_t$	T-bill rate	$R_{3M,t}$		
	T-bill Exp. (long run)	$R_t^e$	$= \bar{\pi}_t + \bar{m}_t - \bar{c}y_t$	
	Long-run Treas.	$R_{20Y,t}$		$+\bar{t}p_t$

$\bar{c}y_t$	Baa Yield	$R_t^{Baa}$	$= \bar{\pi}_t + \bar{m}_t$	$+\bar{t}p_t$
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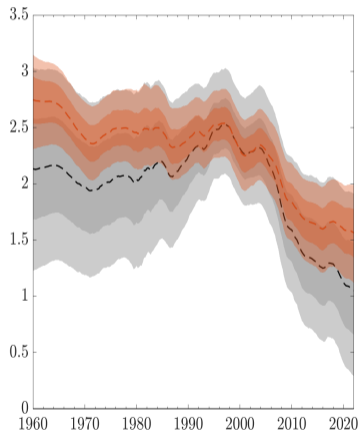
$$\Rightarrow \bar{R}_t^{Baa} - \bar{R}_{80,t} = \bar{c}y_t$$

# Drivers of $r^*$

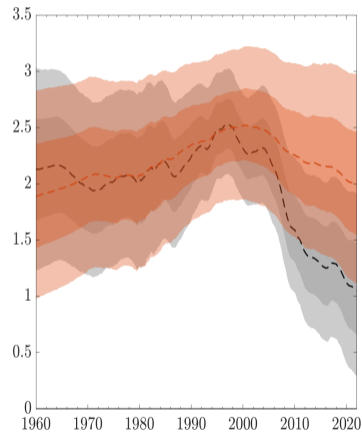
$\bar{r}_t$



$\bar{r}_t$  and  $-\overline{cy}_t$

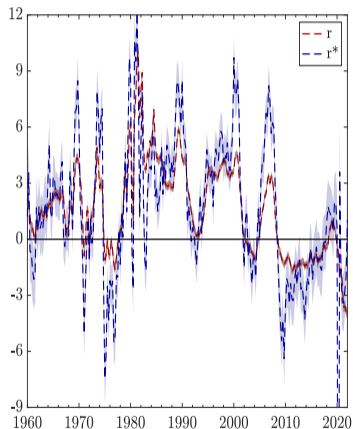


$\bar{r}_t$  and  $\bar{m}_t$

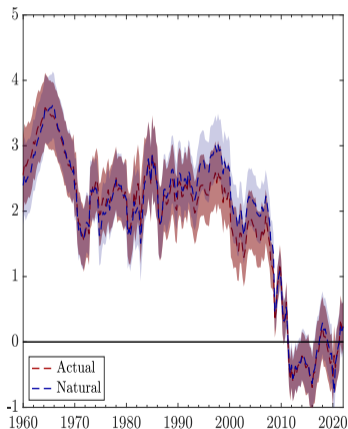


# A structural (DSGE) approach to measuring $r^*$ in the U.S.

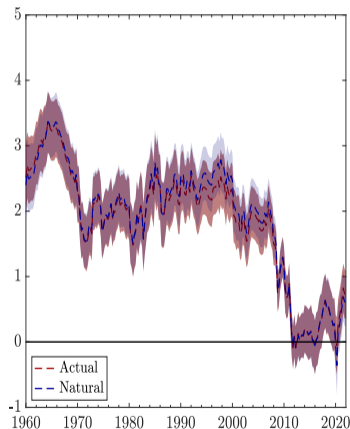
*short run*  $r$  and  $r^*$



$E_t[r_{t+5Y}]$  and  $E_t[r_{t+5Y}^*]$



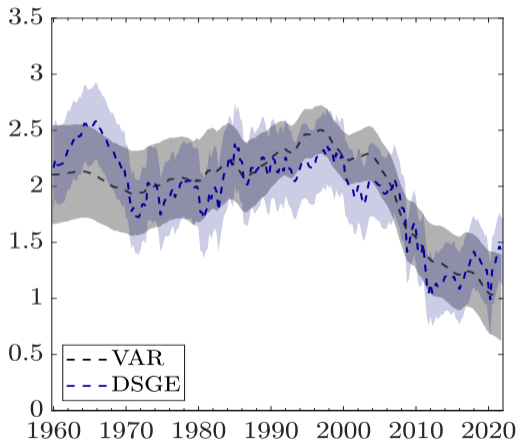
$E_t[r_{t+10Y}]$  and  $E_t[r_{t+10Y}^*]$



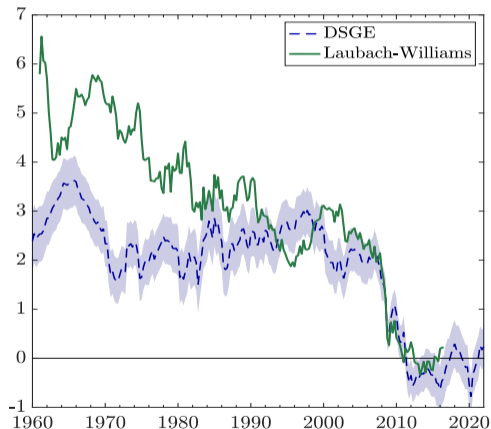


# Structural vs reduced form $r^*$

TVAR  $\bar{r}_t$  vs  $E_t[r_{t+30Y}^*]$

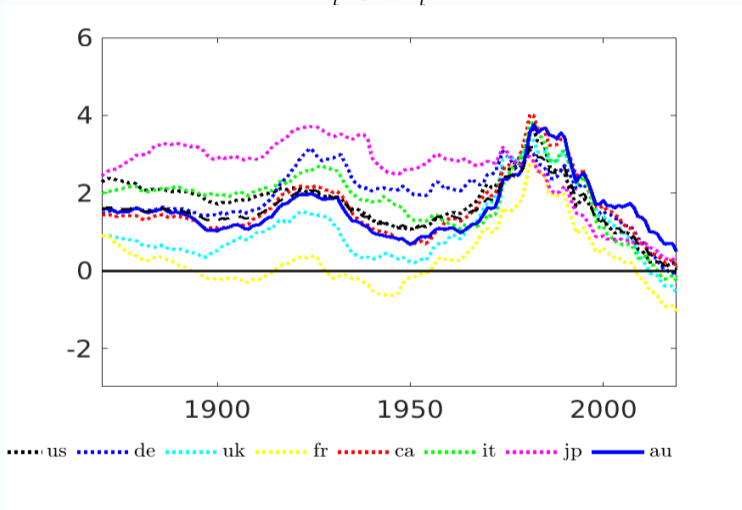


LW  $r^*$  vs  $E_t[r_{t+5Y}^*]$



# Global convergence in $r^*$

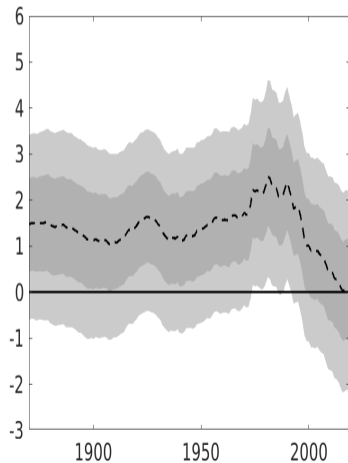
$$\bar{r}_t^W \text{ and } \bar{r}_t^i$$



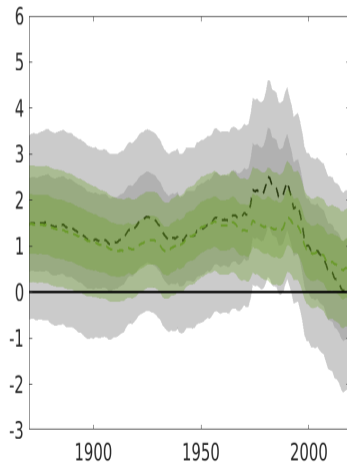
“Global trends in interest rates”, JIE (2019)

## Drivers of global $r^*$

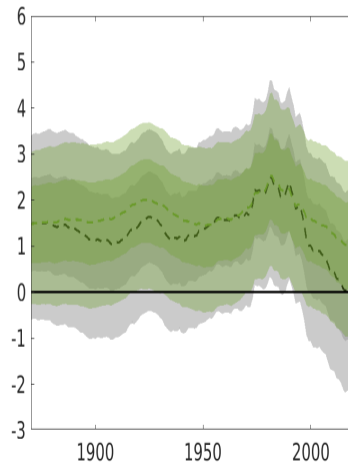
$\bar{r}_t$



$\bar{r}_t$  and  $-\overline{cy}_t$

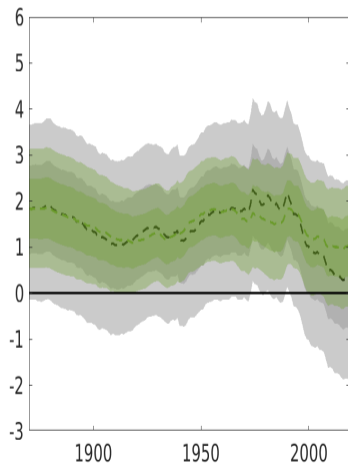


$\bar{r}_t$  and  $\bar{m}_t$

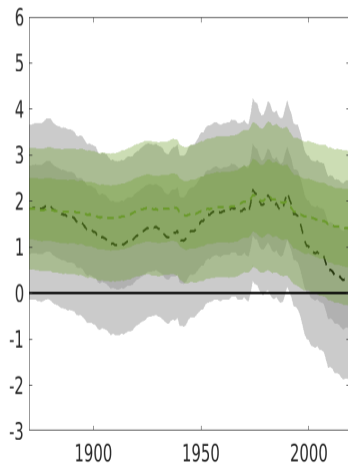


# Drivers of **global** $r^*$ : Model with consumption growth

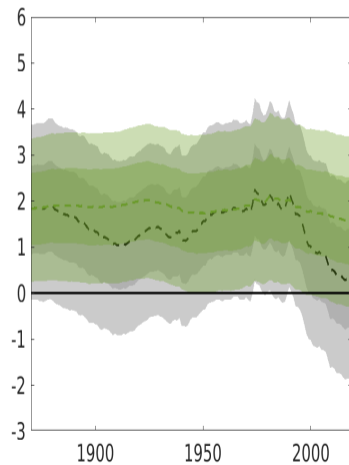
$\bar{r}_t$  and  $-\bar{c}y_t$



$\bar{r}_t$  and  $\bar{g}_t$



$\bar{r}_t$  and  $\bar{\beta}_t$



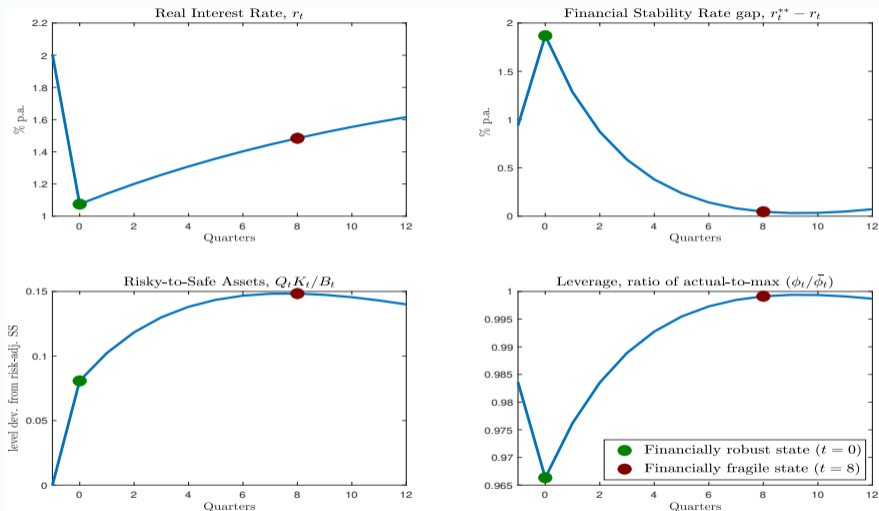
## Low $r^*$ and financial stability

- Based on work with Akinci, Benigno, and Queralto, “The Financial (In)Stability Real Interest Rate,  $R^{**}$ ”
  - ① Define  $r^{**}$ : Map the notion of financial stability onto the interest rate space, and complement  $r^*$  as a guide to policy
  - ② Show that persistently low  $r \rightarrow$  increased financial vulnerability  $\rightarrow$  low  $r^{**} \rightarrow$  reduce the space for monetary policy

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- Simple dynamic macrofinance model with an **occasionally binding** financing **constraint** á la Gertler & Kiyotaki
  - *Tranquil times*
  - *Financial instability/stress*
- The real rate (financial conditions) is one of the state variables determining in which regime we are in  $\rightarrow$   $r^{**}$ : **threshold real rate above which financial instability arises**, ie, the real interest rate that makes the financial constraint *just* bind
- Use  $r^{**}$  as a **summary statistics for financial stability** concerns, just like  $r^*$  is for macroeconomic conditions

# Dynamics of $r^{**}$ : Impulse responses to low interest rates



- Persistently low rates today cause vulnerabilities to build up  $\rightarrow$  low  $r^{**}$   $\rightarrow$  reduced space for monetary policy to raise rates

# Conclusions

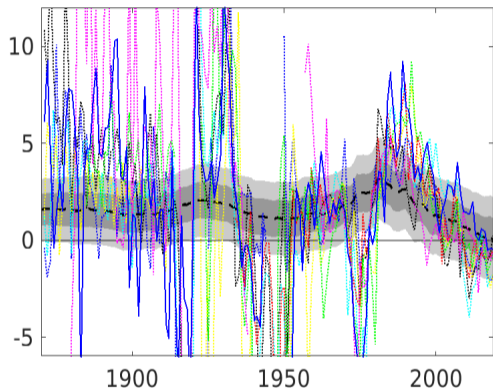
## Why have interest rates been low?

- Interest rates are low because  $r^*$  is low, **globally**
- Since the late 1990s,  $r^*$  begun a *secular* decline, in large part driven by the increase in the **convenience yield** for safe/liquid assets such as Treasuries
- Persistently low  $r^*$  may trigger **financial instability** → low  $r^{**}$  (“financial dominance”)

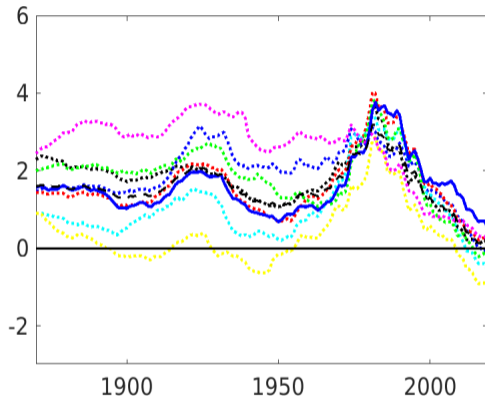


# Appendix

# Global convergence in $r^*$



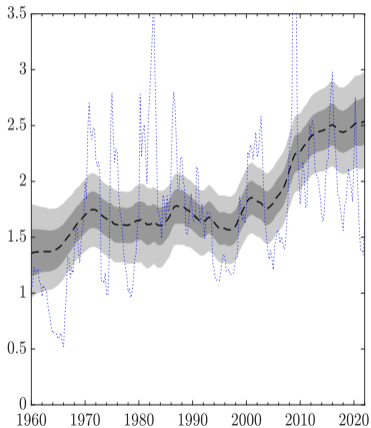
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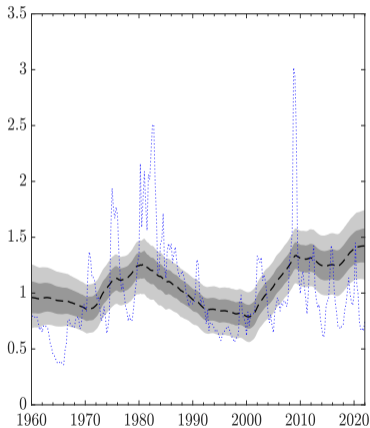
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# Trends in compensation for safety and liquidity, and observables

$\overline{cy}_t$ , and  $R_t^{Baa} - R_{80,t}$



$\overline{cy}_t^s$ , and  $R_t^{Baa} - R_t^{Aaa}$



$\overline{cy}_t^l$ , and  $R_t^{Aaa} - R_{80,t}$

